# D-brane instantons and walls of BPS stability

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Based on I. Garcia-Etxebarria, A.U., arXiv:0711.1430
I. Garcia-Etxebarria, F. Marchesano, A.U. arXiv:0805.0713
A.U., arXiv:0808.2918

## Outline

- Motivation: D-brane instanton effects, globally in moduli space
  - BPS D-brane instanton effects
  - Walls of BPS stability
- Instantons generating superpotentials
  - Multi-instanton processes
- Instantons generating higher F-terms
  - F- vs. D-terms and Beasley-Witten cohomology
- From higher F-terms to superpotentials
  - Lifting fermion zero modes and 4d susy breaking
  - Description 4d effective action
- Conclusions and outlook

## Instanton effects

[Becker's, Strominger; Witten; Harvey, Moore; ...]

D-brane instantons violate certain perturbatively exact U(1) global symmetries

- Consider IIA CY orientifold compactification, and complex structure moduli associated to a 3-cycle C

$$T = t + i\,a = \int_C \mathrm{Re}\,\Omega + i \int_C C_3$$

Peccei-Quinn symmetry $a \longrightarrow a + \lambda$

Violated by euclidean D2-brane instanton wrapped on C $\simeq e^{-T}$

- In models with D-branes, gauging of PQ by U(1) in U(N)

Consider N D6-branes on C', there is a 4d world-volume coupling

$$\int_{C' \times M_4} C_5 \wedge \mathrm{tr}\, F \;\rightarrow\; \int_{M_4} B_2 \wedge \mathrm{tr}\, F \;\rightarrow\; \int d^4x\,(\,\partial_\mu a \,+\, A_\mu\,)^2$$

⇒ Instanton generates terms such that phase rotations compensate

$e^{-T}\,\Phi_1 \ldots \Phi_n$ allows couplings forbidden in pert.th.

(insersions from fermion mode couplings $\int d\lambda\, d\tilde{\lambda}\, e^{-T+\lambda\Phi\tilde{\lambda}} \,=\, e^{-T}\,\Phi$ )

# D-brane instanton effects in 4d N=1

Include standard gauge instantons, plus many more

- Gauge instantons

Instanton D-brane wraps same "cycle" as 4d gauge D-brane

Ex. ADS "fractional" instantons

$$W = (N_c - N_f) \left( \frac{\Lambda^{3N_c - N_f}}{\det M} \right)^{\frac{1}{N_c - N_f}}$$

[..., many authors]

- Non-gauge instantons

General D-brane instantons

In perturbative models, need O(1) Chan-Paton group

The latter provide new sources of interesting 4d operators violating certain perturbatively exact global symmetries
Application to neutrino masses, mu-term, GUT yukawas, ...

[Argurio, Bertolini, Bianchi,Billo,Blumenhagen, Cvetic, Ferretti, Frau,Ibanez, Kiritsis,Lerda, Marotta,Petersson,Richter, Schellekens, Weigand,A.U....]

# Instanton effects and fermion zero modes

Generate different kinds of 4d superspace interactions, according to structure of unlifted fermion zero modes

- Instantons contributing to superpotential

  BPS D-branes with exactly 2 fermion zero modes (goldstinos)

  Generate 4d superpotentials $$\int d^4x\, d^2\theta\, e^{-T}\, \Phi_1 \ldots \Phi_n$$

- Beasley-Witten instantons

  BPS D-branes with more than 2 decoupled fermion zero modes

  Generate multi-fermion F-term, sketchily

$$\int d^4x\, d^2\theta\, w_{\bar{i}_1 \bar{j}_1 \ldots \bar{i}_n \bar{j}_n}(\Phi)\, \bar{D}\bar{\Phi}^{\bar{i}_1}\, \bar{D}\bar{\Phi}^{\bar{j}_1} \ldots \bar{D}\bar{\Phi}^{\bar{i}_n}\, \bar{D}\bar{\Phi}^{\bar{j}_n}$$

- Non-BPS instantons

  Have at least 4 fermion zero modes (goldstinos of 4 broken susys)

  Generate 4d D-terms $$\int d^4x\, d^2\theta\, d^2\bar{\theta}\, f(T, \bar{T}, \Phi, \bar{\Phi})$$

# Instanton effects globally in moduli space

In principle, non-perturbative F-terms at a point in moduli space, from list of BPS instantons at such point

Naive clash between the macro/micro pictures

- 4d N=1 F-terms are defined in terms of holomorphic quantities
- Walls of marginal stability: Real codimension-1 regions in moduli space where spectrum of BPS D-branes changes abruptly

(D-brane instanton becomes unstable and splits into sub-objects)

We find a consistent picture for instantons generating superpotentials, or for instantons generating higher F-terms.

Some interesting lessons all along the way

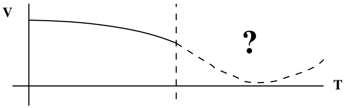
# A “pheno” motivation

Understanding of non-perturbative effects globally in moduli space, crucial in practical applications e.g. to moduli stabilization

E.g. Consider stabilizing Kahler moduli in IIB compactifications via a non-perturbative superpotential, e.g. from gaugino condensates

[Kachru,Kallosh,Linde,Trivedi]

- Sit at a point in Kahler moduli space, where susy D7-branes generate a superpotential
- Minimize scalar potential
- The original D7-brane are NON-susy at the new minimum! Non-zero D-term, D7’s recombine into new susy system
- Does the new system generate the same superpotential? If not, “minimum” is really out of regime of validity of superpotential used to find it!



# Walls of BPS stability (4d N=1)

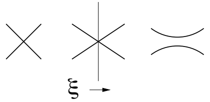
Real codimension-1 loci in moduli space where BPS spectrum changes

Can classify according to decay pattern

- <u>Marginal stability</u>: BPS brane splits, decay products misalign

U(1)xU(1) theory with boson with charges (+1,-1)

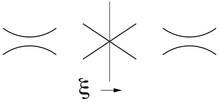
$$V_D = ( |\phi|^2 - \xi )^2$$




- <u>Threshold stability</u>: BPS brane splits, pieces recombine to new BPS

U(1)xU(1) theory with bosons with charges ±(+1,-1)

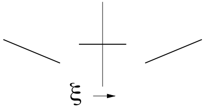
$$V_D = ( |\phi_1|^2 - |\phi_2|^2 - \xi )^2$$




- <u>No-split BPS stability</u>: BPS brane becomes non-BPS, with no splitting

U(1) theory with no boson

$$V_D = \xi^2$$



# Counting Goldstinos

The structure of fermion zero modes already determines the BPS stability properties of the instantons

- An instanton contributing to the superpotential (thus with two fermion zero modes) cannot cross genuine lines of marginal stability and become non-BPS

Not enough fermion zero modes to account for the 4 goldstinos

- An instanton contributing to the superpotential at most can reach a lines of threshold stability, and split into mutually BPS decay products

- An instanton with additional fermion zero modes (thus contributing to higher F-terms) can cross genuine lines of marginal stability and become non-BPS

Two combination of extra fermion zero modes become the two extra goldstinos

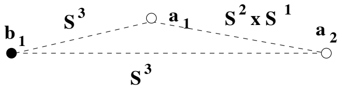
# Useful geometries [Ooguri,Vafa]

Consider non-compact geometries with compact 3-cycles on which we have D2-brane instantons

Double C* fibrations over the complex plane, 3-cycles are double circle fibrations over segments between degenerations

$$xy = \prod_{k=1}^{P} (z - a_k)$$

$$x'y' = \prod_{k'=1}^{P'} (z - b_k)$$

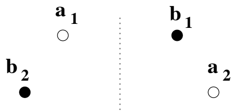



BPS D2' wrap horizontal segments calibrated wrt $\Omega = dz \, \frac{dx}{x} \, \frac{dx'}{x'}$

Useful orientifold, mod by ΩR(-)$^F$ with

$$z \to -\bar{z} \quad ; \quad (x, y) \leftrightarrow (\bar{x}', \bar{y}')$$

O6 in the vertical direction,
exchanging a and b degenerations

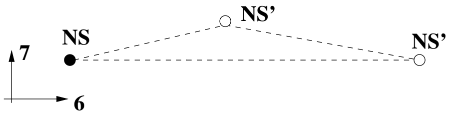


# Useful geometries (cont.)

Useful T-dual realization as D- branes suspended among NS5-branes
[Hanany, Witten]

T-duality along the two circle directions on the fibers

NS along 012345
NS' along 012389
D0 along segment in 67




Picture allows to read out spectrum and interactions of zero modes

e.g. "hypermultiplets" at touching of horizontal segments

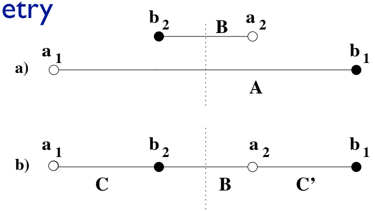
cubic "superpotentials" for adjoint (non-rigidity) zero modes

possible quartic "superpotentials" for bifundamentals

Also useful picture of different orientifold planes
(e.g. O6 on 01236 and 45 degrees in (45,89))

# Superpotentials from non-gauge instantons and threshold stability lines

Orientifolded geometry



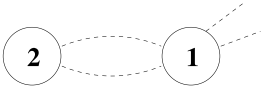

a) Generically there are two O(1) instantons, A and B

$$W = f_1 e^{-T_B} + f_2 e^{-T_A}$$

b) Line of marginal stability, in which instanton A disappears.
One O(1) instanton B and one U(1) instanton C/C'

How is $\exp(-T_A)$ generated?

# The 2-instanton process




## Zero mode analysis

Translational Goldstones $x_1, x_2$; "Goldstinos" $\theta_1, \tilde{\theta}_1, \theta_2$;
bi-fundamental hyperm. $\Phi_{12}, \Phi_{21}$ ie $\varphi_{12}, \varphi_{21}, \chi_{12}, \chi_{21}$

For instantons separated in 4d, too many zero modes:
localization onto $x_1 = x_2$

Couplings for fermions

- $(\chi_{12}\,(\theta_1 - \theta_2))\varphi^*_{12} - (\chi_{21}\,(\theta_1 - \theta_2))\varphi^*_{21} + (\bar{\chi}_{12}\tilde{\theta})\varphi_{12} - (\bar{\chi}_{21}\tilde{\theta})\varphi_{21}$
- $\chi_{12}\varphi_{21}\chi_{12}\varphi_{21} + 2\chi_{12}\chi_{21}\varphi_{12}\varphi_{21} + \varphi_{12}\chi_{21}\varphi_{12}\chi_{21} \quad + \text{h.c.}$

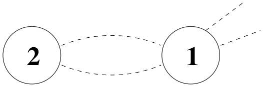
  (from $W \simeq (\Phi_{12}\Phi_{21})^2$)

All fermions couple except for the overall Goldstinos $\theta_1 + \theta_2$

Pull down interactions in $\exp(-S_{inst})$ and soak up zero modes

We recover $S_{4d} \simeq \int d^4x\, d^2\theta\, e^{-(T_B + 2T_C)} \simeq \int d^4x\, d^2\theta\, e^{-T_A}$

# Non-perturbative lifting of fermion zero modes




The O(1) instanton leads to a term in the 4d action, but also leads to a modification of the zero mode action of the U(1) instanton

$$\Delta S_{\rm inst1} = \int d^2\theta_2\, d^4\chi\, d^4\varphi\, \exp[\,(\theta_1 - \theta_2)\,\varphi\,\chi \,+\, \tilde{\theta}_1\,\bar{\varphi}\,\bar{\chi} + \chi^2\,\varphi^2 \,+\, V(\varphi)\,]$$

Sketchily, upon integration over zero modes of O(1) instanton

$$\Delta S_{\rm inst1} \,\simeq\, e^{-T_B}\,\tilde{\theta}_1\tilde{\theta}_1$$

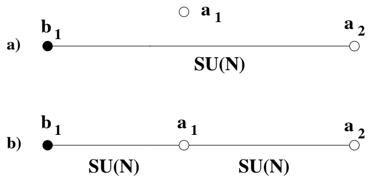
Extra zero modes of U(1) instanton are lifted, it contributes to W

$$S_{4d} \;\simeq\; \int d^4x\, d^2\theta\, d^2\tilde{\theta}\, \exp\,[\,-2T_C \,-\, e^{-T_B}\,\tilde{\theta}\tilde{\theta})\,]$$

$$= \int d^4x\, d^2\theta\, e^{-T_B}\, e^{-2T_C} \,=\, \int d^4x\, d^2\theta e^{-T_A}$$

# Superpotentials from gauge instantons and threshold stability lines

## A SQCD superpotential example




a) SU(N) pure SYM

$$W = N\Lambda^3 \simeq (e^{-T})^{\frac{1}{N}}$$ (from 1/N-instanton)

b) unHiggses to $SU(N)_1 x SU(N)_2$ with bifundamentals $Q_{12}$,$Q'_{21}$ and adjoint $\Phi_2$ with (perturbative) $W=Q_{12}\Phi_2 Q'_{21}$

Is complete superpotential continuous?

## Spacetime analysis

$SU(N)_1$ has $N_f = N_c$ , has quantum deformed moduli space for its mesons $M=Q'_{21}\,Q_{12}$ and barions B
(from Beasley-Witten instanton on 1)

$$W = \Lambda_1 \Phi M + \Lambda_1^{-2N+2}\, X\, (\det M - B\tilde{B} - \Lambda_1^{2N})$$

Mesons are massive, baryons decouple, leaving $SU(N)_2$ pure SYM with dynamical scale

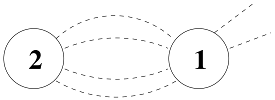
$$\Lambda_2'^{3N} = \Lambda_2^N \Lambda_1^{2N} = e^{-T_2} e^{-T_1}$$

The non-perturbative superpotential
(from 1/N-fractional instanton on 2)

$$W = N\Lambda_2'^3 \simeq (\, e^{-T}\,)^{\frac{1}{N}}$$

Nice agreement!

Microscopic instanton description is a multi-instanton process



# Global picture for non-perturbative superpotentials

- Non-perturbative superpotential is continuous and holomorphic across lines of threshold stability

- Microscopically, thanks to multi-instanton processes

At points where a D-brane instanton becomes unstable against and its contribution disappears, there exists a multi-instanton process (involving the decay products) which reconstructs the same contribution

(our study predates recent beautiful work by Gaiotto, Moore, Neitzke: Kontsevich-Soibelmann BPS wall crossing formula for 4d N=2 theories is equivalent to continuity of the moduli space metric in 3d reduction)

- Many examples, for gauge or non-gauge instantons, see paper

(including Seiberg duality, gauge/non-gauge instanton transitions,...)

- Instantons contributing to the superpotential cannot become non-BPS anywhere in moduli space

Not enough fermion zero modes to account for 4 goldstinos

At worst, lines of threshold stability

What is the story for instantons with extra fermion zero modes, generating higher F-terms?

Instantons with extra fermion zero modes can cross lines of marginal stability and become non-BPS

Seems to lead to a clash with standard wisdom

BPS instantons lead to F-terms, non-BPS instantons lead to D-terms

How can an F-term "become" a D-term?
Holomorphy vs. real codimension-1 walls?

Way out from careful analysis of the structure of higher F-terms

# Higher F-terms and Beasley-Witten cohomology

Higher F-terms in 4d N=1 have a structure

$$\int d^4x\, d^2\theta\, \omega_{\bar{i}_1\cdots\bar{i}_p\,\bar{j}_1\cdots\bar{j}_p}(\Phi)\,\left(\bar{D}_{\dot{\alpha}_1}\bar{\Phi}^{\bar{i}_1}\bar{D}^{\dot{\alpha}_1}\bar{\Phi}^{\bar{j}_1}\right)\cdots\left(\bar{D}_{\dot{\alpha}_p}\bar{\Phi}^{\bar{i}_p}\bar{D}^{\dot{\alpha}_p}\bar{\Phi}^{\bar{j}_p}\right) \equiv \int d^4x\, d^2\theta\, \mathcal{O}_\omega$$

To define a susy operator, need $\bar{D}_{\dot{\alpha}}\mathcal{O}_\omega = 0$

ω holomorphic, i.e. closed under $\bar{\partial}$

If $\mathcal{O}_\omega = \{\bar{Q}_{\dot{\alpha}}, [\bar{Q}^{\dot{\alpha}}, V]\}$ then can write as D-term $\int d^4x\, d^4\theta\, V$

equivalence relation by exact objects

$$\omega_{\bar{i}_1\cdots\bar{i}_p\,\bar{j}_1\cdots\bar{j}_p} \sim \omega_{\bar{i}_1\cdots\bar{i}_p\,\bar{j}_1\cdots\bar{j}_p} + \nabla_{[\bar{i}_1}\xi_{\bar{i}_2\cdots\bar{i}_p]\,\bar{j}_1\cdots\bar{j}_p} + (\bar{i}_k \leftrightarrow \bar{j}_k)$$

Defines a cohomology for tensors in field space

Genuine higher F-terms are defined by forms ω in a non-trivial class of this cohomology

Can be written as D-terms locally in moduli space, but not globally

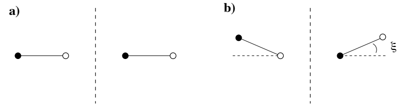
Precisely what we need for instantons which can be BPS/non-BPS

# Example: Non-split marginal stability

Physics of BPS/non-BPS is already visible in ‘no-split’ BPS stability line

see later for splitting

Consider isolated rigid D2-brane on 3-cycle C




BPS stability is controlled by real modulus ξ in chiral multiplet Σ controlling dual 3-cycle

- Phase of the N=2 central charge
- Couples as FI term to instanton world-volume

Zero modes    Translational goldstones $x^\mu$

Goldstinos $\theta'^\alpha, \bar{\tau}'_{\dot{\alpha}}$ of locally felt N=2

Related to susys preserved/broken by the Oplane as

$$\theta' = \cos(\xi/2)\theta + \sin(\xi/2)\tau \qquad \bar{\tau}' = \cos(\xi/2)\bar{\tau} - \sin(\xi/2)\bar{\theta}$$

# The instanton amplitude

## At BPS locus

$$S_{\text{inst}} = t + \theta\,\delta_\theta t + \bar\tau\,\bar\delta_{\bar\tau}\,t + (\theta\,\delta_\theta)\,(\bar\tau\,\bar\delta_{\bar\tau})\,t = T + \bar\tau\,\bar D\bar\Sigma$$

$$\int d^4x\,d^2\theta\,d^2\bar\tau\,e^{-S_{\text{inst.}}} = \int d^4x\,d^2\theta\,e^{-T}\,\bar D\bar\Sigma\,\bar D\bar\Sigma$$

Higher F -term, as expected for BPS instanton

## Away from BPS locus

Non-BPS instanton, D-term is expected: from trading $\bar\tau$ '$\longrightarrow \bar\theta$

$$\int d^4x\,d^2\theta\,d^2\bar\theta\,e^{-\text{Vol}_{\text{inst}}}$$

## In the near-BPS locus Smooth glueing

Instanton amplitude of non-BPS instanton at leading order in ξ-expansion reproduces the BPS higher F-term

Instanton amplitude as function of moduli space is in a non-trivial class of the BW cohomology, with obstruction localized on the BPS locus

## Genuine lines of marginal stability (with split)

Similar lessons apply on lines of marginal stability where instantons split and products become mutuallly non-BPS

Combine lessons from multi-instantons and from non-BPS systems

Ex: two isolated rigid U(1) instantons with 'chiral' intersection

complex scalar m and fermions $\psi, \bar{\psi}$ at intersection

$$V_D = (\,|m|^2 - \xi\,)^2$$

Misaligning e.g. instanton 1

$$S_{2-\text{inst}} = |x_1 - x_2|^2\,|m|^2 + i(x_1^\mu - x_2^\mu)\,\bar{\psi}\,\sigma_\mu\,\psi\, + \\ + \psi\,(\theta_1 - \theta_2)\,\bar{m} + \bar{\psi}\,(\bar{\tau}_1 - \bar{\tau}_2)\,m + \bar{\tau}_1\,\bar{D}\bar{\Sigma}$$

On non-BPS branch all modes saturate but for $\theta_1 + \theta_2,\, \bar{\tau}_1 + \bar{\tau}_2$ with the latter picking up a component along $\bar{\theta}$

Instanton amplitude define a D-term which on BPS locus reduces to $\int d^4x\, d^2\theta\, e^{-T}\, \bar{D}\bar{\Sigma}\, \bar{D}\bar{\Sigma}$

## Global picture for non-perturbative F-terms

- Non-perturbative higher F-terms are continuous across general lines of marginal stability

- Consistency with standard wisdom of
BPS$\Rightarrow$ F-term, non-BPS $\Rightarrow$ D-term

Instanton amplitude defines a 4d operator which is in non-trivial class of the BW cohomology:
- Locally in moduli space, can be written as a D-term
- Obstruction (localized on BPS locus) to write as global D-term

Holomorphy of higher F-term throughout moduli space:
- Full instanton amplitude and expression at BPS locus differ by a global D-term (same in cohomology).
- Non-holomorphies of are in the global D-term part

More examples, for gauge or non-gauge instantons, see paper
e.g. brane realization of Nf=Nc SQCD

# Applications

Useful to think about instanton effects globally in moduli space
But any use?
Let me present two

- A criterion for possibility of lifting extra zero modes

  Intricate relation between lifting of fermion zero modes and 4d susy breaking

- A new viewpoint on effects of fluxes on D-brane instantons

  Interesting role of higher F-terms
  (addresses the question "who cares about higher F-terms")

Both related to processes turning contributions to higher F-terms into superpotential contributions

# Lifting of fermion zero modes and 4d susy breaking

Relation between superpotential and higher F-term by lifting fermion zero modes?

- Consider an instanton which can misalign and become non-BPS
- Introduce a mechanism to lift extra fermion zero modes to make it contribute to the superpotential
- Contradiction with counting of goldstinos is possible only if ...

⇒ 4d supersymmetry breaking upon misalignment due to mechanism lifting fermion zero modes!

Ex: Flavor mass to flow to Nf=Nc-1 SQCD

⇒ D-term on instanton implies a non-zero D-term on 4d branes

Ex: Closed string fluxes

⇒ Mass of extra z.m. ≈ susy variations of gravitino and dilatino

Ex: Lifting by other instantons ⇒ Previous marginal stability turns to threshold stability of new multi-instanton system

[see O(1)xU(1) → O(1) example; also Cvetic, Richter, Weigand]

# Fluxes and D-brane instantons

- Interplay of fluxes and D-brane instantons, at different levels
  - Mutual consistency conditions: Freed Witten anomalies

    Bianchi identity for worldvolume gauge field   $dF=H3$

    D-brane instantons do not break isometries gauged by the flux

    [Kashani-Poor, Tomasiello]

  - Lifting of fermion zero modes of the D-brane instanton
    - Index for a modified Dirac operator   [Bergshoeff, Kallosh, Kashani-Poor, Sorkin, Tomasiello]
    - Lifting computable as G3 $\lambda\lambda$ disk diagram in fluxless CFT

      [Billo, Ferro, Frau, Fucito, Lerda, Morales]

  - Instantons that do not contribute to the superpotential of fluxless compactification can contribute in the presence of fluxes
    - E.g: 3-form flux does not lift N=2 goldstinos of D3-brane instantons but can lift deformation zero modes

  Is there a macroscopic effective field theory description?

## The 4d effective field theory picture [AU]

- Drawbacks of microscopic picture:
  - For fixed CY, need to evaluate superpotential for each flux choice
  - Local in moduli space
  - Requires a microscopic picture of the flux

- There must exist a consistent description in 4d effective theory

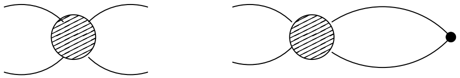
  At large radius, flux scale $\alpha'/R^3$ much smaller than KK scale $1/R$

  Should describe all effects of fluxes as a deformation of the fluxless 4d effective theory (potential in moduli space of exact theory)

- Works indeed if fluxless effective theory includes higher F-terms

  Effects of instantons with additional zero modes

  Upon inclusion of the flux superpotential, they turn into non-perturbative superpotentials, via integration of the massive moduli



- Recovers standard results, and many more

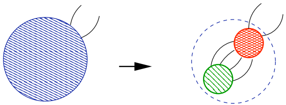
# Conclusions

- Interesting lessons from D-brane instantons and BPS stability

- Instantons with two fermion zero modes do not have genuine lines of marginal stability, at worst lines of threshold stability.

- Non-perturbative superpotential is holomorphic across them

- Microscopically, thanks to contributions from multi-instanton processes



- Extra zero modes of an instanton can be soaked up by others in multi-instanton processes

Non-perturbative lifting of fermion zero modes

Superpotential contributions from U(1) instantons
Evades usual criteria (e.g. of arithmetic genus, etc)

- Many phenomena and systems in this circle of ideas

# Conclusions

- Instantons with additional fermion zero modes can cross genuine lines of marginal stability, beyond which they become non-BPS (possibly multi-instanton) systems

- Instanton amplitude as function over moduli space is in non-trivial Beasley-Witten cohomology class

  Away from BPS locus, can be written as D-term
  (extra zero modes pick up component along $\bar{\theta}$)

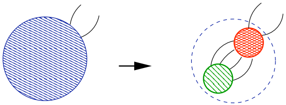
  Global obstruction, localized on BPS locus, where amplitude must be written as F-term

- Agreement with standard wisdom of BPS/non-BPS instantons

- Interesting connection between lifting of extra fermion zero modes and 4d susy breaking

# Outlook

- Universality of contributions to non-perturbative F-terms



(insensitive to D-terms inside instanton world-volumes)
Presumably related to universality of category of holomorf. branes & topological strings

- Any relation to other brane splittings? multicenter bh's [Denef;...]
(in fact, continuity of instanton amplitudes seems to be equivalent to wall crossing formulaes for degeneracies of BPS states)
[Gaiotto, Moore,Neitzke]

- Lifting of zero modes in multi-instanton processes
Define index for multi-instanton systems, robust under splitting?

Expect many other surprises from D-brane instantons