NEUTRINOS FROM THE BIG BANG

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The standard Big Bang cosmology predicts the existence of an, as yet undetected, relic neutrino background, similar to the photons observed in the cosmic microwave background. If neutrinos have mass, then such relic neutrinos are a natural candidate for the dark matter of the universe, and indeed were the first particles to be proposed for this role. This possibility has however been increasingly constrained by cosmological considerations, particularly of large-scale structure formation, thus yielding stringent bounds on neutrino masses, which have yet to be matched by laboratory experiments. Another probe of relic neutrinos is primordial nucleosynthesis which is sensitive to the number of neutrino types (including possible sterile species) as well as to any lepton asymmetry. Combining such arguments with the experimental finding that neutrino mixing angles are large, excludes the possibility of a large asymmetry and disfavours new neutrinos beyond those now known.

1 Relic Neutrinos and the Cosmological Mass Bound

Several years before neutrinos had even been experimentally detected, Alpher, Follin & Hermann[1] noted that they would have been in thermal equilibrium in the early universe "...through interactions with mesons" at temperatures above 5 MeV; below this temperature the neutrinos "...freeze-in and continue to expand and cool adiabatically as would a pure radiation gas". These authors also observed that the subsequent annihilation of $e^{\pm}$ pairs would heat the photons but not the decoupled neutrinos, so by conservation of entropy T_ν/T would decrease from its high temperature value of unity, down to $(4/11)^{1/3}$ at $T \ll m_e$[1].

(Thus the present density of *massless* relic neutrinos (neglecting a possible chemical potential i.e. lepton–antilepton asymmetry) would be

$$\frac{n_\nu}{n_\gamma} = \left(\frac{T_\nu}{T}\right)^3 \left(\frac{n_\nu}{n_\gamma}\right)_{T=T_{\text{dec}}} = \frac{4}{11}\left(\frac{3}{4}\frac{g_\nu}{2}\right),\dots(1)$$

where $g_\nu = 2$ corresponds to left-handed neutrinos and right-handed antineutrinos, and the factor 3/4 reflects Fermi versus Bose statistics. This would also be true for massive neutrinos if the neutrinos are relativistic at decoupling i.e. for $m_\nu \ll T_{\text{dec}}$. For a

present cosmic microwave background (CMB) blackbody temperature[2] $T_0 = 2.725 \pm 0.002$ K, the abundance per flavour should then be $\frac{3}{11} \times \frac{2\zeta(3)}{\pi^2}T_0^3 \simeq 111.9 \,\text{cm}^{-3}$. As long as they remain relativistic, the neutrinos would retain a Fermi-Dirac distribution with phase-space density

$$f_\nu = \frac{g_\nu}{(2\pi)^3}\left[\exp\left(\frac{p}{T_\nu}\right) + 1\right]^{-1}, \qquad \dots(2)$$

since the momentum and temperature would redshift identically.)

Subsequently, Chiu & Morrison[3] found the rate for $e^+e^- \rightleftharpoons \nu_e\bar{\nu}_e$ in a plasma to be $\Gamma_\nu \approx G_F^2 T^5$ for the universal Fermi interaction and Zel'dovich[4] equated this to the Hubble expansion rate in the radiation-dominated era,

$$H = \sqrt{\frac{8\pi G_N \rho}{3}}, \quad \text{with} \quad \rho = \frac{\pi^2}{30}g_* T^4, \dots(3)$$

where g_* counts the relativistic degrees of freedom, to obtain the 'decoupling' temperature below which the neutrinos expand freely without further interactions as $T_{\text{dec}}(\nu_e) \simeq 2$ MeV. (Neutral currents were then unknown so $T_{\text{dec}}(\nu_\mu)$ was estimated from the reaction $\mu \rightleftharpoons e\bar{\nu}_e\nu_\mu$ to be 12 MeV. Later De Graaf[5] noted that these would keep ν_μ's coupled to the plasma down to

the same temperature as ν_e's.[a]) However Zel'dovich[4] and Chiu[10] concluded that relic neutrinos, although nearly as numerous as the blackbody photons, cannot make an important contribution to the cosmological energy density since they are probably massless.

Interestingly enough, some years earlier Pontecorvo and Smorodinski[11] had discussed the bounds set on the cosmological energy density of MeV energy neutrinos (created e.g. by large-scale matter-antimatter annihilation) using data from the Reines–Cowan and Davis experiments.[b]) Not surprisingly these bounds were rather weak so these authors stated somewhat prophetically that "...it is not possible to exclude a priori the possibility that the neutrino and antineutrino energy density in the Universe is comparable to or larger than the average energy density contained in the proton rest mass". Zel'dovich and Smorodinski[12] noted that better bounds can be set by the limit on the cosmological energy density in any form of matter ρ_m ($\equiv \Omega_m \rho_c$) following from the observed present expansion rate H_0 and age t_0 of the universe.[c] Of course they were still discussing *massless* neutrinos. Weinberg[13] even speculated whether a degenerate sea of relic neutrinos can saturate the cosmological energy density bound and noted that such a sea may be detectable by searching for (scattering) events beyond the end-point of the Kurie plot in β-decay experiments. Several years later, Gershteĭn and Zel'dovich[14] made the connection that if relic neutrinos are massive, then a bound on the mass follows from simply requiring that

$$m_\nu n_\nu < \rho_m. \qquad \ldots(4)$$

Using the general relativistic constraint $\Omega t_0^2 H_0^2 < (\pi/2)^2$, they derived $\rho_m < 2 \times 10^{-28}\,\mathrm{gm\,cm^{-3}}$ (just assuming that $t_0 > 5\,\mathrm{Gyr}$, i.e. that the universe is older than the Earth) and inferred that $m_{\nu_e}, m_{\nu_\mu} < 400$ eV for a present photon temperature of 3 K. Their calculation of the relic neutrino abundance was rather ap-

proximate — they adopted $g_\nu = 4$ i.e. assumed massive neutrinos to be Dirac particles with fully populated right-handed (RH) states (even though they commented *en passim* that according to the $V - A$ theory such states are non-interacting and would thus not be in equilibrium at T_{dec}) and moreover they did not allow for the decrease in the neutrino temperature relative to photons due to e^+e^- annihilation. Nevertheless their bound was competitive with the best laboratory bound on m_{ν_e} and 10^4 times better than that on m_{ν_μ}, demonstrating the sensitivity (if not the precision!) of cosmological arguments.

A better bound of $m_{\nu_\mu} < 130$ eV was quoted by Marx and Szalay[15] who numerically integrated the cosmological Friedmann equation from ν_μ decoupling down to the present epoch, subject to the condition $t_0 > 4.5\,\mathrm{Gyr}$. Independently Cowsik and McClleland[16] used direct limits on Ω_m and h to obtain an even more restrictive bound of $m_\nu < 8$ eV, assuming that $m_\nu = m_{\nu_e} = m_{\nu_\mu}$; however they too assumed that $T_\nu = T$ and that RH states were fully populated. As Shapiro, Teukolsky and Wasserman[17] first emphasized, even if massive neutrinos are Dirac rather than Majorana, the RH states have no gauge interactions so should have decoupled much earlier than the left-handed ones. Then subsequent entropy generation by massive particle annihilations would have diluted their relic abundance to a negligible level.[d] Thus we arrive at the modern version of the 'Gershteĭn-Zel'dovich' bound'[20]: the conservative limits $t_0 > 10\,\mathrm{Gyr}$ and $h > 0.4$ imply[21] $\Omega_m h^2 < 1$ i.e. $\rho_m < 10.54\,\mathrm{keV\,cm^{-3}}$; combining this with the relic neutrino number density, which is $\sim 1\%$ larger[9] than in eq.1, gives:

(If neutrinos are non-relativistic at decoupling, then they drop out of chemical equilibrium with an abundance inversely proportional to their self-annihilation cross-section so $\Omega_\nu h^2 \approx (m_\nu/2\,\mathrm{GeV})^{-2}$ for $m_\nu \ll m_Z$[22, 23]. Thus neutrinos with a mass of $\mathcal{O}(\mathrm{GeV})$ can also ac-

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[a] In fact $T_{\mathrm{dec}}(\nu_\mu, \nu_\tau) \simeq 3.5\,\mathrm{MeV}$ while $T_{\mathrm{dec}}(\nu_e) \simeq 2.3\,\mathrm{MeV}$ because of the additional charged current reaction[6, 7]. Actually decoupling is not an instantaneous process so the neutrinos are slightly heated by the subsequent e^+e^- annihilation, increasing the number density (1) by $\approx 1\%$[8, 9].

[b] These authors[11] were also the first to suggest searching for high energy neutrinos by looking for upward going muons in underground experiments, the basis for today's neutrino telescopes.

[c] The critical density is $\rho_c = 3H_0^2/8\pi G_N \simeq 1.879 \times 10^{-29} h^2\,\mathrm{gm\,cm^{-3}}$ where the Hubble parameter $h \equiv H_0/100\,\mathrm{km\,sec^{-1}\,Mpc^{-1}}$, so the present Hubble age is $H_0^{-1} = 9.778 h^{-1}$ Gyr.

[d] Although spin-flip scattering (at a rate $\propto (m_\nu/T)^2$) *can* generate RH states, this can be neglected for $m_\nu \ll 1\,\mathrm{MeV}$. Even if RH neutrinos have new (superweak) interactions, their relic abundance can be no more than $\sim 10\%$ of LH neutrinos so the bound on their mass (5) is relaxed to ~ 1 keV[18] Even if the (post-inflationary) universe was not hot enough to bring such interactions into equilibrium, a cosmologically interesting abundance can still be generated if the RH states have small mixings with LH states[19].

count for the dark matter; however LEP has ruled out such (possible 4th generation) neutrinos upto a mass[2] $\sim m_Z/2$. (Conversely $\Omega_\nu h^2 > 1$ for the mass range $\sim 100\,\text{eV} - 2\,\text{GeV}$, which is thus cosmologically *forbidden* for any stable neutrino having only electroweak interactions[24, 25].) For heavier masses upto the highest plausible value of $\mathscr{O}(\text{TeV})$, the relic abundance decreases steadily due to the increasing annihilation cross-section and remains cosmologically uninteresting[26].)

$$\Omega_\nu h^2 = \sum_i \left(\frac{m_{\nu_i}}{93\,\text{eV}}\right)\left(\frac{g_{\nu_i}}{2}\right) < 1. \qquad \ldots (5)$$

This is a rather conservative bound since galaxy surveys indicate much tighter constraints on the total amount of gravitating (dark) matter in the universe, e.g. the observed 'redshift space distortion'[27] suggests $\Omega_\text{m} \approx 0.3$, averaged over a volume extending several hundred Mpc. Together with the Hubble Key Project determination of $h = 0.72 \pm 0.08$[28], this implies that the sum of all neutrino masses cannot exceed about 15 eV.

Although this has historically been the most restrictive constraint on neutrino masses, it is no longer competetive with the direct laboratory bound on the electron neutrino mass from the Mainz and Troitsk tritium β-decay experiments[29]:

$$m_\nu < 2.2 \text{ eV} (95\% \text{ c.l.}). \qquad \ldots (6)$$

Although the kinematic mass limits on the other neutrino flavours are much weaker (viz. $m_{\nu_\mu} < 190$ keV, $m_{\nu_\tau} < 18.2$ MeV[2]), the bound above now applies in fact to *all* eigenstates[30] given the rather small mass-differences indicated by the oscillation interpretation of the Solar ($\Delta m^2 \simeq 7 \times 10^{-5}\,\text{eV}^2$) and atmospheric ($\Delta m^2 \simeq 3 \times 10^{-3}\,\text{eV}^2$) neutrino anomalies[31]. This means that the laboratory limit on the sum of neutrino masses (for comparison with the new cosmological bounds to be discussed) is presently 6.6 eV; the sensitivity of planned future experiments is at the eV level[32]. Moreover relic neutrinos contribute at least $\Omega_\nu h^2 \sim 0.07/93 \sim 8 \times 10^{-4}$ to the cosmic budget (assuming a mass hierarchy), nearly as much as visible baryons[33].

2 Neutrinos as the Galactic 'Missing Mass'

The cosmological bound (5) assumes conservatively that neutrinos constitute *all* of the (dark) matter permitted by the dynamics of the universal Hubble expansion. Further constraints must be satisfied if they are to cluster on a specified scale (e.g. galactic halos or galaxy clusters) and provide the dark matter whose presence is inferred from dynamical measurements. Cowsik and McClleland[34] were the first to suggest that neutrinos with a mass of a few eV could naturally be the 'missing mass' in clusters of galaxies. This follows from the relation $m_\nu^8 \simeq 1/G_\text{N}^3 r_\text{cl}^3 M_\text{cl}$ (reflecting the Pauli principle) which they obtained by modeling a cluster of mass M_cl as a square potential well of core radius r_cl filled with a Fermi-Dirac gas of neutrinos at zero temperature. Subsequently Tremaine and Gunn[35] noted that this provides a *lower* bound on the neutrino mass. Although the microscopic phase-space density (2) is conserved for collisionless particles, the 'coarse-grained' phase-space density in bound objects can decrease below its maximum value of $g_\nu/2(2\pi)^3$ during structure formation. Modeling the bound system as an isothermal sphere with velocity dispersion σ and core radius $r_\text{cl}^2 = 9\sigma^2/4\pi G_\text{N}\rho(r_\text{cl})$ then gives

$$m_\nu > 120\,\text{eV} \left(\frac{\sigma}{100\,\text{km sec}^{-1}}\right)^{-1/4} \left(\frac{r_\text{cl}}{\text{kpc}}\right)^{-1/2} \ldots (7)$$

This is indeed consistent with the cosmological upper bound (5) down to the scale of galaxies, however there is a conflict for smaller objects, viz. dwarf galaxies which require a *minimum* mass of ~ 100 eV[36, 37]. In fact the central phase space density of observed dark matter cores in these structures decreases rapidly with increasing core radius, rather than being constant as would be expected for neutrinos[38, 39, 40]. Moreover since neutrinos would cluster more efficiently in larger potential wells, there should be a trend of increasing mass-to-light ratio with scale. This was indeed claimed to be the case initially[41] but later it was recognised that the actual increase is far less than expected[42]. Thus massive neutrinos are now disfavoured as the constituent of the 'missing mass' in galaxies and clusters.

3 The Rise and Fall of 'Hot Dark Matter'

Nevertheless such cosmological arguments became of particular interest in the 1980's after the ITEP tritium β-decay experiment claimed a ~ 30 eV mass for the electron neutrino. The attention of cosmologists turned to how the large-scale structure (LSS) of galaxies, clusters and superclusters would have formed if

the universe is indeed dominated by such massive neutrinos. The basic picture[43,44] is that structure grows through gravitational instability from primordial density perturbations; these perturbations were first detected by the *COsmic Background Explorer* (COBE) via the temperature fluctuations they induce in the CMB[45]. On small scales ($\lesssim 10\,$Mpc) structure formation is complicated by non-linear gravitational clustering as well as non-gravitational (gas dynamic) processes but on large scales gravitational dynamics is linear and provides a robust probe of the nature of the dark matter.

Density perturbations in a medium composed of relativistic collisionless particles are subject to a form of Landau damping (viz. phase-mixing through free streaming of particles from high to low density regions) which effectively erases perturbations[46-48] on scales smaller than the free-streaming length $\sim 41\,$Mpc$(m_\nu/30\,$eV$)^{-1}$. This is essentially the (comoving) distance traversed by a neutrino from the Big Bang until it becomes non-relativistic, and corresponds to the scale of superclusters of galaxies. Thus huge neutrino condensations (generically in the shape of 'pancakes'), containing a mass $\sim 3 \times 10^{15}M_\odot(m_\nu/30\,eV)^{-2}$, would have begun growing at a redshift $z_{eq} \sim 7 \times 10^3 (m_\nu/30\,eV)$ when the universe became matter-dominated and gravitational instability set in. This is well before the epoch of (re)combination at $z_{rec} \sim 10^3$ so the baryons were still closely coupled to the photons, while the neutrinos were mildly relativistic ($v/c \sim 0.1$) hence 'hot'. After the universe became neutral, baryonic matter would have accreted into these potential wells, forming a thin layer of gas in the central plane of the pancakes. Thus superclusters would be the first objects to condense out of the Hubble flow in a 'hot dark matter' (HDM) cosmogony, and smaller structures such as galaxies would form only later through the fragmentation of the pancakes.

The gross features of such a 'top-down' model for structure formation are compatible with several observed features of LSS, in particular the distinctive 'voids' and 'filaments' seen in large galaxy surveys. It was also noted that since primordial density perturbations can begin growing earlier than in an purely baryonic universe, their initial amplitude must have been smaller, consistent with extant limits on the isotropy of the microwave background. Detailed studies[49-52] found however that galaxies form too late through

the breakup of the pancakes, at a redshift $z \lesssim 1$, counter to observations of galaxies and quasars at $z > 4$. (Another way of saying this is that galaxies should have formed *last* in an HDM universe, whereas our Galaxy is in fact dynamically much older than the local group[53].) There are other difficulties such as too high 'peculiar' (i.e. non-Hubble) velocities[54], excessive X-ray emission from baryons which accrete onto neutrino clusters[55], and too large voids[56] (although detailed simulations[57-59] showed later that some of these problems had perhaps been exaggerated).

Therefore cosmologists abandoned HDM and turned, with considerably more success, to cold dark matter (CDM), i.e. particles which were non-relativistic at the epoch of matter-domination[60,61]. Detailed studies of CDM universes gave excellent agreement with observations of galaxy clustering and a 'standard CDM' model for large-scale structure formation was established, viz. a critical density CDM dominated universe with an initially scale-invariant spectrum of density perturbations[62-63]. Moreover particle physicists provided plausible candidate particles, notably the neutralino in supersymmetric models with conserved *R*-parity which naturally has a relic abundance of order the critical density[64].

4 COBE and the Advent of 'Mixed Dark Matter'

Nevertheless neutrinos were resuscitated some years later as a possible (sub-dominant) component of the dark matter when the CDM cosmogony itself ran into problems. To appreciate the background to this it is necessary to recapitulate the essential ingredients of a model for cosmic structure formation. A key assumption made concerns the nature of the primordial density perturbations which grow through gravitational instability in the dark matter. Cosmologists usually assume such fluctuations to have a power spectrum of the *scale-free* form:

$$P(k) = \langle |\delta_k|^2 \rangle = Ak^n, \qquad \ldots(8)$$

with $n = 1$ corresponding to the scale-invariant 'Harrison-Zel'dovich' spectrum. Here $\delta_k \equiv \int \frac{\delta\rho(\vec{x})}{\bar{\rho}} e^{-i\vec{k}\cdot\vec{x}} d^3x$ is the Fourier transform of spatial fluctuations in the density field (of wavelength $\lambda = 2\pi/k$). Moreover the perturbations are assumed to be gaussian (i.e. different phases in the plane-wave expansion are uncorrelated) and to be 'adiabatic' (i.e. matter and radiation fluctuate together). Concurrent

with the above studies concerning the nature of the dark matter, powerful support for this conjecture was provided by the development of the 'inflationary universe' model[21, 65]. Here the perturbations arise from quantum fluctuations of a scalar field ϕ, the vacuum energy of which drives a period of accelerated expansion in the early universe. The classical density perturbation thus generated has a spectrum determined by the 'inflaton' potential $V(\phi)$, with a power-law index which is dependent on k[66]:

$$n(k) = 1 - 3M^2 \left(\frac{V'}{V}\right)^2_\star + 2M^2 \left(\frac{V''}{V}\right)_\star \quad \dots (9)$$

where $M \equiv (8\pi G_{\mathrm{N}})^{-1/2} \simeq 2.4 \times 10^{18}\,\mathrm{GeV}$ is the Planck mass and '$\star$' denotes that this is to be evaluated when a mode of wavenumber k crosses the Hubble radius H^{-1}. For a sufficiently 'flat potential' (as is necessary to drive enough e-folds of inflation to solve the problems of the standard cosmology), the spectrum indeed has $n \simeq 1$, with corrections $\propto \ln(k)$.

Gravitational instability sets in only when the universe becomes matter-dominated and this modifies the spectrum on length scales smaller than the Hubble radius at this epoch, viz. for $k > k_{\mathrm{eq}}^{-1} \simeq 80h^{-1}\,\mathrm{Mpc}$. Thus the characteristics of the dark matter can be encoded into a 'transfer function' $T(k)$ which modulates the primordial spectrum; for HDM this is an exponentially falling function while for CDM it is a more gradual power-law. Now the power spectrum inferred from observations may be compared with theoretical models, but another problem arises concerning how we are to normalize the amplitude of the primordial density perturbations,[e] particularly since these are in the dark matter and may differ significantly (i.e. be 'biased') from the observable fluctuations in the density of visible galaxies. Fortunately, the primordial perturbations have another unique observational signature, viz. they induce temperature fluctuations in the CMB through the 'Sachs-Wolfe effect' (gravitational red/blue shifts) on large angular scales $\gtrsim 2^0$, corresponding to spatial scales larger than the Hubble radius on the last scattering surface[67]. It was the COBE measurement of these fluctuations a decade ago that initiated the modern era of cosmological structure formation studies.

The quadrupole anisotropy in the CMB measured by COBE[45] allows a determination of the fluctuation amplitude at the scale, $H_0^{-1} \simeq 3000h^{-1}\,\mathrm{Mpc}$, corresponding to the present Hubble radius. With this normalization it became clear that a $\Omega_\nu \simeq 1$ HDM universe indeed had too little power on small-scales for adequate galaxy formation.[f] However it also became apparent that the 'standard CDM model' when normalized to COBE had too much power on small-scales (see Fig. 1). It was thus a logical step to invoke a suitable mixture of CDM and HDM to try and match the theoretical power spectrum to the data on galaxy clustering and motions[73-75].

In fact the possibility that the dark matter may have both a hot and a cold component had been discussed several years earlier by Shafi and Stecker, motivated by theoretical considerations of SUSY GUTs[77], and the possibilities for solving the problems of the standard CDM model had been noted[78]. In the post-COBE era, a number of detailed studies of mixed dark matter (MDM) universes were performed and a neutrino fraction of about 20% was found to give the best match with observations[79-84]. The implied neutrino mass was $\sim 5\,\mathrm{eV}$, presumably that of the ν_τ given the usual hierarchy implied by the see-saw mechanism[85]. The suppression of small-scale power delays the epoch of galaxy formation, so constraints on the HDM component can also be obtained from the abundance of high redshift objects such as QSOs and Ly-α systems; this gave an *upper* bound of 4.7 eV on the neutrino mass[86]. More baroque schemes in which two neutrinos have comparable masses ($m_{\nu_\mu} \sim m_{\nu_\tau} \sim 2.5\,\mathrm{eV}$) were also constructed[87] seeking to reconcile the LSND report of neutrino oscillations[88] with the atmospheric neutrino anomaly which had just been reported by Kamiokande[89]. This was an exciting time for neutrino cosmology as both laboratory data and astronomical observations supported the possibility that a substantial fraction of the cosmological mass density is in the form of massive neutrinos.

However another way to reconcile a CDM universe with the small-scale observations is to relax the underlying assumption that the primordial spectrum is strictly *scale-invariant*. As shown in Fig.1, a 'tilted' spectrum with $P(k) \propto k^{0.9}$ also gives a good fit to the data[90]. At first sight this might strike one as simply introducing an additional parameter (although this is arguably no worse than introducing an additional form

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[e]   This is determined by the inflaton potential $V(\phi)$ but there is, as yet, no 'standard model' of inflation[66].

[f]   To save HDM would require new sources of small-scale fluctuations, e.g. relic topological defects[68, 69, 70], or *isocurvature* primordial perturbations[71, 72] — to date however there is no evidence for either.
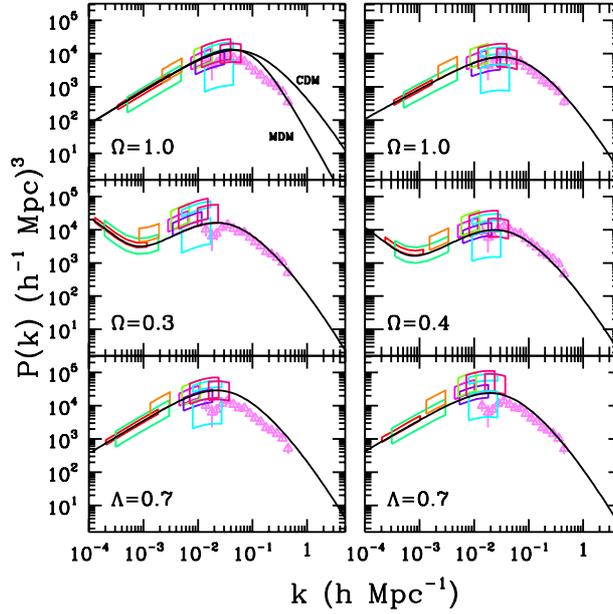
Fig. 1 The matter power spectrum as inferred from LSS and CMB data is compared with the CDM cosmogony and several variants in this figure reproduced from Scott, White and Silk [76]. We see top left that the excess small-scale power in the COBE-normalized 'standard' CDM model with scale-invariant adiabatic fluctuations ($n = 1, \Omega_B = 0.03, h = 0.5$), is reduced in the MDM model which has a substantial neutrino component ($\Omega_\nu = 0.3$). This can also be achieved by 'tilting' the spectrum to $n = 0.9$ as shown top right ($\Omega_B = 0.1, h = 0.45$). The middle panels show the expectations in open universes, while the bottom panels correspond to a flat universe with a cosmological constant (the bottom right figure assumes a significant gravitational wave contribution to the COBE signal).

of dark matter). However one should really ask why the spectrum should be assumed to have a power-law index $n = 1$ in the first place. As indicated in eq.9, n in fact varies slowly with k and is determined by the slope and curvature of the scalar potential at the epoch when the fluctuation at a specified value of k crosses the Hubble radius. The corresponding number of e-folds before the end of inflation is just $N_\star(k) \simeq 51 + \ln\left(\frac{k^{-1}}{3000h^{-1}\,\mathrm{Mpc}}\right)$, for typical choices of the inflationary scale, reheat temperature etc. We see that fluctuations on the scales ($\sim 1 - 3000\,\mathrm{Mpc}$) probed by LSS and CMB observations are generated just $40 - 50$ e-folds before the end of inflation. It is quite natural to expect the inflaton potential to begin curving significantly as the end of inflation is approached, especially in 'new inflation' (small field) models. There are certainly attractive models of inflation in which the spectrum is significantly tilted in this region, in particular an inflationary model based on $N = 1$ supergravity[91] naturally gives $n(k) \simeq (N_\star - 2)/(N_\star + 2) \sim 0.9$ at these scales. With such a *scale-dependent* tilt for the primordial spctrum, the LSS data can be fitted reasonably well, with no need for any HDM component[92]. (Of course in the absence of a 'standard model' of in-

flation, it might be argued that the inflationary spectrum may instead have $n > 1$ thus allowing a *larger* HDM component.)

As Fig.1 shows, yet another way of suppressing small-scale power in the CDM cosmogony is to decrease the matter content of the universe, since this postpones the epoch of matter-radiation equality and thus shifts the peak of power spectrum to larger scales. Furthermore the spatial geometry can be maintained flat if there is a cosmological constant with $\Omega_\Lambda = 1 - \Omega_m \sim 0.7$. Evidence for just such a cosmology (ΛCDM) has come subsequently from observations of the Hubble diagram of Type Ia supernovae which suggest that the expansion is in fact *accelerating*[93, 94], coupled with the observation that Ω_m does not exceed ~ 0.3 even on the largest scales probed[95]. Assuming such a ΛCDM cosmology and a scale-invariant power spectrum gives a limit of $f_\nu \equiv \Omega_\nu/\Omega_m < 0.13$ (95% c.l.) using the power spectrum of galaxy clustering determined from the *Two degree Field galaxy redshift survey* (2dFGRS) as shown in Fig. 2. This corresponds to an upper bound of $\sum m_\nu < 1.8$ eV(95% c.l.) on the sum of neutrino masses, adopting 'concordance' values of Ω_m and h[97]. If the spectral index is al-
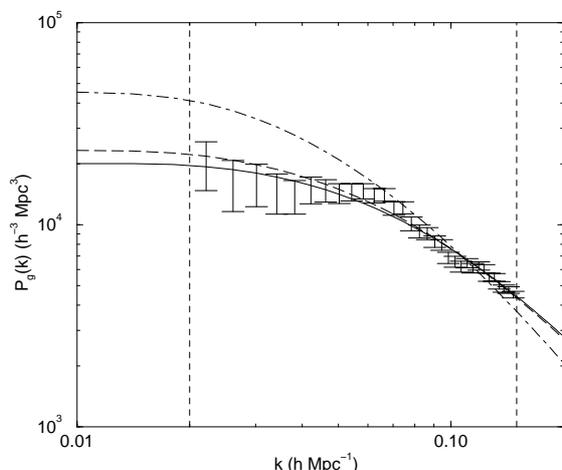
Fig. 2 The 2dFGRS power spectrum of galaxy clustering is compared with the expectations for no neutrino dark matter (solid line), $\Omega_\nu = 0.01$ (dashed line) and $\Omega_\nu = 0.05$ (dot-dashed line). A scale-invariant spectrum is assumed and the cosmological parameters adopted are $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$, $h = 0.7$, $\Omega_B h^2 = 0.02$ (from Elgaroy *et al.* [97]).

lowed to vary in the range $n = 1 \pm 0.1$, then this bound is relaxed to 2.1 eV.

5 CMB Anisotropy and Limits on Neutrino Dark Matter

It is clear that there are uncertainties in the determination of the HDM component from LSS data alone. Fortunately it is possible to reduce these substantially by examining an independent probe of the primordial power spectrum, viz. temperature fluctuations in the CMB.

In general a skymap of the CMB temperature can be decomposed into spherical harmonics

$$T(\theta,\phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_l^m Y_l^m(\theta,\phi), \qquad \ldots (10)$$

where the lth multipole corresponds to an angle $\theta^\circ \sim 200/l$ and probes spatial scales around $k^{-1} \sim 6000 h^{-1} l^{-1}$ Mpc. In inflationary theories, the fluctuations are gaussian so the co-efficients a_l^m are independent stochastic variables with zero mean and variance $C_l = \langle |a_l^m|^2 \rangle$; each C_l has a χ^2 distribution with

$(2l + 1)$ degrees of freedom[60, 98]. For an assumed set of cosmological parameters and given the primordial density perturbation spectrum, the C_l's can be determined by solution of the Einstein-Boltzmann equations which describe how the different components (photons, ions, electrons, neutrinos ...) evolve[99-101]. Thus theoretical estimates of the power at each multipole can be compared with observations. The low multipoles (large spatial scales) are sensitive to the primordial spectrum alone,[g] but the measurements in the region are particularly uncertain, both because of uncertainties in the foreground substraction and also because there are fewer independent measurements for low multipoles ('cosmic variance'). For example, by measuring the first ∼20 multipoles COBE could only fix $n = 1.2 \pm 0.3$[45] so could not discriminate between a scale-invariant and a mildly tilted spectrum. However subsequent ground-based experiments with angular resolution far superior to COBE's have now measured the power at higher multipoles[103]. The dominant features in the power spectrum here are the 'acoustic peaks', the most prominent being at $l \sim 200$, arising from oscillations of the coupled plasma-photon fluids at last scattering[104]. The position of the first peak is a measure of the horizon length at the epoch of (re)combination of the primordial plasma and thus provides a measure of space curvature— observations by the BOOMERanG[105] and MAXIMA[106] experiments were the first to show that this is in fact close to zero. Taken together with the earlier recognition that CDM alone cannot make up the critical density, this has led to the widespread adoption of the ΛCDM model in which $\Omega_m \sim 0.3$, $\Omega_\Lambda \sim 0.7$[95, 96].

As seen in Fig. 3, the expectations for CMB anisotropy in a MDM universe do not differ significantly from a CDM universe having the same initial perturbation spectrum. Thus it is clear that by combining CMB and LSS data, it would be possible to determine whether the suppression of small-scale power is *intrinsic* to the primordial spectrum of inflationary fluctuations, or is induced by a HDM component. However there are additional uncertainties in the input values of the other cosmological parameters, in particular the values of the Hubble parameter and of the baryon density which is usually inferred from considerations of primordial nucleosynthesis. In analysing the observational data, various 'priors' are

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[g] In principle, primordial gravitational waves can also make a contribution here but this is expected to be negligible in 'small-field' inflationary models[91, 102].
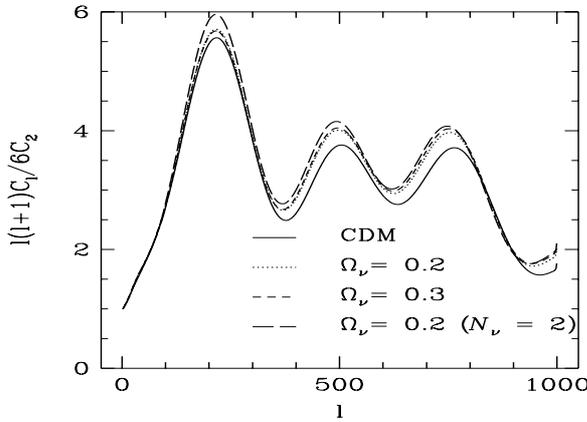
Fig. 3 The angular power spectrum of CMB anisotropy (assuming a scale-invariant spectrum) in CDM and MDM universes, as calculated by Dodelson, Gates and Stebbins [107].

Fig. 4 The marginalized cumulative probability of $\Omega_\nu h^2$ is shown, based on a fit to the WMAP data on CMB anisotropies, together with the 2DFGRS data on of galaxy clustering (from Spergel *et al.* [113], courtesey of WMAP Science Team).

often assumed for these quantities on the basis of other observations. For example a detailed likelihood analysis[108] yielded the neutrino density fraction f_ν to be at most 0.08 from CMB observations alone, decreasing to 0.06 if LSS data from the PSCz survey is added, and further to 0.04 if the value of $h = 0.72 \pm 0.08$[28] is adopted. Another analysis[109] used the more precise 2dFGRS data[97], together with additional constraints on the matter density from the SNIa data ($\Omega_m = 0.28 \pm 0.14$ assuming a flat universe)[93, 94], and on the baryon density from primordial nucleosynthesis ($\Omega_B h^2 = 0.02 \pm 0.002$ (95% c.l.)[111]); using the analytic result for the suppression of the power spectrum, $\Delta P / P \simeq -8 f_\nu$[110], a bound of $\sum m_\nu < 2.5 - 3$ eV was obtained.

It had been estimated[110] that the CMB data expected from the *Wilkinson Microwave Anisotropy Probe* (WMAP)[112], combined with LSS data from the Sloane Digital Sky Survey (SDSS)[118], will provide sensitivity to neutrino mass at the eV level. Indeed the recently announced first results from WMAP, combined with 2dFGRS, have already set the bound $\Omega_\nu h^2 < 0.0076$ corresponding to $\sum m_\nu < 0.7$ eV(95% c.l.), as shown in Fig. 4[113]. This severely restricts[114], but does not altogether rule out[115], a fourth (sterile) neutrino with a mass of $\mathcal{O}(1)$ eV as suggested by the LSND experiment[88]. Combined with the new KamLAND data, this also restricts the accessible range for the observation of neutrinoless $\beta\beta$ decay[116], which has been claimed to have been seen already implying an effective Majorana mass of $|\langle m_\nu \rangle| = 0.39^{+0.45}_{-0.34}$ eV (95% c.l.)[117].

6   The Nucleosynthesis Limit on N_ν

Hoyle and Taylor[119] as well as Peebles[120] had emphasized many years ago that new types of neutrinos (beyond the ν_e and ν_μ then known) would boost the relativistic energy density hence the expansion rate (3) during Big Bang nucleosynthesis (BBN), thus increasing the yield of ^{4}He. Shvartsman[121] noted that new superweakly interacting particles would have a similar effect. Subsequently this argument was refined quantitatively by Steigman, Schramm and Gunn[122]. In the pre-LEP era when the laboratory bound on the number of neutrino species was not very restrictive[123], the BBN constraint was used to argue that at most one new family was allowed[124, 125], albeit with considerable uncertainties[126]. Although LEP now finds $N_\nu = 2.994 \pm 0.012$[2], the cosmological bound is still important since it is sensitive to *any* new light particle[127], not just $SU(2)_L$ doublet neutrinos, so is a particularly valuable probe of new physics, e.g. neutrinos coupled to new gauge bosons expected in string models[126, 128].

BBN limits on neutrinos come mainly from the observational bounds on the primordial ^{4}He abundance, termed Y_p by astronomers. This is proportional to the neutron fraction which 'freezes out' at $n/p \simeq \exp[-(m_n - m_p)/T_{fr}]$ when the weak interaction rate ($\propto G_F^2 T^5$) falls behind the Hubble expansion rate (3), at $T_{fr} \sim (g_* G_N / G_F^4)^{1/6} \approx 1$ MeV. The pres-

ence of additional neutrino flavors (or of any other relativistic species) at this time increases g_*, hence the expansion rate H, leading to a larger value of T_{fr}, n/p, and thus Y_p. In the Standard Model, the number of relativistic particle species at 1 MeV is

$$g_* = 5.5 + \tfrac{7}{4}N_\nu, \qquad \ldots(11)$$

where 5.5 accounts for photons and $e^\pm$, and N_ν is the number of (massless) neutrino flavors. (The energy density of new light fermions i is equivalent to an effective number $\Delta N_\nu = \sum_i (g_i/2)(T_i/T_\nu)^4$ of additional doublet neutrinos, where T_i/T_ν follows from considerations of their (earlier) decoupling.) In Fig. 5, we show the expected abundance of ^{4}He taking $N_\nu = 3$, as a function of the density of baryons normalized to the blackbody photon density:

$$\eta \equiv \frac{n_{\text{B}}}{n_\gamma} \simeq 2.728 \times 10^{-8}\Omega_{\text{B}}h^2, \qquad \ldots(12)$$

with $\eta_{10} \equiv \eta/10^{-10}$. The computed ^{4}He abundance scales linearly with ΔN_ν but it also increases logarithmically with η[129, 130]:

$$\Delta Y \simeq 0.012\Delta N_\nu + 0.01\ln\left(\frac{\eta_{10}}{5}\right). \quad \ldots(13)$$

Thus to obtain a bound on N_ν requires an upper limit on Y_p *and* a lower limit on η. The latter is poorly determined from direct observations of luminous matter[33] so must be derived from the abundances of the other synthesized light elements, D, ^{3}He and ^{7}Li, which are power-law functions of η as seen in Fig. 5. The complication is that these abundances are substantially altered in a non-trivial manner during the chemical evolution of the galaxy, unlike $Y_p(^4$He$)$ which just increases by a few percent due to stellar production. (This can be tagged via the correlated abundance of oxygen and nitrogen which are made *only* in stars.) Even so, chemical evolution arguments have been used to limit the primordial abundances of D and ^{3}He and thus derive increasingly severe bounds on N_ν [131-133], culminating in one[134] *below* 3! However a more conservative view[135] is that there is no 'crisis' with BBN if we recognize that chemical evolution arguments are unreliable and consider only *direct* measurements of light element abundances.

Fig. 5 shows the current status of such measurements, as reviewed in more detail elsewhere[2, 136]. We observe ^{4}He in clouds of ionized hydrogen (H II regions), the most metal-poor of which are in dwarf blue compact galaxies (BCGs). There is now a large body of data on ^{4}He and C, N, O in these systems which confirm that the small stellar contribution to helium is positively correlated with 'metal' production; extrapolating to zero metallicity gives the primordial ^{4}He abundance[137]:

$$Y_p = 0.238 \pm 0.002 \pm 0.005 . \qquad \ldots(14)$$

Here and subsequently, the first error is statistical, and the second is an estimate of the systematic uncertainty. The latter clearly dominates, and is based on the scatter in different analyses of the physical properties of the H II regions[138, 139]. Other extrapolations to zero metallicity give $Y_p = 0.244 \pm 0.002$ [140], and $Y_p = 0.235 \pm 0.003$[141], while the average in the 5 most metal-poor objects is $Y_p = 0.238 \pm 0.003$[142]. The value in eq.14, shown in Fig. 5, is consistent with all these determinations.

The systems best suited for Li observations are hot, metal-poor stars belonging to the halo population (Pop II) of our Galaxy. Observations have long shown that Li does not vary significantly in such stars having metallicities less than 1/30 of Solar — the 'Spite plateau'. Recent precision data suggest a small but significant correlation[144] between Li and Fe[143] which can be understood as the result of Li production from cosmic rays. Extrapolating to zero metallicity one arrives at a primordial value[145]

$$\text{Li/H}|_p = (1.23 \pm 0.06^{+0.68}_{-0.32}{}^{+0.56}) \times 10^{-10}\ldots(15)$$

The last error is our estimate of the maximum upward correction necessary to allow for possible destruction of Li in Pop II stars, due to e.g. mixing of the outer layers with the hotter interior[146, 147]. Such processes can be constrained by the absence of significant scatter in the Li-Fe correlation plot[143], and through observations[144] of the even more fragile isotope ^{6}Li.

In recent years, high-resolution spectra have revealed the presence of D in quasar absorption systems (QAS) at high-redshift, via its isotope-shifted Lyman-α absorption. It is believed that there are no astrophysical sources of deuterium, so any measurement of D/H provides a lower limit to the primordial abundance and thus an upper limit on η; for example, the local interstellar value[148] of $\text{D}/\text{H} = (1.5 \pm 0.1) \times 10^{-5}$ requires $\eta_{10} \leq 9$. Early reports of $\text{D}/\text{H} > 10^{-4}$ towards 2 quasars (Q0014+813[149] and PG1718+4807[150]) have been undermined by later
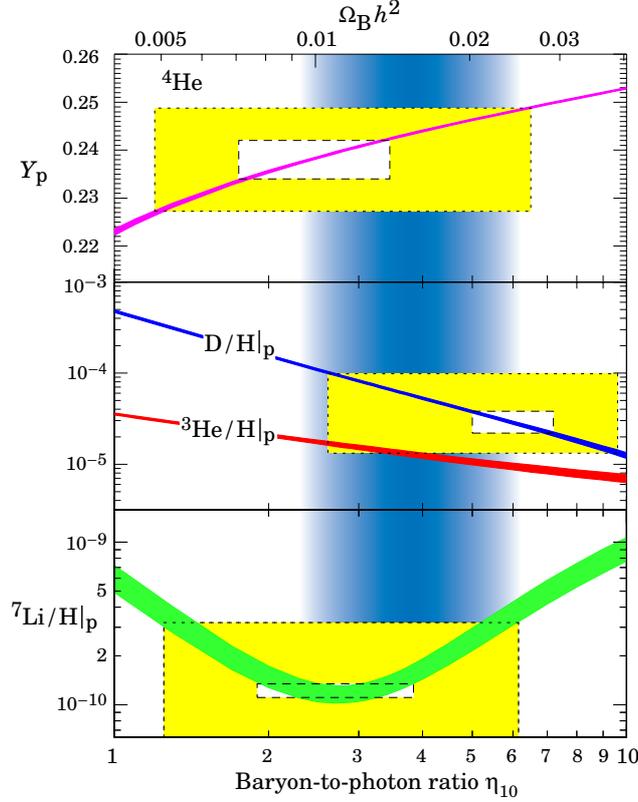
Fig. 5 The primordial abundances of ^{4}He, D, ^{3}He and ^{7}Li as predicted by the standard BBN model compared to observations — smaller boxes: 2σ statistical errors; larger boxes: $\pm 2\sigma$ statistical and systematic errors added in quadrature (from Fields and Sarkar [2]).

analyses[151, 152]. Three high quality observations yield $D/H = (3.2 \pm 0.3) \times 10^{-5}$ (PKS1937-1009), $(4.0 \pm 0.7) \times 10^{-5}$ (Q1009+2956), and $(2.5 \pm 0.2) \times 10^{-5}$ (HS0105+1619); their average value

$$D/H = (3.0 \pm 0.4) \times 10^{-5} \qquad \dots (16)$$

has been widely promoted as *the* primordial abundance[153]. Recently, the same group[154] have provided another measurement: $D/H = (2.42^{+0.35}_{-0.25}) \times 10^{-5}$ (HS 243+3057). However the observed dispersion in the measurements suggests either that systematic uncertainties have been underestimated, or that there is intrinsic dispersion in the D abundance in QAS. Other values have been reported in different (damped Lyman-α) systems which have a higher column density of neutral H, viz. $D/H = (2.24 \pm 0.67) \times 10^{-5}$ (Q0347-3819)[155] and $D/H = (1.65 \pm 0.35) \times 10^{-5}$ (Q2206-199)[156]. Moreover, allowing for a more complex velocity structure than assumed in these analyses raises the inferred abundance by upto $\sim 50\%$[157]. Even the ISM value of D/H now shows unexpected scatter[158] of a factor of 2. All this may indicate significant processing of the D abundance even at high redshift.

Given these uncertainties, it is prudent to bound the primordial abundance with an upper limit set by the non-detection of D absorption in a high-redshift system (Q0130-4021)[159], and the lower limit set by the local interstellar value[148], both at 2σ:

$$1.3 \times 10^{-5} < D/H|_p < 9.7 \times 10^{-5}. \quad \dots (17)$$

For ^{3}He, the only observations available are in the Solar system and (high-metallicity) H II regions in our Galaxy[160]. This makes inference of the primordial abundance difficult, a problem compounded by the fact that stellar nucleosynthesis models for ^{3}He are in conflict with observations[161]. Such conflicts can perhaps be resolved if a large fraction of low mass stars destroy ^{3}He by internal mixing driven by stellar rotation, consistent with the observed ^{12}C/^{13}C ratios[162]. The observed abundance 'plateau' in H II regions then implies a limit[160] on the primordial value of ^{3}He/H $< (1.9 \pm 0.6) \times 10^{-5}$, which is consistent with the other abundance constraints we discuss.

The overlap in the η ranges spanned by the larger boxes in Fig. 5 indicates overall concordance between the various abundance determinations. Accounting for theoretical uncertainties as well as the statistical

and systematic errors in observations, there is acceptable agreement among the abundances when[2]

$$2.6 \leq \eta_{10} \leq 6.2 . \qquad \ldots (18)$$

However the agreement is far less satisfactory if we use only the quoted statistical errors in the observations. As seen in Fig. 5, ^{4}He and ^{7}Li are consistent with each other but favor a value of η which is *discrepant* by $\sim 2\sigma$ from the value $\eta_{10} = 5.9 \pm 0.4$ indicated by the D abundance (16) alone. It is important to note that it is the latter which essentially provides the widely-quoted 'precision' determination of the baryon density: $\Omega_B h^2 = 0.02 \pm 0.002$ (95% c.l.)[111]. Additional studies are required to clarify if this discrepancy is real.

The recent WMAP data on CMB fluctuations implies a baryon abundance of $\eta_{10} = 6.5^{+0.4}_{-0.3}$ if the primordial perturbations are assumed to have a power-law form[113]; the value decreases by $\sim 9\%$ if the spectral index is allowed to vary with scale, as is now indicated by the observations. Although broadly consistent with the determinations based on primordial abundances, it is clear that this exacerbates the tension with the ^{4}He and ^{7}Li measurements mentioned above. It has been proposed that the CMB determination of η_{10} can be used as an input into BBN calculations[163]. However it would be advisable to await further LSS data from SDSS[118] to pin down more precisely the primordial fluctuation spectrum, to which such determinations are sensitive. For example, allowing for a 'step' in the primordial spectrum at $k \sim 0.05h$ Mpc^{-1}, as was indicated by data from the APM galaxy survey[164, 165], can *decrease* the baryon density inferred from the WMAP+2DFGRS data by upto $\sim 40\%$[166].

Given this situation, it is still neccessary to be conservative in evaluating bounds on N_ν from BBN. An analysis based on simple χ^2 statistics and taking into account the correlated uncertainties of the elemental yields, gives[167]

$$2 < N_\nu < 4 \text{ (95\% c.l.)}. \qquad \ldots (19)$$

Tighter bounds can of course be obtained under less conservative assumptions, e.g. adopting the D abundance in eq.16 requires[168] $N_\nu < 3.2$. If true, such bounds rule out the possibility of 'sterile' neutrinos since these would be brought into equilibrium through mixing with the active neutrinos in all currently viable schemes[169, 170]. The WMAP determination of

the baryon density seems to require N_ν less than 3[171] but is consistent with the Standard Model taking systematic uncertainties in the elemental abundances into account[114, 172]. Moreover as emphasized above, the WMAP determination of η will have to be lowered if the primordial spectrum is not scale-free and this will considerably ease the present tension. (The WMAP+2dFGRS data by themselves restrict $1.8 < N_\nu < 5.7$ (95% c.l.) assuming a flat universe[173, 174].)

The above limits hold for the standard BBN model and one can ask to what extent they can be modified if plausible changes are made to the model. For example, there may be an initial excess of electron neutrinos over antineutrinos, parametrised by a 'chemical potential' ξ_e in the distribution function: $f_\nu \propto [\exp(p/T - \xi_e) + 1]^{-1}$. Then $n - p$ equilibrium is shifted in favour of less neutrons, thus reducing the ^{4}He abundance[129]. by a factor $\approx \exp(-\xi_e)$. However the accompanying increase in the relativistic energy density speeds up the expansion rate and increases the n/p ratio at freeze-out, leading to more ^{4}He, although this effect is smaller. For neutrinos of other flavours which do not participate in nuclear reactions, only the latter effect was presumed to operate, allowing the possibility of balancing a small chemical potential in ν_e by a much larger chemical potential in $\nu_{\mu, \tau}$, and thus substantially enlarging[175] the concordance range of η_{10}. However the recent recognition from Solar and atmospheric neutrino experiments that the different flavours are maximally mixed, no longer permits such a hierarchy of chemical potentials[176–179], thus ruling out this possible loophole. Consequently the relic neutrino abundance cannot be significantly different from the value in eq.1. A small chemical potential in electron neutrinos of $\xi_e \lesssim 0.08$ is however still possible and could be another explanation for why the ^{4}He abundance seems to be lower than the value that would be expected on the basis of the measurements of the D abundance or the CMB anisotropy.

Another possible change to standard BBN is to allow inhomogeneities in the baryon distribution, created e.g. during the QCD (de)confinement transition. If the characteristic inhomogeneity scale exceeds the neutron diffusion scale during BBN, then increasing the average value of η increases the synthesised abundances such that the observational limits essentially rule out such inhomogeneities. However fluctuations in η on smaller scales will result in neutrons escaping from the high density regions leading to spatial vari-

ations in the n/p ratio which might allow the upper limit to η to be raised substantially[180]. Recent calculations show that D and ^{4}He can indeed be matched even when η is raised by a factor of ~ 2 by suitably tuning the amplitude and scale of the fluctuations, but this results in unacceptable overproduction[181] of ^{7}Li. A variant on the above possibility is to allow for regions of antimatter which annihilate during or even after BBN; however the ^{7}Li abundance again restricts the possibility of raising the limit on η substantially[182]. Finally the synthesised abundances can be altered if a relic massive particle decays during or after BBN generating electromagnetic and hadronic showers in the radiation-dominated plasma. Interestingly enough the processed yields of D, ^{4}He and ^{7}Li can then be made to match the observations even for a universe closed by baryons[183]; however the production of ^{6}Li is excessive and argues against this possibility.

In summary, standard BBN appears to be reasonably robust (although non-standard possibilities cannot be *definitively* ruled out) and consistent with $N_\nu = 3$, leaving little room for new physics. Systematic un-certainties in the elemental abundance determinations need to be substantially reduced in order to make further progress.

Conclusions

The study of neutrinos in cosmology has had a long history. However as both theory and observations have improved, the fascinating possibilities that had been raised in early work have gradually been eliminated. In particular neutrino (hot) dark matter is no longer required by our present understanding of large-scale structure. Cosmological bounds on neutrino masses continue to be more restrictive than laboratory limits and all known species are now required to be sufficiently light that the rich phenomenology of unstable neutrinos is now largely of historical interest. Moreover there is no motivation from considerations of primordial nucleosynthesis for invoking new sterile neutrinos. Nevertheless such neutrinos can exist (e.g. in the 'bulk' [184, 185]) and there may yet be surprises from the Big Bang in store for us.

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