Exploring General Gauge Mediation

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Buican, Meade, Seiberg, DS (to appear)

The LHC is coming...

- What will we discover?
- Low-energy supersymmetry is still the most theoretically well-motivated possibility.
  - Hierarchy problem
  - Gauge coupling unification
  - Dark matter (?)

The MSSM

- SUSY in its minimal incarnation is known as the MSSM.

Names		spin 0	spin 1/2	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
squarks, quarks	Q	$(\widetilde{u}_L\ \ \widetilde{d}_L)$	$(u_L\ \ d_L)$	$(\,\mathbf{3},\ \mathbf{2}\,,\ \frac{1}{6})$
(×3 families)	$\overline{u}$	$\widetilde{u}^*_R$	$u^\dagger_R$	$(\,\overline{\mathbf{3}},\ \mathbf{1},\ -\frac{2}{3})$
	$\overline{d}$	$\widetilde{d}^*_R$	$d^\dagger_R$	$(\,\overline{\mathbf{3}},\ \mathbf{1},\ \frac{1}{3})$
sleptons, leptons	L	$(\widetilde{\nu}\ \ \widetilde{e}_L)$	$(\nu\ \ e_L)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ -\frac{1}{2})$
(×3 families)	$\overline{e}$	$\widetilde{e}^*_R$	$e^\dagger_R$	$(\,\mathbf{1},\ \mathbf{1},\ 1)$
Higgs, higgsinos	H_u	$(H_u^+\ \ H_u^0)$	$(\widetilde{H}_u^+\ \ \widetilde{H}_u^0)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ +\frac{1}{2})$
	H_d	$(H_d^0\ \ H_d^-)$	$(\widetilde{H}_d^0\ \ \widetilde{H}_d^-)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ -\frac{1}{2})$

Table 1.1: Chiral supermultiplets in the Minimal Supersymmetric Standard Model. The spin-0 fields are complex scalars, and the spin-1/2 fields are left-handed two-component Weyl fermions.

The MSSM

- SUSY in its minimal incarnation is known as the MSSM.

Names	spin 1/2	spin 1	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
gluino, gluon	$\widetilde{g}$	g	$(\ \mathbf{8},\ \mathbf{1}\ ,\ 0)$
winos, W bosons	$\widetilde{W}^{\pm}\quad \widetilde{W}^{0}$	$W^{\pm}\quad W^{0}$	$(\ \mathbf{1},\ \mathbf{3}\ ,\ 0)$
bino, B boson	$\widetilde{B}^{0}$	B^{0}	$(\ \mathbf{1},\ \mathbf{1}\ ,\ 0)$

Table 1.2: Gauge supermultiplets in the Minimal Supersymmetric Standard Model.

The MSSM

- Despite its theoretical successes, the MSSM also has many puzzling aspects.
  - SUSY flavor problem
  - SUSY CP problem
  - Little hierarchy problem
  - mu problem
- All of these problems are related in some way to SUSY breaking.

~~SUSY~~ in the MSSM

$$\mathcal{L}_{soft} = -\frac{1}{2}\sum_{i=1}^{3} M_i \lambda_i \lambda_i - \sum_{\tilde{f}=\tilde{Q},\tilde{u},\tilde{d},\tilde{L},\tilde{e}} \tilde{f}^\dagger m^2_{\tilde{f}} \tilde{f} + (Higgs)$$

Complex

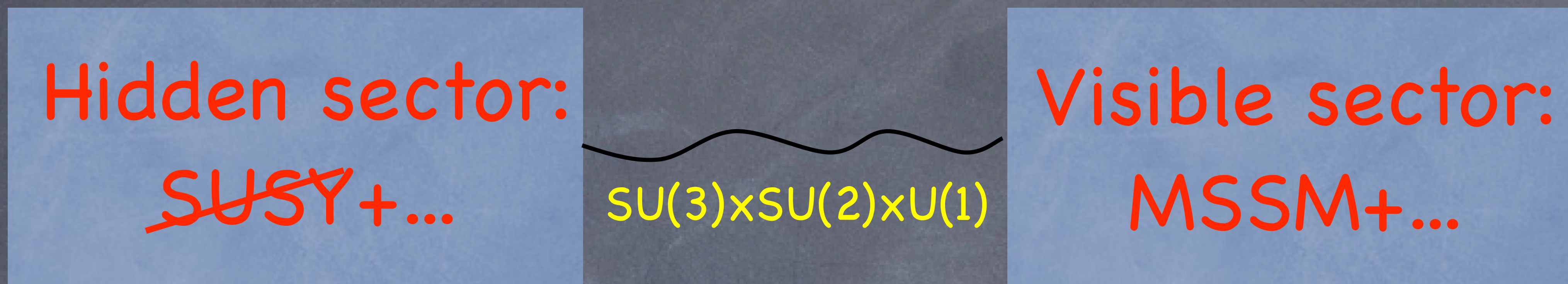
3x3 Hermitian matrices

- "Soft ~~SUSY~~" Lagrangian guarantees that quadratic divergences are not re-introduced
- Over 100 new parameters in addition to SM!
  - Can lead to major flavor and CP violation

SUSY flavor and CP

- The soft Lagrangian must originate from a theory of spontaneous SUSY breaking.
- This theory should have far fewer parameters than the full soft Lagrangian.
- In particular, this theory should avoid the SUSY flavor and CP problems.

Gauge Mediation



- Gauge mediation is a promising framework for communicating SUSY-breaking to the SSM.
- Its advantages include:
  - Automatic flavor universality (no FCNCs)
  - Viable spectrum
  - Calculability
  - Distinctive phenomenology

Messenger Paradigm

- Introduce weakly-coupled "messengers" that couple directly to the SUSY-breaking sector and are charged under the SM gauge interactions.
- Popular ansatz for model building -- can decouple details of SUSY breaking sector
- (But not the most general construction!)

Minimal Gauge Mediation

(Dine, Nelson, Nir, Shirman, ...)

$$W = \lambda X \phi \tilde{\phi}, \quad \langle X \rangle = M + \theta^2 F$$

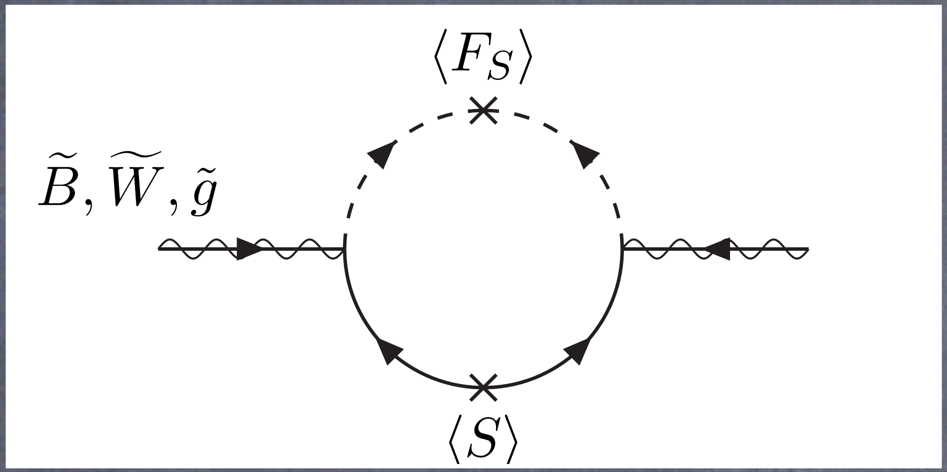
- X: spurion for hidden sector SUSY breaking and R-symmetry breaking.
- ϕ, $\tilde{\phi}$: messengers in irreps of G_{SM} . They receive tree-level SUSY breaking mass splittings through their direct coupling to X.

$$M_\phi = M, \quad M_\phi^2 = \begin{pmatrix} |M|^2 & F \\ F^* & |M|^2 \end{pmatrix} \to |M|^2 \pm F$$

"F-type" messenger masses

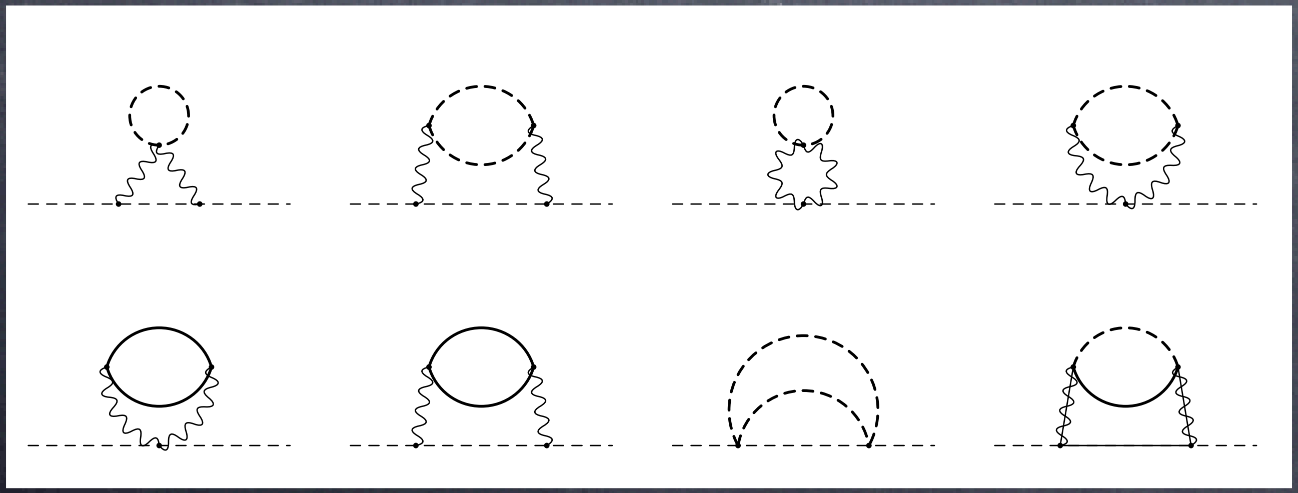
MGM Soft Masses

- 1-loop gaugino masses:



$$M_{r=1,2,3} \sim \frac{\alpha_r}{4\pi}\frac{F}{M}$$

- 2-loop sfermion mass-squareds:



$$m_{\tilde{q}}^2 \sim \left(\frac{\alpha_r}{4\pi}\right)^2 \left(\frac{F}{M}\right)^2$$

MGM Phenomenology

- MGM soft masses are controlled by only a few parameters.
- This leads to many specific and well-known "predictions" of gauge mediation:
  - Gaugino unification
  - Sfermion mass hierarchy
  - Bino or slepton NLSP
  - ....

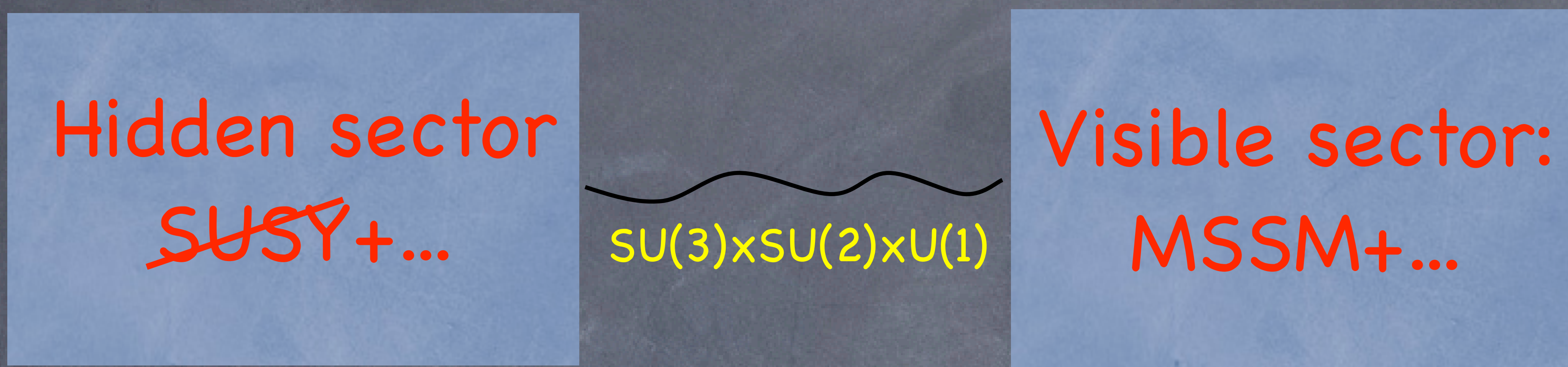
Motivation for GGM

- What are the most general predictions/parameters of gauge mediation?
- Especially important question in the LHC era.
- To date many models of gauge mediation have been constructed.
- However, it has not been clear up to now which features of these models are general and which are specific.

Plan of the Talk

- Introduction and Motivation
- General Gauge Mediation
  - Currents and Correlators
  - Soft masses
  - Sum Rules
- Constraints on GGM
- Covering the Parameter Space

General Gauge Mediation



- Theory decouples into separate hidden and visible sectors in g->0 limit.
- (Messengers, if present, are part of the hidden sector.)
- Hidden sector:
  - spontaneously breaks SUSY at a scale M
  - has a weakly-gauged global symmetry

$$G \supset G_{SM}$$

General Gauge Mediation

- All the information we need about the hidden sector is encoded in the currents of G and their correlation functions.
- Philosophy: work exactly in the hidden sector but to leading order in g.
- Start by analyzing the hidden sector at g=0. Assume for simplicity G=U(1).

Current Supermultiplet

- Current sits in a real linear supermultiplet defined by:

$$\mathcal{J} = \mathcal{J}(x, \theta, \bar{\theta}), \qquad D^2 \mathcal{J} = \bar{D}^2 \mathcal{J} = 0$$

SUSY generalization of current conservation

- In components:

$$\mathcal{J} \;=\; J + i\theta j - i\bar{\theta}\bar{j} - \theta\sigma^\mu\bar{\theta} j_\mu + \frac{1}{2}\theta\theta\bar{\theta}\bar{\sigma}^\mu\partial_\mu j - \frac{1}{2}\bar{\theta}\bar{\theta}\theta\sigma^\mu\partial_\mu\bar{j} - \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box J$$

ordinary U(1) current, satisfies

$$\partial_\mu j^\mu = 0$$

Current correlators

$$\mathcal{J} = J + i\theta j - i\bar{\theta}\bar{j} - \theta\sigma^{\mu}\bar{\theta}j_{\mu} + \ldots$$

- Nonzero two-point functions constrained by Lorentz invariance, current conservation:

Dim'less

Real

$$C_0(p^2/M^2) = \langle J(p)J(-p)\rangle$$

$$C_{1/2}(p^2/M^2) = \frac{1}{p^2}p^{\mu}\sigma_{\mu}^{\alpha\dot{\alpha}}\langle j_{\alpha}(p)\bar{j}_{\dot{\alpha}}(-p)\rangle$$

$$C_1(p^2/M^2) = \frac{1}{p^2}\langle j^{\mu}(p)j_{\mu}(-p)\rangle$$

Complex

$$B(p^2/M^2) = M^{-1}\langle j_{\alpha}(p)j_{\beta}(-p)\rangle$$

- (M = scale of ~~SUSY~~ in hidden sector)

SUSY limit

- If SUSY is unbroken, can show:

$$C_0 = C_{1/2} = C_1, \qquad B = 0$$

- More generally, SUSY must be restored in the UV

$$\lim_{x \to 0} C_0(x),\ C_{1/2}(x),\ C_1(x) = c\ ; \quad \lim_{x \to 0} B(x) = 0$$

Can show: determines the hidden sector contribution to the beta function.

Coupling to visible sector

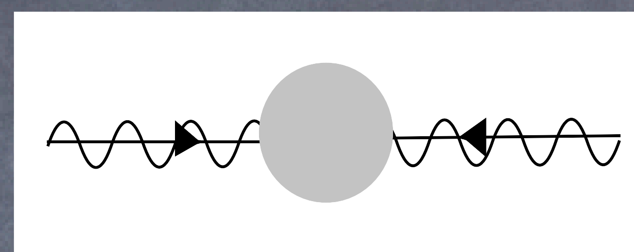
- Now weakly gauge G=U(1)

$$\mathcal{L}_{int} = 2g \int d^4\theta \mathcal{J}\mathcal{V} + \cdots = g(JD - \lambda j - \bar{\lambda}\bar{j} - j^\mu V_\mu) + \ldots$$

- Integrate out hidden sector exactly; work to leading order in gauge coupling.
- Soft masses can be related to the current-current correlators.

Soft Masses

$$\mathcal{L}_{int} = 2g\int d^4\theta \mathcal{J}\mathcal{V} + \dots = g(JD - \lambda j - \bar{\lambda}\bar{j} - j^\mu V_\mu) + \dots$$

- Gaugino: j_α  j_α

$$M_\lambda = g^2 M B(p=0)$$

- Scalars:

$$m_{\tilde{f}}^2 = g^4 A$$

Why does this integral converge?
Not obvious...

$$A \equiv -\int \frac{dp^2}{(2\pi)^4}\Big(3C_1(p^2/M^2) - 4C_{1/2}(p^2/M^2) + C_0(p^2/M^2)\Big)$$

Rewriting the soft masses

- Analogy with chiral superfield:

$$\bar{D}\Phi = 0 \quad \Leftrightarrow \quad Q\phi = 0$$

- An equivalent formulation of the current s'multiplet is to start with the defining relation:

$$Q^2 J = \bar{Q}^2 J = 0$$

- It follows that

$$j_\alpha \equiv Q_\alpha J$$
$$\bar{j}_{\dot{\alpha}} \equiv \bar{Q}_{\dot{\alpha}} J$$
$$\sigma^\mu_{\alpha\dot{\alpha}} j_\mu \equiv [Q_\alpha, \bar{Q}_{\dot{\alpha}}] J$$

Rewriting the soft masses

Using action of supercharges, can show:

$$\begin{aligned}\langle Q^2 J(p) J(-p)\rangle &= \langle Q^\alpha J(p) Q_\alpha J(-p)\rangle \\ &= \langle j^\alpha(p) j_\alpha(-p)\rangle \\ &= MB(p)\end{aligned}$$

Similar manipulations lead to

$$\langle Q^2 \bar{Q}^2 J(p) J(-p)\rangle = p^2\Big(3C_1(p^2/M^2) - 4C_{1/2}(p^2/M^2) + C_0(p^2/M^2)\Big)$$

Rewriting the soft masses

- Thus:

$$M_\lambda = g^2 \langle Q^2 J(0) J(0) \rangle$$
$$m_{\tilde{f}}^2 = g^4 \int \frac{dp^2}{p^2} \langle Q^2 \bar{Q}^2 J(p) J(-p) \rangle$$

- Comments on the result:
  - Check: vanish when SUSY is unbroken.
  - Generalization of small F-term SUSY-breaking relations (Distler & Robbins; Intriligator & Sudano)

$$M_\lambda \sim F, \qquad m_{\tilde{f}}^2 \sim |F|^2$$

Rewriting the soft masses

- Thus:

$$M_\lambda = g^2 \langle Q^2 J(0) J(0) \rangle$$

$$m_{\tilde{f}}^2 = g^4 \int \frac{dp^2}{p^2} \langle Q^2 \bar{Q}^2 J(p) J(-p) \rangle$$

- Comments on the result:
  - At high momentum, only the OPE of J with itself matters! Can use this to prove convergence of the scalar mass integral.

An aside on the sign of A

$$m_{\tilde{f}}^2 = g^4 A$$

$$A \equiv -\int \frac{dp^2}{(2\pi)^4}\Big(3C_1(p^2/M^2) - 4C_{1/2}(p^2/M^2) + C_0(p^2/M^2)\Big)$$

- Notice that A is a linear combination of two-point functions with different signs -- it is not obviously positive

- Indeed, simple models with A<0 already exist in the literature...

  Poppitz & Trivedi; Nakayama, Taki, Watari, Yanagida

Messengers with D-terms

Poppitz & Trivedi; Nakayama, Taki, Watari, Yanagida

- Messengers $\phi,\ \tilde{\phi}$ with charge +1, -1 under a U(1)′.
- If the U(1)′ breaks SUSY via an FI term,

$$V \supset V_D = (D/2 + |\phi|^2 - |\tilde{\phi}|^2)^2$$

the messengers receive "D-type" SUSY-splittings

$$M_F = m, \qquad M_B^2 = \begin{pmatrix} m^2 + D & 0 \\ 0 & m^2 - D \end{pmatrix}$$

- Then explicit calculation shows that:

$$A = -D^4/M^6 + \cdots < 0$$

An aside on the sign of A

- Important consequence of the indefiniteness of the sign of A: one cannot be sure that a given gauge mediation model is consistent unless the sfermion masses are calculable.
- In particular, many incalculable, strongly-coupled "direct gauge mediation" models built in the past are now of questionable validity.

Sum Rules

$$m^2_{\tilde{f}} = \sum_{r=1}^{3} g_r^4 \, c_2(f;r) A_r$$

Quadratic Casmir

- Trivial to generalize from U(1) to SU(3)xSU(2)xU(1)
- Five MSSM sfermion masses f=Q,U,D,L,E are given in terms of 3 parameters $A_{r=1,2,3}$
- So there must be 2 relations...

$$\mathrm{Tr}\, Y m^2 = \mathrm{Tr}\,(B-L) m^2 = 0$$

- Corrections: sum rules true at the scale M. (Small) corrections from RG and EWSB.

Parameter space

- The GGM parameter space consists of 9 real parameters:

$$A_{1,2,3}, \qquad |B_{1,2,3}|, \qquad \arg(B_{1,2,3})$$

- Note: GGM in general has a SUSY CP problem!
- Contrast with MGM parameter space -- many more parameters in general

Parameter space

- Question: are there simple models of weakly coupled messengers that cover the entire parameter space?
- We are looking for an ``existence proof”

Phenomenological Constraints on GGM

Messenger Parity

- We have related the soft masses to the current two-point functions. However, we ignored the possible contribution of the one-point function (FI parameter):

$$\langle J \rangle = \zeta \neq 0$$

- This can be nonzero for $U(1)_Y$ without breaking gauge symmetry.

Messenger Parity

- It is dangerous because it contributes to the scalar masses:

$$\delta m_{\tilde{f}}^2 = g_1^2 Y_f \zeta$$

- Not positive definite and $\mathcal{O}(g^2)$ (vs. $\mathcal{O}(g^4)$ for usual GM contributions).
- So if zeta is too large this can cause some scalars (esp. sleptons) to become tachyonic!

Messenger Parity

- Thus we would like the hidden sector to be invariant under a symmetry that forbids J one-point functions.
- The simplest such symmetry is a parity:

$$\mathcal{J} \to -\mathcal{J}$$

- Examples of this symmetry in the context of minimal gauge mediation have been discussed in the literature. (Dine & Fischler; Dimopoulos & Giudice)

Messenger Parity

- E.g. in models with weakly-coupled messengers,

$$J = \phi_i^\dagger \phi_i - \tilde{\phi}_i^\dagger \tilde{\phi}_i$$

- So can always choose a basis in which messenger parity is explicitly realized as:

$$\phi_i \leftrightarrow \tilde{\phi}_i$$

- Couplings of the hidden sector must be invariant under this transformation.

CP phases

- The B's are complex and independent in GGM. However, B's with arbitrary phases would typically lead to an unacceptable level of CP violation.
- So either the hidden sector is CP invariant, or its CP violation is somehow shielded from the visible sector.
- We will assume some mechanism at work, and take the gaugino masses to be real.

Unification

- We would like the hidden sector to be compatible with 3-2-1 gauge coupling unification.
- The beta functions come from the real correlators C. In general they have nothing to do with the complex correlator B.
- So gaugino unification is not tied to gauge coupling unification.

Covering the parameter space of GGM

Parameter space

- Question: are there simple models of weakly coupled messengers that cover the entire parameter space and satisfy the phenomenological constraints?
    - Messenger parity
    - CP invariance
    - Gauge couple unification
- We are looking for an ``existence proof"

Parameter space

- Carpenter, Dine, Festuccia & Mason studied this question recently in the context of messenger models with small F-type SUSY breaking.
- They found models with the right number of parameters (6) but which did not cover the entire parameter space.

Setup

- We also consider models with messengers with tree-level SUSY splittings, but allow for the possibility of D-type splittings.
- Such splittings could come from e.g. a U(1)′, or from non-Abelian hidden sector dynamics such as in the model of Seiberg, Volansky & Wecht.

Warmup: G=U(1)

- As a warmup, let us consider the parameter space covering problem for a U(1) gauge group.

- Here there is only one A and one B parameter; so we would like a theory that covers the range

$$\frac{A}{|B|^2} \in (0, \infty)$$

Warmup: G=U(1)

- Case 1: One messenger.

$$M_F = M, \qquad M_B^2 = \left(\begin{array}{cc} M^2 & F \\ F & M^2 \end{array} \right) \left(\begin{array}{c} \phi^* \\ \tilde{\phi} \end{array} \right)$$

$$\left(\begin{array}{cc} \phi & \tilde{\phi}^* \end{array} \right)$$

- Messenger parity => MGM

- Here there are two parameters (M,F), but can show that they do not cover the entire parameter space:

$$\frac{A}{|B|^2} \in (0.37, 1)$$

Warmup: G=U(1)

- Case 2: Two messengers.

$$M_F = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}$$

$$M_B^2 = \begin{pmatrix} M_1^2+\xi & 0 & F_1 & 0 \\ 0 & M_2^2-\xi & 0 & F_2 \\ F_1 & 0 & M_1^2+\xi & 0 \\ 0 & F_2 & 0 & M_2^2-\xi \end{pmatrix} \begin{pmatrix} \phi_1 \\ \tilde{\phi}_1^* \\ \phi_2 \\ \tilde{\phi}_2^* \end{pmatrix}$$

Messenger parity => allows for D-type splitting

$$B \sim \frac{F_1}{M_1} + \frac{F_2}{M_2}$$

$$A \sim \left(\frac{F_1}{M_1}\right)^2 + \left(\frac{F_2}{M_2}\right)^2 + \xi \log \frac{M_1}{M_2}$$

With nonzero xi, can now cover the entire parameter space!

General Result

- Consider a collection of vectorlike messengers all transforming in the same irrep $(R, \tilde{R})$ of 3-2-1. Then they contribute to the soft masses

$$\delta A_r = a_{R,r} A(R), \qquad \delta B_r = b_{R,r} B(R)$$

  - $a_{R,r}$, $b_{R,r}$: group theory factors
  - $A(R)$, $B(R)$: functions of messenger masses and couplings
- So on general grounds, need at least three different 3-2-1 irreps.

Finding the Model

D L

$$5 \rightarrow (\bar{3}, 1, 1/3) \oplus (1, 2, -1/2)$$

$$10 \rightarrow (3, 2, 1/6) \oplus (\bar{3}, 1, -2/3) \oplus (1, 1, 1)$$

Q U E

- Case 1: any number of $(5, \bar{5})$ (not necessarily OGM) -- only two irreps (D,L) => can cover at most a 4d subspace
- Case 2: single $(10, \bar{10})$ -- right # of irreps, but messenger parity allows only MGM => cannot cover entire space (cf. CDFM).

Finding the Model

- Case 3: single $(10,\bar{1}0)$+$(5,\bar{5})$ -- same as case 2
- Case 4: that leaves

$$(10,\bar{1}0) + 2(5,\bar{5}) \quad \text{and} \quad 2(10,\bar{1}0)$$

as the minimal possibilities. By including D-type SUSY breaking as in the U(1) example, one can cover the entire parameter space of GGM.

So the entire parameter space of GGM is physical, and its phenomenology should be studied!

Summary

- We constructed a framework for analyzing general models of gauge mediation: arbitrary hidden sectors coupled to the MSSM via SM gauge interactions.
- Using our framework, we derived general properties of gauge mediation. These include:
  - Parameter space: 3 complex parameters (gaugino masses) and 3 real parameters (sfermion masses)
  - Two sum rules for sfermion masses
  - SUSY CP problem in general

Summary

- We presented weakly-coupled messenger models which satisfy all phenomenological constraints and cover the entire GGM parameter space.
- Our framework is well-suited for analyzing strongly-coupled hidden sectors.
  (cf. Ooguri, Ookouchi, Park & Song)

Outlook

- Detailed study of entire GGM parameter space at colliders (cf. recent work of L. Carpenter)
- The formulas for the soft masses in terms of Q^2 and Q^4 are quite pretty. What else can be done with them?
- Is there a theorem for positivity of the sfermion masses for pure F-term breaking?
- mu/Bmu still an important open problem...