Exploration prospects of a long baseline beta beam neutrino experiment with an iron calorimeter detector

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Abstract

A high intensity source of a single neutrino flavour with known spectrum is most desirable for precision measurements, the consensus direction for the future. The beta beam is an especially suitable option for this. We discuss the prospects of a very long baseline beta beam experiment with a magnetized iron calorimeter detector. In particular, with the source at CERN and the detector at the proposed India-based Neutrino Observatory (INO) the baseline is near the ‘magic’ value where the effect of the CP phase is small. We observe that this experiment will be well suited to determine the sign of $m_{2}^{2} - m_{3}^{2}$ and will be capable of probing $\theta_{13}$ down to about 1°.

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1. Introduction

There is now compelling evidence in support of neutrino mass and mixing [1] from a number of atmospheric [2], solar [3], reactor [4], and long-baseline [5] neutrino experiments. The neutrino mass eigenvalues and the Pontecorvo, Maki, Nakagawa, Sakata (PMNS) mixing matrix [6] provide a natural framework for formulating the scenario for three active neutrinos. At present, information is available on two neutrino mass-square differences and two mixing angles: from atmospheric neutrinos one gets the best-fit values with 3σ error $^{1}$ $|\Delta m_{23}^{2}| \simeq 2.12^{+1.09}_{-0.81} \times 10^{-3}$ eV$^2$, $\theta_{23} \simeq 45.0^{+10.5}_{-8.3}^\circ$ while solar neutrinos tell us $\Delta m_{12}^{2} \simeq 7.9 \times 10^{-5}$ eV$^2$, $\theta_{12} \simeq 33.21^\circ$ [7]. At the moment, the sign of $\Delta m_{23}^{2}$ is not known. It determines whether the neutrino mass spectrum is direct or inverted hierarchical. The two large mixing angles and the relative oscillation frequencies could be useful for measurement of CP-violation in the neutrino sector, if the third mixing angle, $\theta_{13}$, and the CP phase, $\delta$, are not van-

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$^{1}$ Here $\Delta m_{ij}^{2} = m_{j}^{2} - m_{i}^{2}$. 

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ishingly small. The current bound on the former is \( \sin^2 \theta_{13} < 0.05 \) (3\( \sigma \)) \( [8,9] \) while the latter is unconstrained.

A number of possible high-precision neutrino oscillation experiments are being designed to shed light on \( \theta_{13}, \delta \), and the sign of \( \Delta m^2_{32} \); among these are super-beams (very intense conventional neutrino beams) \( [10–12] \), neutrino factories (neutrino beams from boosted-muon decays) \( [13] \), improved reactor experiments \( [14] \), and more recently \( \beta \) beams (neutrinos from boosted-ion decays) \( [15–17] \).

Here we focus on a long baseline (\( \sim \) several thousand km) \( \beta \) beam experiment in conjunction with a magnetized iron calorimeter detector with charge identification capability. The proposal for a detector of this type (ICAL) is being evaluated by the INO Collaboration \( [18] \). We consider the beta beam source to be located at CERN. To maintain collimation over such long baselines, the beta beam has to be boosted to high \( \gamma \). The longer baseline captures a matter-induced contribution to the neutrino parameters, essential for probing the sign of \( \Delta m^2_{32} \). The CERN–INO distance happens to be near the so-called ‘magic’ baseline \( [19] \) for which the results are relatively insensitive to the yet unconstrained CP phase. This permits such an experiment to make precise measurements of the mixing angle \( \theta_{13} \) avoiding the degeneracy issues \( [20] \) which plague other baselines.

2. The \( \beta \) beam

The beta beam, an idea put forward by Zucchelli \( [15] \), is connected with the production of a pure, intense, collimated beam of electron neutrinos or their antiparticles via the beta decay of accelerated radioactive ions circulating in a storage ring \( [21] \). In particular, such a beam can be produced with the help of the existing facilities at CERN. An intense proton driver and a hippodrome-shaped decay ring are the essential requirements for this programme.

It has been proposed to produce \( \nu_e \) beams through the decay of highly accelerated \( ^{18}\text{Ne} \) ions and \( \bar{\nu}_e \) from \( ^{6}\text{He} \) \( [21] \). Using the SPS accelerator at CERN, it will be possible to access \( \gamma \sim 100 \) for completely ionized \( ^{18}\text{Ne} \) and \( \gamma \sim 60 \) for \( ^{6}\text{He} \). The ratio between the two boost factors is fixed by the necessity of using the same ring for both ions. It is envisaged to have both beams simultaneously in the ring. Such an arrangement will result in a \( \nu_e \) as well as a \( \bar{\nu}_e \) beam pointing towards a distant target. Higher values of \( \gamma \), as required for longer baselines to INO from CERN, for example, can be achieved by upgrading the SPS with superconducting magnets or by making use of the LHC. The reach of the LHC will be \( \gamma = 2488 \) \( (^{6}\text{He}) \) and \( \gamma = 4158 \) \( (^{18}\text{Ne}) \) \( [22] \). The beta beam is almost systematic free.

\( \bar{\nu}_e \) are produced by the super-allowed \( \beta^- \) transition \( ^{6}\text{He} \rightarrow ^{3}\text{Li} + e^- + \bar{\nu}_e \). The half-life of \( ^{6}\text{He}^{++} \) is \( 0.807 \) s and the \( Q \) value of the reaction is \( E_0 = 3.507 \) MeV. Neutrino beams can be produced by the super-allowed \( \beta^+ \) transition \( ^{16}\text{Ne} \rightarrow ^{18}\text{F} + e^+ + \nu_e \), having the half-life \( 1.672 \) s and the \( Q \) value, \( E_0 = 3.424 \) MeV. According to feasibility studies \( [23,24] \), the number of injected ions in case of antineutrinos can be \( 2 \times 10^{18} \) yr\(^{-1} \) and for neutrinos \( 1.1 \times 10^{18} \) yr\(^{-1} \).

3. Neutrino fluxes

Neglecting small Coulomb corrections, the differential width of \( \beta \)-decay is described by

\[
\frac{d^2 I^*}{d\Omega^* dE^*_{\nu}} \frac{\ln 2}{4\pi} \frac{m_e^2}{m^2_{\nu}} f_{t/2}(E_0 - E^*_{\nu}) E^*_{\nu}^2 \times \sqrt{(E_0 - E^*_{\nu})^2 - m^2_e},
\]

where \( m_e \) is the electron mass and \( E^*_{\nu} \) is the neutrino energy.\(^2\) Here \( E_0 \) represents the electron end-point energy, \( t_{1/2} \) is the half life of the decaying ion in its rest frame and

\[
f(y_e) \equiv \frac{1}{60y_e^3} \left\{ \sqrt{1 - y_e^2(2 - 9y_e^2 - 8y_e^4)} + 15y_e^4 \log \left[ \frac{y_e}{1 - \sqrt{1 - y_e^2}} \right] \right\}.
\]

where \( y_e = m_e/E_0 \).

Since the spin of the parent nucleus is zero, it decays isotropically and there is no angular dependence in its rest frame. The Jacobian, \( J = [\gamma(1 - \beta \cos \theta)]^{-1} \), connects the rest frame quantities \((\cos \theta^*, E^*_{\nu})\) to the lab frame ones \((\cos \theta, E_{\nu})\).

\(^2\) The quantities with (without) * refer to the rest (lab) frame.
The flux \( N \) is related to \( \Gamma \) by the radioactive decay law
\[
\frac{d^2 N}{dE_\nu \, dt} = g\gamma \tau \frac{d\Gamma}{dE_\nu},
\]
where \( g \) is the number of injected ions per unit time and \( \tau \) is the lifetime of that ion in its rest frame.

We replace \( d\Omega \) by \( dA/L^2 \), where \( dA \) is the small area of the detector and \( L \) is the distance between the source and the detector. So, using Eqs. (1) and (3), the number of electron neutrinos, within the energy range \( E_\nu \) to \( E_\nu + dE_\nu \), hitting unit area of the detector located at a distance \( L \) aligned with the straight sections of the storage ring in time \( dt \) is given by
\[
\frac{d^3 N}{dA \, dE_\nu \, dt} \bigg|_{lab} = \frac{1}{4\pi L^2} \frac{\ln 2}{m_5 f_{1/2}} \frac{g\tau}{\gamma (1 - \beta \cos \theta)} \times (E_0 - E_\nu^*) E_\nu^{*2} \sqrt{(E_0 - E_\nu^*)^2 - m_\nu^2},
\]
where \( E_\nu^* = \gamma E_\nu (1 - \beta \cos \theta) \).

From a technical point of view, it is not difficult to achieve designs aiming at higher \( \gamma \) by direct extrapolation of existing facilities [22,23]. The neutrino parameters we are interested to explore require a “high” \( \gamma \) option (\( \gamma \geq 1500 \)) which would be accessible, as noted earlier, in the LHC era at CERN.

We discuss below the physics reach of a \( \beta \) beam using a magnetized iron calorimeter detector, of the type being considered by the India-based Neutrino Observatory (INO) Collaboration. For the long baselines suitable for a rich physics harvest, the iron calorimeter detector (ICAL) being examined for INO provides a favourable target. The site for this Observatory has been narrowed down to one of two possible locations (a) Rammam in the Darjeeling Himalayas (latitude = 27°2’N, longitude = 88°16’E) or (b) Singara (PUSHEP) in the Nilgiris (latitude = 11°5’N, longitude = 76°6’E). In the following, we show that ICAL will be an attractive choice for a very long baseline \( \beta \) beam experiment, with the source at CERN, Geneva (\( L = 6937 \) km (Rammam), 7177 km (PUSHEP)). The unoscillated neutrino and anti-neutrino fluxes reaching the detector are depicted in Fig. 1.

It is noteworthy that the large CERN to INO distance ensures a significant matter effect contribution enabling a determination of the mass hierarchy. At the same time, it matches the so-called ‘magic’ baseline [19] where the results become insensitive to the unknown CP phase \( \delta \). This permits a clean measurement of \( \theta_{13} \).

4. Three flavour oscillations

Here we briefly summarize the notations and conventions that will be followed. The neutrino flavour states \( |\nu_\alpha\rangle (\alpha = e, \mu, \tau) \) are linear superpositions of the mass eigenstates \( |\nu_i\rangle (i = 1, 2, 3) \) with masses \( m_i \), i.e., \( |\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle \). Here \( U \) is the \( 3 \times 3 \) unitary matrix parametrized as (ignoring Majorana phases):
\[
U = V_{23} W_{13} V_{12}.
\]
where

\[ V_{12} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \]

\[ W_{13} = \begin{pmatrix} c_{13} & 0 & s_{13} \mathrm{e}^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} \mathrm{e}^{i\delta} & 0 & c_{13} \end{pmatrix}, \]

\[ V_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}, \]

(6)

where \( c_{12} = \cos \theta_{12} \), \( s_{12} = \sin \theta_{12} \), etc., and \( \delta \) denotes the CP-violating (Dirac) phase. The probability that an initial \( \nu_f \) of energy \( E \) gets converted to a \( \nu_g \) after traveling a distance \( L \) in vacuum is

\[
P(\nu_f \rightarrow \nu_g) = \delta_{fg} - 4 \sum_{j>i} \text{Re}(U_{ji}^\ast U_{gj} U_{fj} U_{gj}^\ast) \times \sin^2 \left( \frac{1.27 \Delta m_{ij}^2 L}{E} \right) \\
+ \pm 2 \sum_{j>i} \text{Im}(U_{ji}^\ast U_{gj} U_{fj} U_{gj}^\ast) \times \sin \left( \frac{2.54 \Delta m_{ij}^2 L}{E} \right).
\]

(7)

In the above, \( L \) is expressed in km, \( E \) in GeV and \( \Delta m_{ij}^2 \) in eV\(^2\). The ‘−’ (‘+’) refers to neutrinos (antineutrinos).

Neutrino interactions in matter modify the oscillation probability. Interactions of the \( \nu_e \) occur through both charged and neutral weak currents making an additional contribution to its mass while the muon- and tau-neutrinos get contributions only through the neutral interaction. This alters both the mass splittings as well as the mixing angles. The general expression for the oscillation probability is messy. The appearance probability \((\nu_e \rightarrow \nu_\mu)\) in matter, up to second order in the small parameters \( \alpha \equiv \Delta m_{12}^2/\Delta m_{13}^2 \) and sin \( 2\theta_{13} \), is [25]:

\[
P_{\nu e \mu} \simeq \sin^2 2\theta_{13} \sin^2 \theta_{23} \times \frac{\sin^2[(1 - \hat{A})\Delta]}{(1 - \hat{A})^2} \\
\pm \alpha \sin 2\theta_{13} \xi \sin \delta \sin \Delta \\
\times \frac{\sin(\hat{A}\Delta) \sin[(1 - \hat{A})\Delta]}{\hat{A}} \\
+ \alpha \sin 2\theta_{13} \xi \cos \delta \cos \Delta \\
\times \frac{\sin(\hat{A}\Delta) \sin[(1 - \hat{A})\Delta]}{\hat{A}} \\
+ \alpha \sin 2\theta_{13} \xi \cos \delta \cos \Delta,
\]

(8)

where

\[
\Delta \equiv \frac{\Delta m_{13}^2 L}{4E}, \quad \xi \equiv \cos \theta_{13} \sin 2\theta_{12} \sin 2\theta_{23},
\]

\[
\hat{A} \equiv \pm \frac{2\sqrt{2} G_F n_e E}{\Delta m_{13}^2}.
\]

\( G_F \) and \( n_e \) are the Fermi coupling constant and the electron density in matter, respectively. The sign of the second term is positive (negative) for \( \nu_e \rightarrow \nu_\mu \) (\( \nu_\mu \rightarrow \nu_e \)). The sign of \( \hat{A} \) is positive (negative) for neutrinos (antineutrinos) with normal hierarchy and it is opposite for inverted hierarchy. We have checked numerically that for low \( \theta_{13} (\lesssim 4^\circ) \) the results from the above approximate expression agree well with those from the exact three flavor oscillation formula. For higher values of \( \theta_{13} \) though agreement of a qualitative nature remains, the actual results differ by upto \( \sim 35\% \).

One of the complications which needs to be addressed in the extraction of the neutrino properties is the issue of parameter degeneracies [20]; namely, that different sets of values of these parameters can result in the same predictions. It is imperative therefore to identify situations where this degeneracy problem can be circumvented or evaded. For example, in Eq. (8), if one chooses \( \sin(\hat{A}\Delta) = 0 \), the \( \delta \) dependence disappears and thus a clean measurement of the hierarchy and \( \theta_{13} \) is possible without any correlation with the CP phase \( \delta \) [19]. The first non-trivial solution for this condition is \( \sqrt{2} G_F n_e L = 2\pi \). For an approximately isoscalar (one electron per two nucleons) medium of constant density \( \rho \) this equation gives an estimate of the size of this ‘magic’ baseline \( L_{\text{magic}} \):

\[
L_{\text{magic}} [\text{km}] \approx 32726 \frac{1}{\rho [\text{g/cm}^3]}.
\]

(9)

In particular, for the CERN–INO path, the neutrino beam passes through the mantle of the earth where the density can be considered to be constant to a reasonable accuracy. The appropriately averaged density turns out to be \( \rho = 4.15 \text{ g cm}^{-3} \) for which \( L_{\text{magic}} = 7886 \text{ km} \). The results presented in the course of our discussion are obtained by numerically solving the full
three-flavour neutrino propagation equation based on the framework of Barger et al. [26], including the CP phase \( \delta \) and reflect the expectations for a near-‘magic’ baseline.

Simulation for the ICAL design has shown excellent energy determination and charge identification capability for muons with the energies relevant here. We focus therefore on the muon neutrino appearance mode, i.e., \( \nu_e \to \nu_\mu \) and \( \bar{\nu}_e \to \bar{\nu}_\mu \) transitions. Even though it is possible to increase the ion energy to achieve the threshold necessary for \( \tau \) production \( (\nu_\tau \to \nu_\mu) \) appearance, it would require a very large storage ring and an enhanced storage time because of the lifetime dilatation.

5. Cross sections, detector

Following the standard approach, the neutrino-nucleus interaction cross section is obtained by including contributions from the exclusive channels of lower multiplicity (quasi-elastic scattering [27] and single-pion production [28]), while all additional channels are incorporated as part of the deep-inelastic scattering [29] cross section:

\[
\sigma_{CC} = \sigma_{QE} + \sigma_{1\pi} + \sigma_{DIS}. \tag{10}
\]

At the low energy end, quasi-elastic events are dominant and the cross section grows rapidly for \( E_\nu \leq 1 \) GeV, while at the higher energies \( (E_\nu \geq \text{a few GeV}) \), mostly deep-inelastic scattering occurs and the cross section increases linearly with neutrino energy. At intermediate energies, both types of events contribute. In addition, resonant channels dominated by the \( \Delta(1232) \) resonance [28] also take part in the process. We include all of the above. Because the neutrino energy extends up to about 20 GeV most events are deep-inelastic. There is about 10\% contribution of quasi-elastic and single-pion production events each.

The detector is assumed to be made of magnetized iron slabs with interleaved active detector elements as in the MINOS [30], and proposed ICAL detector at INO [18]. For ICAL, glass resistive plate chambers have been chosen as the active elements. In these proposals the detector mass is almost entirely (\( >98\% \)) due to its iron content. Here we follow the present ICAL design—a 32 kt iron detector with an energy threshold around 800 MeV. The signature for the \( \nu_e \to \nu_\mu \) and \( \bar{\nu}_e \to \bar{\nu}_\mu \) transitions is the appearance of prompt muons whose tracks inside the detector will be reconstructed to give the direction and energy. Simulations have shown that the charge identification efficiency is around 95\% at ICAL. So \( \nu_\mu \) and \( \bar{\nu}_\mu \) will be readily distinguished. For this analysis the detector is taken to be of perfect efficiency and with no backgrounds.\(^3\) Since we are very far from the source and the storage ring, the geometry of the storage ring will not play a vital role in the calculation of flux.

6. Results

6.1. Determination of the \( \text{sign} (\Delta m^2_{23}) \)

The CERN to INO distance is close to the ‘magic’ baseline (\( \sim 7000 \) km) where matter effects are significant and the impact of the CP phase is negligible.\(^4\) Over such long baselines, measurement of the neutrino mass hierarchy becomes possible, as matter effects become sizable. Within the three neutrino mixing framework, the results on solar neutrinos prefer the dominant mass eigenstates in \( \nu_e \) to have the hierarchy \( m^2_2 > m^2_1 \) so that the mass-squared difference \( \Delta m^2_{12} = m^2_2 - m^2_1 > 0 \). The sign of \( \Delta m^2_{23} \) is a remaining missing piece of information to pin down the structure of the neutrino mass matrix. The beta beam can make good progress in this direction.

Fig. 2 shows the number of events over a five-year period as a function of \( \theta_{13} \), taking the direct \( (m^2_2 - m^2_1 > 0) \) and inverted \( (m^2_2 - m^2_1 < 0) \) hierarchies.\(^5\) The noteworthy feature of this analysis is that for the direct hierarchy, the number of events obtained from a neutrino beam could be substantial while that for the antineutrino beam would be strongly suppressed, while the opposite will be true for the inverted hierarchy. Such an asymmetry would be easy to detect using the charge identification abilities of ICAL.

\[^{3}\] Atmospheric neutrino and other backgrounds will be eliminated by the directionality cut imposed in event selection.

\[^{4}\] For the results which we present, we have used the CERN to Rammam \( (L = 6937 \text{ km}) \) baseline. We have checked that if the baseline for the alternate PUSHEP site \( (L = 7177 \text{ km}) \) is used, the results vary by less than 5\%.

\[^{5}\] Unless specified otherwise, where necessary, we use the best-fit values of the mixing parameters.
The mass hierarchy can be probed at the 4.4 (4.8)σ level with a neutrino (antineutrino) beam for values of θ_{13} as low as ∼1°. As seen from Fig. 2, the sensitivity increases dramatically with θ_{13}. This sensitivity will also depend on the precise value of Δm_{23}^2. For example, for Δm_{23}^2 within the present 1σ interval 1.85−2.48 × 10^{-3} eV^2, this significance varies within 3.5−5.3σ (4.6−5.1σ) for neutrinos (antineutrinos). In the above, the CP phase δ is chosen to be 90°. As checked below, the CERN–INO baseline, close to the ‘magic’ value, ensures essentially no dependence of the final results on δ.

In this calculation, we have considered an uncertainty of 2% [16] in the knowledge of the number of ions in the storage ring. Following the standard practice, we have assumed a 10% fluctuation in the cross section, σ. The statistical error has been added to the above in quadrature. We have neglected nuclear effects.

6.2. Precision measurement of θ_{13}

Aside from the neutrino mass hierarchy, the other major unknown in the neutrino sector is the mixing angle θ_{13}. Here also the results for the long baseline beta beam set-up are encouraging.

In Fig. 3, we plot the number of events in 5 years as a function of θ_{13} for two extreme values of δ. Results are shown for neutrinos and antineutrinos. The dependence on δ is seen to be very mild—a reflection of the near ‘magic’ baseline. The growth of the number of events with increasing θ_{13} is consistent with Eq. (8). For these plots we have chosen Δm_{23}^2 > 0. In this case, as already noted in Fig. 2, the number of events for
the $\bar{\nu}_e$ beam is quite small while for $\nu_e$ it is substantial. Therefore, for this mass hierarchy, the neutrino run must be used to extract $\theta_{13}$ and values as small as 1° can be probed. For the opposite hierarchy, the antineutrino beam will give the larger number of events which can be used to determine $\theta_{13}$.

The estimated $3\sigma$ errors on $\theta_{13}$ measured to be $1^\circ$ ($5^\circ$) are $\pm 0.6^\circ$ ($\pm 2.2^\circ$) with $\delta = 0^\circ$ for neutrinos. The results are somewhat worse for antineutrinos for the direct hierarchy. $\Delta m^2_{23} > 0$, considered here. For the inverted hierarchy, antineutrino beams provide the better measurement.

In the extraction of $\theta_{13}$, a major role is played by the value of $\Delta m^2_{23}$. For illustration, with a neutrino beam and for $\delta = 90^\circ$, the 1σ error of $\Delta m^2_{23}$ translates to uncertainties of $\sim \pm 1^\circ$ at $\theta_{13} = 5^\circ$ and less than $\pm 4^\circ$ at $\theta_{13} = 1^\circ$. This is for the normal hierarchy. In principle, the long baseline beta beam experiment can narrow down the permitted range of $\Delta m^2_{23}$. However, it is very likely that this improvement will be achieved in the meanwhile by other experiments.

The effect of $\delta$ is negligibly small for neutrinos as seen in Fig. 3 (left). To estimate the effect of $\delta$ for the case of antineutrinos we vary it over its whole range in the meanwhile by other experiments.

7. Conclusions

We have discussed the prospects of obtaining information on the mixing angle $\theta_{13}$ and the sign of $\Delta m^2_{23}$ using a magnetized iron calorimeter detector, such as the proposed ICAL detector at INO, and a high $\gamma$ beta beam source. It appears that such a combination of a high intensity $\nu_e$, $\bar{\nu}_e$ source and a magnetized iron detector is well-suited for this purpose. We have focused on the CERN to INO baseline, which is close to the ‘magic’ value, and found that it should be possible to determine the sign of $\Delta m^2_{23}$ and probe $\theta_{13}$ down to 1° in a five-year run. The effect of the CP phase $\delta$ is quite mild.

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