Euclidean 3-algebra dynamics

Costis Papageorgakis
Tata Institute of Fundamental Research

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Based on:

- "M2 to D2",
  Sunil Mukhi and CP,
  arXiv:0803.3218 [hep-th], JHEP 0805:085 (2008)
- "M2-branes on M-folds",
  Jacques Distler, Sunil Mukhi, CP and Mark van Raamsdonk
  arXiv:0804.1256 [hep-th], JHEP 0805:038 (2008)
- "D2 to D2",
  Bobby Ezhuthachan, Sunil Mukhi and CP
  arXiv:0806.1639 [hep-th], JHEP 0807:041 (2008)
- Bobby Ezhuthachan, Sunil Mukhi and CP
  in progress

In the Beginning...

Recently, Bagger-Lambert and Gustavsson proposed a candidate theory for describing multiple M2-branes.

Achieved via introduction of CS gauge fields for closure of $\mathcal{N}=8$ supersymmetry algebra and 3-algebras [Filipov]

$$[T^A, T^B, T^C] = f^{ABC}{}_D T^D \qquad \text{with} \qquad h^{AB} = \mathrm{Tr}(T^A T^B)$$

satisfying

$$f^{ABCD} = f^{[ABCD]} \qquad \text{and} \qquad f^{[ABC}{}_G f^{E]FG}{}_D = 0 \quad \text{(F.I.)}$$

With this write a 3d action

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}D_\mu X^{A(I)} D^\mu X_A^{(I)} + \frac{i}{2}\overline{\Psi}^A \Gamma^\mu D_\mu \Psi_A + \frac{i}{4} f_{ABCD}\overline{\Psi}^B \Gamma^{IJ} X^{C(I)} X^{D(J)} \Psi^A \\ &- \frac{1}{12}\left(f_{ABCD}X^{A(I)}X^{B(J)}X^{C(K)}\right)\left(f_{EFG}{}^D X^{E(I)}X^{F(J)}X^{G(K)}\right) \\ &+ \frac{1}{2}\epsilon^{\mu\nu\lambda}\left(f_{ABCD}A_\mu{}^{AB}\partial_\nu A_\lambda{}^{CD} + \frac{2}{3}f_{AEF}{}^G f_{BCDG} A_\mu{}^{AB} A_\nu{}^{CD} A_\lambda{}^{EF}\right)\end{aligned}$$

This has the following exciting features

- Manifest $\mathrm{SO}(8)$ R-symmetry
- Maximally $(\mathcal{N}=8)$ superconformal

However

- $\exists$ only one example with positive definite metric: $\mathcal{A}_4$-theory with

$$f^{ABCD} = f\epsilon^{ABCD} \in \mathrm{SO}(4)$$

  [Papadopoulos, Gauntlett-Gutowski]

- $\exists$ a discrete free parameter coming from the quantised CS level

$$f = \frac{2\pi}{k}$$

  Intriguing but can be used as expansion parameter

- $\mathcal{A}_4$ can be re-cast as $\mathrm{SU}(2) \times \mathrm{SU}(2)$ CS-matter theory [van Raamsdonk]
- M-theory interpretation? Role of 3-algebra?

Moduli space analysis suggests 2 M2s on generalised orbifold ('M-fold'). [Lambert-Tong, Distler-CP-Mukhi-van Raamsdonk]

- Moduli space topology: $\frac{\mathbb{R}^8\times\mathbb{R}^8}{D_{2k}} \simeq \frac{\mathbb{R}^8\times\mathbb{R}^8}{\mathbb{Z}_2\ltimes\mathbb{Z}_{2k}}$
- Preserves $\mathcal{N}=8$ for any value of k
- Not conventional M-theory orbifold $\frac{(\mathbb{R}^8/\mathbb{Z}_k)^2}{S_2}$ - but agrees for $k=2$
- Precise spacetime interpretation remains unknown. Possible mysterious enhancement from $\mathcal{N}=6$?

ABJM generalised the above by relaxing manifest $\mathcal{N} = 8$ to $\mathcal{N} = 6$. They found a $\mathrm{U}(N) \times \mathrm{U}(N)$ CS-matter theory describing N M2-branes on $\mathbb{C}^4/\mathbb{Z}_k$. [Aharony-Bergman-Jafferis-Maldacena, Gaiotto-Yin, Gaiotto-Witten, Hosomichi-Lee^3-Park]

Use $\lambda = N/k$ as 't Hooft expansion parameter: Gauge theory now weakly coupled for $\lambda \ll 1$. Good gravity description for $\lambda \gg 1$ as $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$ or $\mathrm{AdS}_4 \times \mathbb{CP}^3 \longrightarrow \mathrm{AdS}_4/\mathrm{CFT}_3$ correspondence.

ABJM can also be thought as 'relaxed' 3-algebra: not totally antisymmetric structure constants – modified version of FI. Leads to a classification of $\mathcal{N} = 6$ CS-matter theories. [Bagger-Lambert, Schnabl-Tachikawa]

In both BLG and ABJM, there exists a novel version of the Higgs mechanism corresponding to giving a diagonal vev to a scalar, *i.e.* simultaneously taking the M2-branes away from orbifold.

For sufficiently large vevs one of the CS gauge fields can be integrated out while the other one becomes dynamical.

Resulting theory is SYM in 3d and can be thought as low energy theory on equal number of D2-branes in flat space.

Outline

Higgsing the $\mathcal{A}_4$-theory

The Lagrangean for the $\mathcal{A}_4$-theory is in van Raamsdonk's formulation

$$S_{\mathcal{A}_4} = \frac{k}{2\pi}\int d^3x\,\mathrm{Tr}\Big[-(\tilde{D}^\mu X^I)^\dagger \tilde{D}_\mu X^I - \tfrac{8}{3}\,X^{IJK\dagger}X^{KJI} + \text{fermions}$$
$$+\tfrac{1}{2}\,\epsilon^{\mu\nu\lambda}\Big(A^{(1)}_\mu\partial_\nu A^{(1)}_\lambda + \tfrac{2}{3}A^{(1)}_\mu A^{(1)}_\nu A^{(1)}_\lambda - A^{(2)}_\mu\partial_\nu A^{(2)}_\lambda - \tfrac{2}{3}A^{(2)}_\mu A^{(2)}_\nu A^{(2)}_\lambda\Big)\Big]$$

where

$$X^{IJK} = X^{[I}X^{J\dagger}X^{K]}$$

The matter fields are complex-valued but obey the reality condition

$$X_{a\dot{b}} = \epsilon_{ab}\,\epsilon_{\dot{b}\dot{a}}\,X^{\dagger\dot{a}b}$$

They transform in the bi-fundamental representation $(\mathbf{2}, \bar{\mathbf{2}})$

$$\tilde{D}_\mu X^I = \partial_\mu X^I + A_\mu^{(1)} X^I - X^I A_\mu^{(2)}$$

As a first step create linear combinations of the gauge fields

$$\begin{aligned} A_\mu &= \tfrac{1}{2}\big(A_\mu^{(1)} + A_\mu^{(2)}\big) \\ B_\mu &= \tfrac{1}{2}\big(A_\mu^{(1)} - A_\mu^{(2)}\big) \end{aligned}$$

The covariant derivative is now

$$\tilde{D}_\mu X^I = D_\mu X^I - \{B_\mu, X^I\}$$

with

$$\begin{aligned} D_\mu X^I &= \partial X^I + [A_\mu, X^I] \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu] \end{aligned}$$

The bosonic part of the action becomes

$$
\begin{aligned}
S^B &= \frac{k}{2\pi}\int d^3x\,\mathrm{Tr}\Big[-(D^\mu X^I)^\dagger D_\mu X^I - \tfrac{8}{3}\,X^{IJK\dagger}X^{KJI} \\
&\quad +\{B_\mu, X^I\}\{B^\mu, X^{I\dagger}\} + D_\mu X^{I\dagger}\,\{B_\mu, X^I\} - \{B^\mu, X^{I\dagger}\}\,D_\mu X^I \\
&\quad +\epsilon^{\mu\nu\lambda}\Big(B_\mu F_{\nu\lambda} + \tfrac{1}{3}B_\mu B_\nu B_\lambda\Big)\Big]
\end{aligned}
$$

Note that the new gauge field A_μ is in the diagonal subgroup of $\mathrm{SU}(2)\times\mathrm{SU}(2)$ and has an adjoint action on the Xs.

Expand the scalars in suitable basis and give one a vev v, *e.g.* X^8

$$
\begin{aligned}
X^8 &= \tfrac{1}{2}(v + \tilde{x}^8)\, \mathbb{1} + \boldsymbol{x}^8 \\
X^i &= \tfrac{1}{2}\tilde{x}^i\, \mathbb{1} + \boldsymbol{x}^i
\end{aligned}
$$

where

$$\boldsymbol{x}^I = i\, x^{Ia}\, \frac{\boldsymbol{\sigma}^a}{2}$$

with $\boldsymbol{\sigma}^a$ the Pauli matrices. Have performed decomposition of bi-fundamental scalars into trace and traceless part,

We will be interested in the limit of large vev, $v \to \infty$.

Substitute back into the action to get

$$\begin{aligned} S^B &= \frac{k}{2\pi}\int d^3x\Big\{-\tfrac{1}{2}\partial^\mu\tilde{x}^I\partial_\mu\tilde{x}^I + \mathrm{Tr}\Big(D^\mu\boldsymbol{x}^I D_\mu\boldsymbol{x}^I + \tfrac{v^2}{2}[\boldsymbol{x}^i,\boldsymbol{x}^j][\boldsymbol{x}^i,\boldsymbol{x}^j] \\ &\quad + 2vB^\mu D_\mu\boldsymbol{x}^8 + v^2B^\mu B_\mu + \epsilon^{\mu\nu\lambda}B_\mu F_{\nu\lambda}\Big)\Big\} + \text{higher-order} \end{aligned}$$

The higher-order terms will be suppressed in powers of $\frac{1}{v}$.

The gauge field B_μ has acquired a mass term by the Higgs mechanism. Since it has no kinetic term, it can be integrated out.

This will render A_μ dynamical! The corresponding Goldstone boson is $\boldsymbol{x}^8$, as is evident by grouping terms depending on $\boldsymbol{x}^8$ and B_μ

$$v^2\left(B_\mu + \tfrac{1}{v}D_\mu \boldsymbol{x}^8\right)^2 + \epsilon^{\mu\nu\lambda}\left(B_\mu + \tfrac{1}{v}D_\mu \boldsymbol{x}^8\right)F_{\nu\lambda}$$

and shifting $B_\mu \to B_\mu - \frac{1}{v}D_\mu \boldsymbol{x}^8$. The equation of motion for B_μ is

$$B_\mu = -\frac{1}{2v^2}\epsilon^{\mu\nu\lambda}F_{\nu\lambda}$$

Eliminating B_μ from the action and rescaling $\tilde{x} \to \frac{1}{v}\tilde{x}$, $\boldsymbol{x} \to \frac{1}{v}\boldsymbol{x}$

$$\begin{aligned} S^B =& \frac{k}{2\pi v^2}\int d^3x \Big\{ -\tfrac{1}{2}\partial^\mu \tilde{x}^I \partial_\mu \tilde{x}^I + \mathrm{Tr}\Big(\tfrac{1}{2}F^{\mu\nu}F_{\mu\nu} \\ & + D^\mu \boldsymbol{x}^i D_\mu \boldsymbol{x}^i + \tfrac{1}{2}[\boldsymbol{x}^i, \boldsymbol{x}^j][\boldsymbol{x}^i, \boldsymbol{x}^j]\Big)\Big\} \end{aligned}$$

As last step, combine seven singlet scalars $\tilde{x}^i$ with $\mathrm{SU}(2)$ adjoints $\boldsymbol{x}^i$ to make $\mathrm{U}(2)$ adjoints

$$\hat{X}^i = \tfrac{i}{2}\tilde{x}^i\ \mathbb{1} + \boldsymbol{x}^i$$

Left with singlet scalar $\tilde{x}^8$, to be dualised into an abelian gauge field

$$-\int d^3x\ \tfrac{1}{2}\partial^\mu\tilde{x}^8\partial_\mu\tilde{x}^8 \to \int d^3x\ \left(-\tfrac{1}{4}F^{\mu\nu}_{\mathrm{U}(1)}F^{\mathrm{U}(1)}_{\mu\nu} + \tfrac{1}{2}\epsilon^{\mu\nu\lambda}\partial_\mu\tilde{x}^8 F^{\mathrm{U}(1)}_{\nu\lambda}\right)$$

Integrating out $\tilde{x}^8$ on RHS gives Bianchi for $F^{U(1)}_{\mu\nu}$ and the new abelian gauge field combines with $\mathrm{SU}(2)$ gauge field to form $\mathrm{U}(2)$ gauge field

$$\begin{aligned}\hat{A}_\mu &= \tfrac{i}{2}A^{U(1)}_\mu\ \mathbb{1} + A_\mu \\ \hat{F}_{\mu\nu} &= \tfrac{i}{2}F^{U(1)}_{\mu\nu}\ \mathbb{1} + F_{\mu\nu}\end{aligned}$$

We have ended up with

$$S^B = \frac{k}{2\pi v^2} \int d^3x \, \mathrm{Tr}\Big(\tfrac{1}{2} \hat{F}^{\mu\nu} \hat{F}_{\mu\nu} + D^\mu \hat{X}^i D_\mu \hat{X}^i + \tfrac{1}{2} [\hat{X}^i, \hat{X}^j][\hat{X}^i, \hat{X}^j] \Big)$$

The higher-order terms have decoupled for $v \to \infty$ as they are of higher order $\frac{1}{v}$ but have same k-dependence as leading terms.

Fermions follow and we recover maximally supersymmetric $\mathrm{U}(2)$ YM with coupling constant $g_{YM}^2 = v^2/k \to$ fixed & small, when also $k \to \infty$: the low-energy theory on 2 D2s.

For finite k the SYM theory becomes strongly coupled. By definition theory on 2 M2s $\to$ suggests that $\mathcal{A}_4$ is related to membrane physics.

Summary and effective Higgs rules

Summarising the Higgs process

- Give vev to a bi-fundamental scalar X^8
- One non-dynamical gauge field integrated out while other becomes dynamical in diagonal of $\mathrm{SU}(2) \times \mathrm{SU}(2)$
- Decompose scalars into trace and traceless part
- X^8 traceless part is the Goldstone mode
- X^8 trace part becomes abelian gauge field after duality
- Seven remaining scalars become $\mathrm{U}(2)$ adjoint scalars
- Everything combines into $\mathrm{U}(2)$ $\mathcal{N} = 8$ SYM in 3d

Net result can be captured by set of effective rules:

- For CS gauge fields

$$\mathcal{L}_{CS} \to -\tfrac{2}{v^2} f^\mu f_\mu \qquad \text{with} \qquad f^\mu = \frac{1}{2}\epsilon^{\mu\nu\lambda}\hat{F}_{\nu\lambda}$$

- For scalars

$$\begin{aligned} \tilde{D}^\mu X^8 &\to \tfrac{1}{v} f^\mu \,, & \qquad X^{ij8} &\to -\tfrac{1}{4v}[\hat{X}^i, \hat{X}^j] \,, \\ \tilde{D}^\mu X^i &\to \tfrac{1}{v} D^\mu \hat{X}^i \,, & \qquad X^{ij8\dagger} &\to \tfrac{1}{4v}[\hat{X}^i, \hat{X}^j] \end{aligned}$$

We have gone from M2 to D2! This has not involved conventional compactification to IIA but is reminiscent of similar effects in models of deconstruction. [Mukhi-CP, Distler-Mukhi-CP-van Raamsdonk]

Higgsing ABJM

Higgs mechanism extends to ABJM with minor modifications.
[Aharony-Bergman-Jafferis-Maldacena, Pang-Wang, Li-Liu-Xie]

The fields are now in $\mathrm{U}(N) \times \mathrm{U}(N)$ and decompose as

$$Y^A = Y^{A0}T^0 + iY^{Aa}T^a \quad \text{with} \quad Y^A = X^A + iX^{A+4}$$

and the vev is $\langle Y^{40} \rangle = \sqrt{k}v$.

The dynamical gauge field is now in the $\mathrm{U}(N)$ diagonal subgroup.
Need to cancel one more degree of freedom.

After implementing the Higgsing find that traceless part of X^4 and trace part of X^8 cancel out.

The degrees of freedom work out right $\rightarrow$ no need for abelian duality.

Implies compactification of X^8 transverse to X^4 which developed the vev. [Ezhuthachan-Mukhi-CP – in progress]

Get $\mathrm{U}(N)$ theory of N D2-branes with YM coupling $g_{YM}^2 = v^2/k$. The theory is weakly coupled when $Ng_{eff}^2 \ll 1$, implying $k \gg N$.

Knowing the geometric orbifold action it is easy to see the effective compactification. Start with

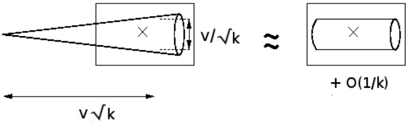
$$(Y^1, Y^2, Y^3, Y^4) \sim e^{\frac{2\pi i}{k}} (Y^1, Y^2, Y^3, Y^4)$$

For large k, v we can expand around the large vev

$$Y^4 \sim Y^4 + \frac{2\pi i}{k}(\sqrt{k} v \, \ell_{pl}^{3/2}) + \ldots$$

In components

$$\begin{aligned} X^4 &\sim X^4 \\ X^8 &\sim X^8 + 2\pi R \quad \text{with} \quad R = \frac{v}{\sqrt{k}} \ell_{pl}^{3/2} = g_{YM} \ell_{pl}^{3/2} \end{aligned}$$



Four-derivative corrections to $\mathcal{A}_4$

Many applications of this in ABJM context. Serves as check for consistency of proposals relating to weakly-coupled gauge theory.

Another application is the determination of four-derivative terms in the $\mathcal{A}_4$ action. This result also applies to finite k.

Same problem solved for Lorentzian BLG-theories.

The four-derivative corrections obtained by applying the dNS procedure to α'^2-corrected D2-brane action. [Ezhuthachan-Mukhi-CP, de Wit-Nicolai-Samtleben, Alishahiha-Mukhi]

Corrections arranged themselves in terms of 3-brackets and it was conjectured that the same should hold for Euclidean BLG-theories.

With this assumption, the Higgs mechanism determines the form of the four-derivative corrections to the BLG $\mathcal{A}_4$-theory uniquely.
[Ezhuthachan-Mukhi-CP – in progress]

There is a limit in which ABJM reduces to Lorentzian BLG. This should also extend to higher-derivative corrections.
[Honma et al., Antonyan-Tseytlin]

The non-abelian '3BI'

What is the form of the most general operator one can add?

The non-linear form of the action for the single membrane points to ℓ_{pl}^3-corrections.

Thus the full non-abelian 3BI action will have an expansion

$$S_{\mathrm{3BI}} = S_{\mathcal{A}_4/\mathrm{ABJM}} + S_{\ell_p^3} + \ldots$$

Look for dimension 6 operators. In the bosonic sector, take the most general gauge-invariant operator built out of 3-algebra building blocks.

- No CS-terms for gauge fields (not gauge invariant)
- No F^2, F^4 terms for gauge fields (extra d.o.f. - break susy)

Left with all legal index contractions for

$$
\begin{aligned}
(\tilde{D}X)^4 : \quad & k^2 \ell_{pl}^3 \,\mathrm{STr}\Big(\mathbf{a}\, \tilde{D}^\mu X^I\, \tilde{D}_\mu X^{J\dagger}\, \tilde{D}^\nu X^J\, \tilde{D}_\nu X^{I\dagger} \\
& +\mathbf{b}\, \tilde{D}^\mu X^I\, \tilde{D}_\mu X^{I\dagger}\, \tilde{D}^\nu X^J\, \tilde{D}_\nu X^{J\dagger}\Big) \\
X^{IJK}(\tilde{D}X)^3 : \quad & k^2 \ell_{pl}^3 \epsilon^{\mu\nu\lambda}\,\mathrm{STr}\Big(\mathbf{c}\, X^{IJK}\tilde{D}_\mu X^{I\dagger}\tilde{D}_\nu X^J\, \tilde{D}_\lambda X^{K\dagger} + \mathbf{c}\; h.c.\Big) \\
(X^{IJK})^2(\tilde{D}X)^2 : \quad & k^2 \ell_{pl}^3 \mathrm{STr}\Big(\mathbf{d}\, X^{IJK}\, X^{IJK\dagger}\, \tilde{D}_\mu X^L\, \tilde{D}^\mu X^{L\dagger} \\
& +\mathbf{e}\, \tilde{X}^{IJK}\, X^{IJL\dagger}\, \tilde{D}_\mu X^K\, D^\mu X^{L\dagger}\Big) \\
(X^{IJK})^4 : \quad & k^2 \ell_{pl}^3\, \mathrm{STr}\Big(\mathbf{f}\; X^{IJK}\, X^{IJK\dagger}\, X^{LMN}\, X^{LMN\dagger} \\
& +\mathbf{g}\, X^{IJM}\, X^{KLM\dagger}\, X^{IKN}\, X^{JLN\dagger} \\
& +\mathbf{h}\; X^{IJK}\, X^{IJL\dagger}\, X^{MNK}\, X^{MNL\dagger}\Big)
\end{aligned}
$$

Higgs and fix numerical coefficients by comparing with D2-brane effective action at $\mathcal{O}(\alpha'^2)$.

Latter obtained using symmetrised trace in non-abelian DBI. [Tseytlin, Bergshoeff-Bilal-de Roo-Sevrin, Cederwall-Nielsson-Tsimpis]

$$
\begin{aligned}
S^B_{\alpha'^2} = & \frac{\alpha'^2}{g^2_{YM}} \int d^3x \, \mathrm{STr}\Big[\tfrac{1}{4}\hat{F}_{\mu\nu}\hat{F}^{\nu\rho}\hat{F}_{\rho\sigma}\hat{F}^{\sigma\mu} - \tfrac{1}{16}\hat{F}^{\mu\nu}\hat{F}_{\mu\nu}\hat{F}^{\rho\sigma}\hat{F}_{\rho\sigma} \\
& \quad - \tfrac{1}{4}D_\mu\hat{X}^i D^\mu\hat{X}^i D_\nu\hat{X}^j D^\nu\hat{X}^j + \tfrac{1}{2}D_\mu\hat{X}^i D^\nu\hat{X}^i D_\nu\hat{X}^j D^\mu\hat{X}^j \\
& \quad + \tfrac{1}{4}\hat{X}^{[ij]}\hat{X}^{[jk]}\hat{X}^{[kl]}\hat{X}^{[li]} - \tfrac{1}{16}\hat{X}^{[ij]}\hat{X}^{[ij]}\hat{X}^{[kl]}\hat{X}^{[kl]} \\
& -\hat{F}_{\mu\nu}\hat{F}^{\nu\rho}D_\rho\hat{X}^i D^\mu\hat{X}^i - \tfrac{1}{4}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu}D_\rho\hat{X}^i D^\rho\hat{X}^i - \tfrac{1}{8}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu}\hat{X}^{[kl]}\hat{X}^{[kl]} \\
& -\tfrac{1}{4}D_\mu\hat{X}^i D^\mu\hat{X}^i\hat{X}^{[kl]}\hat{X}^{[kl]} - \hat{X}^{[ij]}\hat{X}^{[jk]}D^\mu\hat{X}^k D_\mu\hat{X}^i \\
& \quad - \hat{F}_{\mu\nu}D^\nu\hat{X}^i D^\mu\hat{X}^j\hat{X}^{[ij]}\Big]
\end{aligned}
$$

3BI to DBI

Proceed with Higgsing as before. The e.o.m. for the gauge field B^μ get corrected

$$B^\mu = -\frac{1}{v^2} f^\mu - \frac{\ell_{pl}^3}{2v} \delta_{B_\mu}(\mathcal{L}_{\ell_{pl}^3})$$

Two ways to get ℓ_{pl}^3 terms:

- Plug in lowest-order e.o.m. to four-derivative action
- Plug in four-derivative correction to lowest-order action – these vanish

  $\longrightarrow$ Higgs rules extend to this order.

So use

$$\tilde{D}^\mu X^8 \to \tfrac{1}{v} f^\mu \,, \qquad X^{ij8} \to -\tfrac{1}{4v}[\hat{X}^i, \hat{X}^j] \,, \qquad \tilde{D}^\mu X^i \to \tfrac{1}{v} D^\mu \hat{X}^i$$

The terms we get are weighted by: $\ell_{pl}^3 k^2/v^4 = g_s \ell_s^3/g_{YM}^4 = \alpha'^2/g_{YM}^2$ which is exactly what we want.

But 3 $(X^{IJK})^4$ terms in 3BI while 2 $(\hat{X}^{[ij]})^4$ in DBI. Saved by $\mathrm{SU}(2)$ structure leading to identities for DBI and 3BI respectively

$$\mathrm{STr}\Big(\hat{X}^{[ij]}\hat{X}^{[jk]}\hat{X}^{[kl]}\hat{X}^{[li]}\Big) = \frac{1}{2}\mathrm{STr}\Big(\hat{X}^{[ij]}\hat{X}^{[ij]}\hat{X}^{[kl]}\hat{X}^{[kl]}\Big)$$

Also

$$\begin{aligned}\mathrm{STr}\Big(X^{IJK}X^{IJL\dagger}X^{MNK}X^{MNL\dagger}\Big) =& 2\mathrm{STr}\Big(X^{IJM}X^{KLM\dagger}X^{IKN}X^{JLN\dagger}\Big)\\ =& \frac{1}{3}\mathrm{STr}\Big(X^{IJK}X^{IJK\dagger}X^{LMN}X^{LMN\dagger}\Big)\end{aligned}$$

Can fix the coefficients uniquely

$$\mathbf{a}=\tfrac{1}{2}\,,\quad \mathbf{b}=-\tfrac{1}{4}\,,\quad \mathbf{c}=-\tfrac{2}{3}$$
$$\mathbf{d}=\tfrac{4}{3}\,,\quad \mathbf{e}=-8\,,\quad \mathbf{f}=\tfrac{16}{9}$$

Corrections including fermions work out in a similar way.

We have obtained the full four-derivative correction to the $\mathcal{A}_4$-theory for any k.

The full answer is

$$
\begin{aligned}
S_{\mathcal{A}_4+\ell_{pl}^3} =& \frac{k}{2\pi}\int d^3x\,\mathrm{Tr}\Big[-(\tilde{D}^\mu X^I)^\dagger \tilde{D}_\mu X^I - \tfrac{8}{3}\,X^{IJK\dagger}X^{KJI} \\
&+ \tfrac{1}{2}\,\epsilon^{\mu\nu\lambda}\Big(A_\mu^{(1)}\partial_\nu A_\lambda^{(1)} + \tfrac{2}{3}A_\mu^{(1)}A_\nu^{(1)}A_\lambda^{(1)} - A_\mu^{(2)}\partial_\nu A_\lambda^{(2)} - \tfrac{2}{3}A_\mu^{(2)}A_\nu^{(2)}A_\lambda^{(2)}\Big)\Big] \\
&+\frac{k^2\ell_p^3}{4\pi^2}\int d^3x\,\mathrm{STr}\Big[\tfrac{1}{2}\tilde{D}^\mu X^I\,\tilde{D}_\mu X^{J\dagger}\,\tilde{D}^\nu X^J\,\tilde{D}_\nu X^{I\dagger} \\
&- \tfrac{1}{4}\tilde{D}^\mu X^I\,\tilde{D}_\mu X^{I\dagger}\,\tilde{D}^\nu X^J\,\tilde{D}_\nu X^{J\dagger} + \tfrac{16}{9}X^{IJK}\,X^{KJI\dagger}\,X^{LNP}\,X^{PNL\dagger} \\
&+ \tfrac{4}{3}X^{IJK}\,X^{KJI\dagger}\,\tilde{D}_\mu X^L\,\tilde{D}^\mu X^{L\dagger} - 8\,X^{IJK}\,X^{KJL\dagger}\,\tilde{D}_\mu X^L\,\tilde{D}^\mu X^{I\dagger} \\
&- \tfrac{2}{3}\epsilon^{\mu\nu\lambda}\Big(\tilde{D}_\mu X^{I\dagger}\tilde{D}_\nu X^J\,\tilde{D}_\lambda X^{K\dagger}\,X^{IJK} + \tilde{D}_\mu X^I\,\tilde{D}_\nu X^{J\dagger}\,\tilde{D}_\lambda X^K\,X^{IJK\dagger}\Big)\Big] \\
&+ \text{fermions}
\end{aligned}
$$

...which are far worse.

...and look something like this

$$
\begin{aligned}
S^F_{\ell_p^3} = \frac{k^2\ell_p^3}{4\pi^2}\int d^3x\, \mathrm{STr}\Big[& \hat{A}\ \bar{\Psi}^\dagger\Gamma^{IJ}[X^K, X^{L\dagger}, \Psi]\bar{\Psi}^\dagger\Gamma^{KL}[X^I, X^{J\dagger}, \Psi] \\
& + \hat{B}\ \bar{\Psi}^\dagger\Gamma^\mu\tilde{D}^\nu\Psi\tilde{\Psi}^\dagger\Gamma_\nu\tilde{D}_\mu\Psi + \hat{C}\ \bar{\Psi}^\dagger\Gamma^\mu[X^I, X^{J\dagger}, \Psi]\bar{\Psi}^\dagger\Gamma^{IJ}\tilde{D}_\mu\Psi \\
& + \hat{D}\ \bar{\Psi}^\dagger\Gamma_\mu\Gamma^{IJ}\tilde{D}_\nu\Psi\tilde{D}^\mu X^{I\dagger}\tilde{D}^\nu X^J + \hat{E}\ \bar{\Psi}^\dagger\Gamma_\mu\tilde{D}^\nu\Psi\tilde{D}^\mu X^{I\dagger}\tilde{D}_\nu X^I \\
& + \hat{F}\ \bar{\Psi}^\dagger\Gamma^{IJKL}\tilde{D}_\nu\Psi\ X^{[IJK]\dagger}\tilde{D}^\nu X^L + \hat{G}\ \bar{\Psi}^\dagger\Gamma^{IJ}\tilde{D}_\nu\Psi\ X^{[IJK]\dagger}\tilde{D}^\nu X^K \\
& + \hat{H}\ \bar{\Psi}^\dagger\Gamma^{IJ}[X^J, X^{K\dagger}, \Psi]\tilde{D}^\mu X^{I\dagger}\tilde{D}_\mu X^K + \hat{I}\bar{\Psi}^\dagger[X^I, X^{J\dagger}, \Psi]\tilde{D}^\mu X^{I\dagger}\tilde{D}_\mu X^J \\
& + \hat{J}\ \bar{\Psi}^\dagger\Gamma^{\mu\nu}[X^I, X^{J\dagger}, \Psi]\tilde{D}_\mu X^{I\dagger}\tilde{D}_\nu X^J + \hat{K}\ \bar{\Psi}^\dagger\Gamma_{\mu\nu}\Gamma^{IJ}[X^J, X^{K\dagger}, \Psi]\tilde{D}^\mu X^{I\dagger}\tilde{D}^\nu X^K \\
& + \hat{L}\ \bar{\Psi}^\dagger\Gamma_\mu\Gamma^{IJ}[X^K, X^{L\dagger}, \Psi]\tilde{D}^\mu X^{I\dagger}X^{[JKL]} + \hat{M}\ \bar{\Psi}^\dagger\Gamma_\mu[X^I, X^{J\dagger}, \Psi]\tilde{D}^\mu X^{K\dagger}X^{[IJK]} \\
& + \hat{N}\ \bar{\Psi}^\dagger\Gamma_\mu\Gamma^{IJKL}[X^L, X^{M\dagger}, \Psi]X^{[IJK]\dagger}\tilde{D}^\mu X^M + \hat{O}\ \bar{\Psi}^\dagger\Gamma_\mu\Gamma^{IJ}[X^K, X^{L\dagger}, \Psi]X^{[IJK]\dagger}\tilde{D}^\mu X \\
& + \hat{P}\ \bar{\Psi}^\dagger\Gamma^{IJKL}[X^M, X^{N\dagger}, \Psi]X^{[IJL]\dagger}X^{[KMN]} + \hat{Q}\ \bar{\Psi}^\dagger\Gamma^{IJ}[X^K, X^{L\dagger}, \Psi]X^{[IJM]\dagger}X^{[KLM]} \\
& + \text{Hermitian conjugates with the same coefficients}\Big]
\end{aligned}
$$

Summary & Conclusions

- We have reviewed an interesting version of the Higgs mechanism in the context of M2 worldvolume theories
- For large vevs one gets effective compactification and the theory is equivalent to that of D2-branes
- This connection holds beyond linear order and helped us establish the form of the four-derivative correction to the $\mathcal{A}_4$-theory
- Expect the same to work for ABJM

- Crucial test would be to show that these corrections are compatible with $\mathcal{N} = 8$ supersymmetry
- In that case it would show that the novel Higgs mechanism can tell us something about these theories at finite k
- 3-algebra structure again evident
- Relation between 3-algebras and supersymmetry?