Intersecting branes and Nambu–Jona-Lasinio model

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December 7, 2008

Nobel for NJL

2008: The Year of Nobel Prize for NJL model

References

This talk is based on an ongoing project with Partha Nag.
Other references, which are of immediate relevance, are:

- Antonyan, Harvey, Jensen and Kutasov, *NJL and QCD from string theory*, hep-th/0604017
- Antonyan, Harvey and Kutasov, *Chiral symmetry breaking from intersecting D-branes*, Nucl. Phys. **B784** (2007) 1, hep-th/0608177
- Sakai and Sugimoto, *Low energy hadron physics in holographic QCD*, Prog. Theor. Phys. **113**, 843 (2005), hep-th/0412141

Prehistory

- The NJL model - First example of dynamical chiral symmetry breaking and fermion mass generation in a simple effective field theory setting.
  - **Y. Nambu** and G. Jona-Lasinio, *Dynamical Model Of Elementary Particles Based On An Analogy with Superconductivity. I*, Phys. Rev. **122** (1961) 345
- Original model - Fermions in the four-fermi interaction are nucleons. Interest in the model has endured for two main reasons:
  - It appears to give a rather accurate description of chiral symmetry breaking and its consequences for low-energy hardron phenomenology
  - Appropriately replacing the original nucleons by coloured quarks, the model can be argued to describe all of the low-energy physics of QCD, including the anomaly term

Prehistory

First papers to discuss NJL for quarks and derive meson lagrangians, with the correct anomaly terms

- Dhar and Wadia, *Nambu–Jona-Lasinio Model: An Effective Lagrangian for Quantum Chromodynamics at Intermediate Length Scales*, Phys. Rev. Lett. **52** (1984) 959.
- Dhar, Shankar and Wadia, *Nambu–Jona-Lasinio–type effective Lagrangian: Anomalies and nonlinear Lagrangian of low-energy large-N QCD*, Phys. Rev. **D31** (1985) 3256.

Plan

What I plan to do

- NJL from QCD
  - local approximation, critical coupling
  - generalization to a nonlocal model
- Nonlocal NJL in Sakai-Sugimoto intersecting brane configuration - motivation from noncompact limit
  - weak coupling, hierarchy of length scales
  - integrating out Yang-Mills degrees of freedom
- χSB in nonlocal NJL
  - nonlinear gap equation
  - scale of χSB, the condensate
- Summary

NJL from QCD

Yang-Mills with $U(N_c)$ gauge group and N_f massless quark flavours confines and develops a mass scale Λ

$$\Lambda \sim m\, e^{-1/\beta_0 g_4^2}, \qquad \beta_0 = \frac{1}{24\pi^2}(11N_c - 2N_f)$$

- At energies below the confining scale, an effective NJL model for quarks captures the dynamics
- There is no systematic way of integrating out the Yang-Mills degrees of freedom from QCD to get an effective action for quarks

NJL from QCD: A scenario

- Start from a nonperturbative formulation of Yang-Mills, say lattice, with massless quarks (masses much smaller than the confining scale)
- Integration of Yang-Mills degrees of freedom would lead to effective multi-quark interactions
- The range of these interactions must be short, of the order of $1/\Lambda$, because of confinement
- At energies below Λ a local approximation would be adequate. The NJL interaction between gauge-invariant quark bilinears is the leading term compatible with gauge symmetry and global symmetries of QCD

χSB in NJL

$$S_{\rm NJL} = \int d^4x \Big(q_L^{\dagger\alpha}(x)\bar{\sigma}^\mu i\partial_\mu q_L^\alpha(x) + q_R^{\dagger\alpha}(x)\sigma^\mu i\partial_\mu q_R^\alpha(x) + g_{\rm eff}^2 [q_L^{\dagger\alpha}(x) q_R^\beta(x)][q_R^{\dagger\beta}(x) q_L^\alpha(x)] \Big)$$

- Standard Gaussian trick

$$T^{\alpha\beta}(x) = g_{\rm eff}^2 [q_R^{\dagger\beta}(x) q_L^\alpha(x)]$$

- Effective action for scalar fields is obtained by integrating fermions from the resulting quadratic action

χSB in NJL

- For vacuum configurations, set $T^{\alpha\beta}(x) = T_0 \delta^{\alpha\beta}$.
- T_0 is the mass gap. It is determined by the gap equation

$$\frac{T_0}{N_c g_{\text{eff}}^2} = 2T_0 \int^M \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + T_0^2}$$

- Two possible solutions: $T_0 = 0$ and $T_0 \neq 0$
- For $g_{\text{eff}}^2 > g_c^2 = \frac{8\pi^2}{N_c M^2}$, the solution with $T_0 \neq 0$ is stable to small fluctuations, while the solution with $T_0 = 0$ is unstable
- $\Rightarrow$ χSB for $g_{\text{eff}} > g_c$

Nonlocal NJL and QCD

- Integrating out Yang-Mills degrees of freedom would generally lead to a nonlocal effective theory
- In QCD, the χSB scale M_χ coincides with the confinement scale Λ. So, at energies below Λ a local approximation is adequate
- Suppose, however, the two scales are separated by some new physics, as happens in the SS intersecting brane configuration
- In this case, for studying χSB we need the effective four-fermi theory at energies larger than the mass gap Λ

Nonlocal NJL and QCD

- Suppose $M_\chi >> \Lambda$, i.e. the energies of quarks involved in the four-fermi interaction are much larger than Λ
- In this case, we can present a more precise derivation of the effective interaction, because of asymptotic freedom.
- The leading contribution to the interaction comes from a one-gluon exchange approximation, which can be calculated exactly:

$$\begin{aligned} S_{\rm NJL} &= g_4^2 \int d^4x\, d^4y\, \Delta(x-y)[q_L^\dagger(x) q_R(y)][q_R^\dagger(y) q_L(x)] \\ \Delta(x) &= \int \frac{d^4k}{(2\pi)^4} \frac{e^{ik.x}}{k^2} \end{aligned}$$

Nonlocal NJL and QCD

- The effective four-fermi theory resulting from integrating out the gluon degrees of freedom is

$$\begin{aligned} S_{\rm NJL} &= \int d^4x \Big(q_L^\dagger(x)\bar{\sigma}^\mu i\partial_\mu q_L(x) + q_R^\dagger(x)\sigma^\mu i\partial_\mu q_R(x) \Big) \\ &+ g_4^2 \int d^4x \; d^4y \; G_0(x-y)[q_L^\dagger(x) q_R(y)][q_R^\dagger(y) q_L(x)] \end{aligned}$$

$$G_0(x) = \int \frac{d^4k}{(2\pi)^4} \; e^{ik.x} \; \tilde{G}_0(k)$$

Nonlocal NJL and QCD

- $\tilde{G}_0(k)$ satisfies:
  - Confinement $\Rightarrow$ for $k << \Lambda$, $\tilde{G}_0(k) \sim$ constant $\sim 1/\Lambda^2$
  - Asymptotic freedom $\Rightarrow$ for $\Lambda << k << M_\chi$, $\tilde{G}_0(k) \sim 1/k^2$
- A simple example of a function $\tilde{G}_0(k)$ which satisfies these two properties is (with a cut-off $k \lesssim M_\chi$)

$$\tilde{G}_0(k) = \frac{1}{k^2 + \Lambda^2}$$

- A version with smooth cut-off is

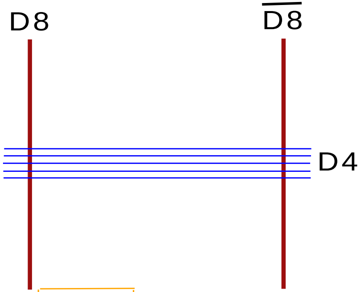
$$\tilde{G}_0(k) = \frac{e^{-kM_\chi}}{k^2 + \Lambda^2}$$

Nonlocal NJL and QCD

- Integrating out gluon degrees of freedom from QCD (or any other confining theory) is expected to give rise to a far more complicated effective fermion action
- However, one may expect this simple model to be more amenable to analytical treatment of qualitative questions about χSB
- Alternatively, one may consider it as an interesting model to study in its own right

Weakly coupled SS intersecting brane configuration

- N_c $D4$-branes wrapping a thermal circle of radius $R \Rightarrow$ colour degrees of freedom
- open strings between $D4$-branes and $D8$ and anti-$D8$-branes separated by $L \leq \pi R$ apart on the circle $\Rightarrow$ flavour degrees of freedom



	0	1	2	3	4	5	6	7	8	9
D4	x	x	x	x	x					
D8	x	x	x	x		x	x	x	x	x

Length scales

- The SS model has several length scales:

$$l_s,\ R,\ L,\ g_5^2$$

  where $g_5^2 = 4\pi^2 g_s l_s$ is the 5-d YM coupling

- Alternatively, there is the 5-dimensional 't Hooft coupling $\lambda_5 = g_5^2 N_c$

- There is also the confinement length scale

$$\Lambda \sim (\pi R)^{-1}\ e^{-2\pi R/\beta_0 g_5^2}$$

Hierarchy of length scales

$$\lambda_5 << l_s << L \leq \pi R << \Lambda^{-1}$$

- Stringy corrections to low-energy action are small if $\lambda_5/l_s << 1$
- For a metastable brane-anti-brane system we need $l_s << L$
- For separation L between the flavour branes, strings of mass L can get excited. Corrections due to these are small if $\lambda_5/L << 1$

Low-energy, weakly coupled SS model

At scales much smaller that the string length, the dynamics of the weakly coupled SS model is governed by the action

$$
\begin{aligned}
S &= -\frac{1}{4g_5^2}\int d^4x \int_0^{2\pi R} dx^4 \, (F^a_{MN}(x,x^4))^2 \\
&\quad + \int d^4x \; q_L^{\alpha\dagger}(x)\bar{\sigma}^\mu\Big(i\partial_\mu + t^a A^a_\mu(x,-L/2)\Big)q_L^\alpha(x) \\
&\quad + \int d^4x \; q_R^{\alpha\dagger}(x)\sigma^\mu\Big(i\partial_\mu + t^a A^a_\mu(x,L/2)\Big)q_R^\alpha(x)
\end{aligned}
$$

$$
\begin{aligned}
A^a_M(x,x^4) &= \sum_{n=1}^{\infty}\Big(A^{a(n)}_M(x)e^{inx^4/R} + A^{a(n)^*}_M(x)e^{-inx^4/R}\Big) \\
&\quad + A^{a(0)}_M(x)
\end{aligned}
$$

Integrating out YM degrees of freedom

$$S = S_0 + S_1 + \cdots$$

$\Rightarrow S_0$ is QCD action with $g_4^2 = g_5^2/2\pi R$

$$S_1 = g_4^2 \sum_{n=1}^{\infty} \int d^4x\ d^4y\ \Delta_n(x-y) J^a_{n\mu}{}^*(x) J^{a\mu}_n(y)$$

$$J^{a\mu}_n(x) = \left(q^\dagger_L(x)\bar{\sigma}^\mu t^a q_L(x) e^{inL/2R} + q^\dagger_R(x)\sigma^\mu t^a q_R(x) e^{-inL/2R} \right)$$

$$\Delta_n(x) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{ik.x}}{(-k^2 + \frac{n^2}{R^2})}$$

Nonlocal NJL from SS

$$S_1 = g_4^2 \int d^4x \; d^4y \; G_1(x-y)[q_L^\dagger(x) q_R(y)][q_R^\dagger(y) q_L(x)]$$

$$\begin{aligned} G_1(x) &= \int \frac{d^4k}{(2\pi)^4} \; e^{ik.x} \; \tilde{G}_1(k) \\ \tilde{G}_1(k) &= \frac{\pi R \cosh k(\pi R - L)}{k \sinh k\pi R} - \frac{1}{k^2} \end{aligned}$$

- This result is valid for all values of L and R, since we have summed over the exchange of all the Kaluza-Klein modes. In particular, in the limit $R \to \infty$, keeping L fixed, we get

$$\tilde{G}_1(k) \to \frac{\pi R}{k} e^{-kL}$$

Nonlocal NJL model

$$S_{\rm eff} = \int d^4x \Big(q_L^\dagger(x)\bar{\sigma}^\mu i\partial_\mu q_L(x) + q_R^\dagger(x)\sigma^\mu i\partial_\mu q_R(x) \Big)$$
$$+ g_4^2 \int d^4x\, d^4y\, G(x-y)[q_L^\dagger(x) q_R(y)][q_R^\dagger(y) q_L(x)]$$

- $\tilde{G}(k)$ satisfies:
  - For $k \sim \Lambda$, $\tilde{G}(k) \sim 1/(k^2+\Lambda^2)$. This follows from the hierarchy of scales $\Lambda << 1/\pi R \leq 1/L$
  - For $\Lambda << k << 1/\pi R$, $\tilde{G}(k) \sim 1/k^2$. Asymptotically free regime of 4-d
  - For $k >> 1/\pi R$, $\tilde{G}(k) \sim \pi R e^{-kL}/k$. Here 4-d $\Rightarrow$ 5-d. The nonlocal NJL interaction has a short-distance cut-off at a scale of order L

Nonlocal NJL model

- A simple function that contains all the three scales and capture all the essential features of $\tilde{G}(k)$ is

$$\tilde{G}(k) = \left(\frac{1+\pi R k}{k^2+\Lambda^2}\right) e^{-kL}$$

  Remember the hierarchy of scales is $L << \pi R << \Lambda^{-1}$

- $G(x)$ can be computed
  - For $|x| = r \sim L$,

$$G(x) \sim \frac{1}{(r^2+L^2)^{3/2}}$$

  - For $r >> \Lambda^{-1}$,

$$G(x) \sim \frac{e^{-r\Lambda}}{r^{3/2}}$$

χSB in nonlocal NJL

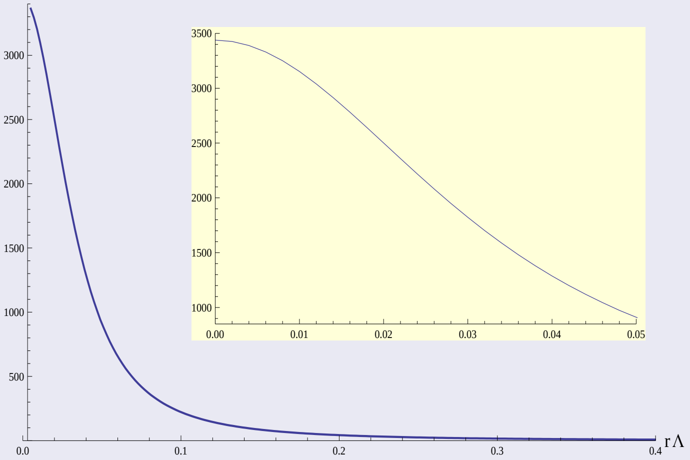


Figure: $G(x)$ as a function of $r\Lambda$

The order parameter

- The next step is to introduce scalars

$$T^{\alpha\beta}(x,y) = g_4^2 \; G(x-y) \; q_R^{\dagger\beta}(y) q_L^{\alpha}(x)$$

- For classical ground state solutions, it is sufficient to take

$$T^{\alpha\beta}(x,y) = \delta^{\alpha\beta} T(|x-y|)$$

- The order parameter is

$$\phi(x) = \frac{1}{N_c} \langle q_L^{\dagger\alpha}(x) q_R^{\alpha}(0) \rangle = \frac{T(x)}{g_4^2 N_c G(x)}$$

Gap equation

- Integrating out the fermions gives an effective action for $T(x)$:

$$\frac{S_{\text{eff}}}{V N_c N_f} = \int d^4x \, \frac{|T(x)|^2}{g_4^2 N_c G(x)} - \int \frac{d^4k}{(2\pi)^4} \, \ln\left(1 + \frac{|\tilde{T}(k)|^2}{k^2}\right)$$

- The equation of motion - the gap equation

$$\int \frac{d^4x}{(2\pi)^4} e^{-ik.x} \frac{T(x)}{g_4^2 N_c G(x)} = \tilde{\phi}(k) = \frac{\tilde{T}(k)}{k^2 + |\tilde{T}(k)|^2}$$

- $T(x) = 0$ is a solution with $S_{\text{eff}} = 0$.

Gap equation

- On-shell value of S_{eff}:

$$\frac{S_{\text{eff}}}{VN_cN_f} = \int \frac{d^4k}{(2\pi)^4}\left[\frac{|\tilde{T}(k)|^2}{k^2+|\tilde{T}(k)|^2} - \ln\left(1+\frac{|\tilde{T}(k)|^2}{k^2}\right)\right]$$

- As a function of $|\tilde{T}(k)|/k$, the integrand always decreases. Since for $|\tilde{T}(k)|/k = 0$, the integrand vanishes, it is always negative for any value of k

- $\Rightarrow T(x) \neq 0$ solution has negative $S_{\text{eff}} \Rightarrow$ χSB for arbitrarily weak coupling

- For finiteness of S_{eff}, need the asymptotic behaviour

$$|\tilde{T}(k)| \sim k^{-\gamma}, \qquad \gamma > 0$$

Solving gap equation

- For sufficiently large k, we may assume $|\tilde{T}(k)| << k$. (This must be so for $k >> L^{-1}$). Then the nonlinearities can be neglected and the gap equation can be approximated as

$$k^2\tilde{\phi}(k) = \tilde{T}(k), \qquad k >> L^{-1}$$

- In coordinate space, on spherically symmetric functions, it takes the form

$$\partial_r^2\phi(r) + \frac{3}{r}\partial_r\phi(r) + g_4^2 N_c G(r)\phi(r) = 0$$

- ϕ_0, the normalization of ϕ is not determined. But ϕ_0 determines beyond what value of k this approximation can be used.

Solving gap equation

- Need to use nonlinearity of gap equation to determine ϕ_0
- For sufficiently small k, $|\tilde{T}(k)| >> k$. In this case, the gap equation becomes

$$\tilde{\phi}(k) = 1/\tilde{T}^*(k), \qquad k << \Lambda$$

- This must certainly be the case for $k << \Lambda$ since in this region we expect the order parameter to decay (in coordinate space) as $e^{-r\Lambda}$
- Take as ansatz: $\phi(r) = \phi_0 x^{-\delta} e^{-rL_X}$. One can now determine both $\tilde{\phi}(k)$ and $|\tilde{T}(k)|$ (both turn out to be constants) and hence ϕ_0.

Solving gap equation

- We have obtained the solution analytically in these two limits:

$$\tilde{T}(k) \sim k^{-\gamma}, \qquad k >> 1/L$$

and

$$\tilde{T}(k) \sim \text{constant}, \qquad k << \Lambda$$

- The scale of χSB can be obtained from the solution which interpolates between these two. Because of nonlinearity of the gap equation, this can be done only numerically.
- Work on the numerical solution is in progress and should be completed soon

$R \to \infty$ limit

- The nonlocal NJL model of Antonyan, Harvey, Jensen and Kutasov was obtained from the weakly coupled SS model in which the circle is noncompact
- The confining scale, which is related to R as

$$\Lambda \sim \frac{1}{\pi R} e^{-2\pi R/\beta_0 g_5^2}$$

  vanishes in this limit, giving rise to a long range interaction of the fermions
- This makes the limit subtle and it seems that a consistent model cannot be obtained in the limit

Summary

- Nolocal NJL models arise as effective low-energy theories in a variety of situations, e.g. low-energy QCD, low-energy string theory on intersecting brane systems, etc.
- Unlike in the original local NJL model, these models may have χSB at arbitrarily weak coupling. Also, if the underlying microscopic theory is confining, these effective theories may have the scale of χSB separated from the confinement scale.
- We proposed and discussed in detail a nonlocal NJL model, which has features similar to the nonlocal NJL model one gets from the SS model at weak coupling. We argued that a vacuum solution with the above properties exists. We are working on explicitly obtaining a complete solution to the nonlinear gap equation.