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NEUTRINOS AND MAGNETIC FIELDS: A SHORT REVIEW

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Neutrinos have no electric charge, but a magnetic field can indirectly affect neutrino properties and interactions through its effect on charged particles. After a brief field-theoretic discussion of charged particles in magnetic fields, we discuss two broad kinds of magnetic field effects on neutrinos. First, effects which come through virtual charged particles and alter neutrino properties. Second, effects which alter neutrino interactions through charged particles in the initial or final state. We end with some discussion about possible physical implications of these effects.

1 Motivation

Neutrinos have no electric charge. So they do not have any direct coupling to photons in any renormalizable quantum field theory. The standard Dirac contribution to the magnetic moment, which comes from the vector coupling of a fermion to the photon, is therefore absent for the neutrino. In the Standard Model of electroweak interactions, the neutrinos cannot have any anomalous magnetic moment either. The reason is simple: anomalous magnetic moment comes from chirality-flipping interactions $\overline{\psi}\sigma_{\mu\nu}\psi F^{\mu\nu}$, and neutrinos cannot have such interactions because there are no right-chiral neutrinos in the Standard Model. The bottom line is: neutrinos do not interact with the magnetic field at all in the Standard Model.

Why then should we discuss the relation between neutrinos and magnetic fields? There are several reasons, which will be discussed in the rest of this section.

We now know that neutrinos are not massless as the Standard Model presupposes. Inclusion of neutrino mass naturally takes us beyond the Standard Model, where the issue of neutrino interactions with a magnetic field must be reassessed. If the massive neutrino turns out to be a Dirac fermion, its right-chiral

projection must be included in the fermion content of the theory, and in that case an anomalous magnetic moment of a neutrino automatically emerges when quantum corrections are taken into account. In the simplest extension of the Standard Model including right-chiral neutrinos, the magnetic moment arises from the diagrams in Fig. 1 and is given by[1]

$$\mu_\nu = \frac{3eG_F m_\nu}{8\sqrt{2}\pi^2} = 3 \times 10^{-19}\mu_B \times \left(\frac{m_\nu}{1\,\text{eV}}\right), \quad \ldots(1)$$

where m_ν is the mass of the neutrino and μ_B is the Bohr magneton.

If, on the other hand, neutrinos have Majorana masses,[a] i.e., they are their own antiparticles, they cannot have any magnetic moment at all, because CPT symmetry implies that the magnetic moments of a particle and its antiparticle should be equal and opposite. However, even in this case there can be transition magnetic moments, which are co-efficients of effective operators of the form $\overline{\psi}_1\sigma_{\mu\nu}\psi_2 F^{\mu\nu}$, where ψ_1 and ψ_2 denote two different fermion fields. These will also indicate some sort of interaction with the magnetic field, associated with a change of the fermion flavor.

The question of the neutrino magnetic moment assumed immense importance when it was suggested that it can be a potential solution for the solar neutrino puzzle[3–5]. A viable solution required a neutrino

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[a] For a detailed discussion on Dirac and Majorana masses of neutrinos, see, e.g., ref.[2].
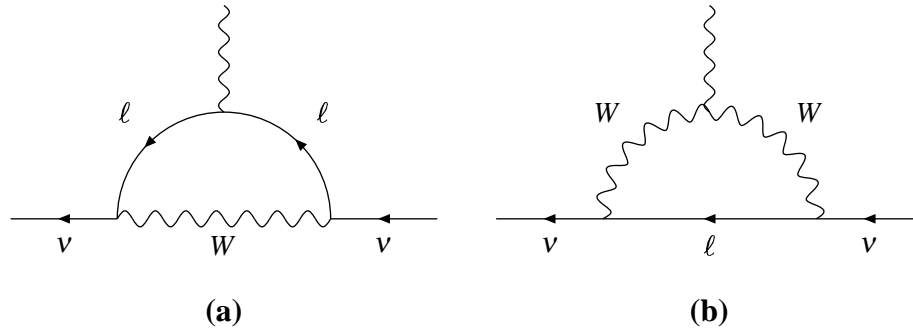
Fig. 1 One-loop diagrams that give rise to neutrino magnetic moment in standard model aided with right-handed neutrinos. The lines marked ν are generic neutrino lines, whereas those marked ℓ are generic charged leptons. The external vector boson line is the photon. In renormalizable gauges, there are extra diagrams where any of the W lines can be replaced by the corresponding unphysical Higgs scalar.

magnetic moment around $10^{-10}\mu_B$, orders of magnitude larger than that given by eq.1, knowing that the neutrino masses cannot be very large. However, such a magnitude could not be ruled out by direct laboratory experiments. A lot of research was carried out to explore possible ways of evading the proportionality between the neutrino mass and magnetic moment as shown in eq.1. Voloshin[6] showed that there may be symmetries to forbid neutrino mass but not its magnetic moment. Thus, if such a symmetry remained unbroken, even massless neutrinos could have had a large magnetic moment. However, the proposed symmetries had to be broken in the real world in order to meet other phenomenological constraints, but in the end it was possible to envisage models where the ratio between the magnetic moment and the mass of the neutrino is much larger than that predicted by eq.1.[b]

In this article, we will not follow the theoretical ideas outlined above, mainly because the phenomenological motivation has become thin. After the various recent solar neutrino experiments, especially the data from the Sudbury Neutrino Observatory[8], no one believes that neutrino magnetic moments solve the solar neutrino problem. However, there may be celestial objects other than the Sun where the interaction between neutrinos and magnetic fields hold the keys to some important questions.

It is not difficult to guess that the most important objects for this purpose are the ones where very high magnetic fields are available. Neutron stars have strong magnetic fields. In fact, the surface magnetic fields are typically of the order of 10^{12} Gauss. In the

core, the field might be larger. Such high magnetic fields exist also in the proto-neutron star, and its interaction with the neutrino might have important effects on the supernova explosion. There are also objects called magnetars whose magnetic field is much higher than that in ordinary neutron stars. Even a small magnetic moment can have a large effect in such systems.

But effects need not come through magnetic moment alone. There may be other physical quantities which, like the neutrino coupling to the photon in Fig. 1, contain charged particles in virtual lines. Calculation of such a diagram would be affected by a background magnetic field through the propagator of virtual charged particles, even if the external lines contain only neutrinos and possibly other uncharged particles like the photon. The simplest example of physical quantities of this sort is the neutrino self-energy. Due to electrons in the internal lines, it is affected by a background magnetic field. Many other such examples can be given, and some will be discussed later in this review.

We can also think of a different class of effects, where a process involving neutrinos contains charged particles in either the initial or the final state. Since the asymptotic states of a charged particle are affected by the presence of a magnetic field, the rates of such processes would depend on the magnetic field. This also can have important physical implications.

On top of all these considerations, there is another very important one. If the background magnetic field is seeded in a material medium, there can be extra effects coming from the density or the temperature of

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[b] For an introduction to such models, see e.g., ref.[7].

the medium. This also opens up many new interesting possibilities, as we will see later in this review.

No matter which class of problems one considers, at a basic level one must tackle the interaction of charged particles with magnetic fields. We therefore start with a short introduction to a field theoretical discussion of charged particles in magnetic fields.

2 Charged Particles in Magnetic Fields

Spinor Solutions

Unless otherwise mentioned, we will always talk about a homogeneous and static magnetic field. The background field tensor will be denoted by $B_{\mu\nu}$, the magnetic field 3-vector by **B**, and its magnitude by B. Without loss of generality, the magnetic field can be assumed to direct in the z-direction. In quantum theory, the vector potential would appear directly in the equations. The vector potential **A** can be chosen in many equivalent ways. For example, one can choose[c]

$$A_0 = A_y = A_z = 0, \qquad A_x = -yB, \qquad \dots(2)$$

or

$$A_0 = A_x = A_z = 0, \qquad A_y = xB, \qquad \dots(3)$$

or more complicated ones where both A_x and A_y would be non-zero. We will work with the choice of eq.2. The stationary state solutions of the Dirac equations have the energy eigenvalues [9]

$$E^2 = m^2 + p_z^2 + 2NeB, \qquad \dots(4)$$

where N is a non-negative integer. For a fixed value of p_z, the energy eigenvalues are thus quantized. The quantum number N is called the Landau level, because eq.4 is the generalization of a similar formula obtained by Landau in the non-relativistic regime. As is obvious from the energy relation, the validity of the non-relativistic approximation requires not only that the momentum must be small compared to the mass, but also $|eB| \ll m^2$. Since the lightest charged particle is the electron, the ratio

$$B_e = m_e^2/e = 4.4 \times 10^{13}\,\mathrm{G} \qquad \dots(5)$$

can be taken as a benchmark value for the magnetic field beyond which the relativistic effects cannot be ignored.[d] Since the potential applications involve stellar

objects where the magnetic fields can be comparable to, or larger than, this benchmark value B_e, we will always use the relativistic formulas.

Note that the components of the momentum perpendicular to the magnetic field do not enter the dispersion relation. The eigenfunctions corresponding to the positive and negative roots of E can be written as

$$e^{-ip \cdot X_{\backslash}} U_s(y, N, \mathbf{p}_{\backslash}) \quad \text{and} \quad e^{ip \cdot X_{\backslash}} V_s(y, N, \mathbf{p}_{\backslash}), \quad \dots(6)$$

where U and V are spinors, whose explicit forms will be given shortly. The coordinate 4-vector has been represented by X^μ (in order to distinguish it from x, which is one of the components of X^μ). The symbol $X_{\backslash}^\mu$ stands for the same 4-vector, with the difference that the y-coordinate has been set to zero. Thus, for example,

$$p \cdot X_{\backslash} = Et - p_x x - p_z z. \qquad \dots(7)$$

The exponential factors in the eigenfunctions therefore do not contain the y-coordinate. However, the y-coordinate appears in the spinor, along with the non-y components of momentum. The components of the spinors can be conveniently expressed in terms of the dimensionless variable ξ defined by

$$\xi = \sqrt{eB}\left(y + \frac{p_x}{eB}\right), \qquad \dots(8)$$

and by defining the following function of ξ:

$$I_N(\xi) = \left(\frac{\sqrt{eB}}{N!\,2^N\sqrt{\pi}}\right)^{1/2} e^{-\xi^2/2} H_N(\xi), \qquad \dots(9)$$

where $H_N(\xi)$ denote Hermite polynomials, and the normalizing factor ensures that the functions $I_N(\xi)$ satisfy the following completeness relation:

$$\sum_N I_N(\xi) I_N(\xi_\star) = \sqrt{eB}\,\delta(\xi - \xi_\star) = \delta(y - y_\star)\dots(10)$$

In terms of these notations, it is now easy to write down the spinors appearing in eq.6. Using the shorthand

$$M_N = \sqrt{2NeB}, \qquad \dots(11)$$

the U-spinors can be written as

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[c] We employ the notation that a lettered subscript would mean the contravariant component of a 4-vector. If the covariant component has to be used, it will be denoted by a numbered subscript.

[d] Many authors denote this value by B_c and call it the 'critical field'. This is misleading. There is nothing critical about this value. Magnetic field effects exist both below and above this value.

$$U_+(y,N,\mathbf{p}_x) = \begin{pmatrix} I_N(\xi) \\ 0 \\ \dfrac{p_z}{E_N+m}I_N(\xi) \\ \dfrac{M_N}{E_N+m}I_{N-1}(\xi) \end{pmatrix},$$

$$U_-(y,N,\mathbf{p}_x) = \begin{pmatrix} 0 \\ I_{N-1}(\xi) \\ \dfrac{M_N}{E_N+m}I_N(\xi) \\ -\dfrac{p_z}{E_N+m}I_{N-1}(\xi) \end{pmatrix}. \quad \dots(12)$$

While using this and other formulas for $N = 0$, one should put $I_{N-1} = 0$. This implies that only the U_+ solution exists for $N = 0$. Similarly, the V-spinors are given by

$$V_+(y,N,\mathbf{p}_x) = \begin{pmatrix} \dfrac{p_z}{E_N+m}I_N(\widetilde{\xi}) \\ -\dfrac{M_N}{E_N+m}I_{N-1}(\widetilde{\xi}) \\ I_N(\widetilde{\xi}) \\ 0 \end{pmatrix},$$

$$V_-(y,N,\mathbf{p}_x) = \begin{pmatrix} -\dfrac{M_N}{E_N+m}I_N(\widetilde{\xi}) \\ -\dfrac{p_z}{E_N+m}I_{N-1}(\widetilde{\xi}) \\ 0 \\ I_{N-1}(\widetilde{\xi}) \end{pmatrix}. \quad \dots(13)$$

where

$$\widetilde{\xi} = \sqrt{eB}\left(y - \frac{p_x}{eB}\right). \quad \dots(14)$$

Propagator

In a field theoretic calculation, the spinors given above should be used if the charged particle appears in the initial or the final state of a physical process. If, on the other hand, the charged particle appears in the internal lines, we should use its propagator.

There are two ways to write the propagator. The first is to start with the fermion field operator $\psi(X)$ written in terms of the spinor solutions and the creation and annihilation operators, and construct the time ordered product, as is usually done for finding the propagator of a free fermion field in the vacuum. The algebra is straight forward and yields the result

$$\begin{aligned} S_B(X,X') \\ &= i\sum_N \int \frac{dp_0\,dp_x\,dp_z}{(2\pi)^3}\frac{E+m}{p_0^2 - E^2 + i\varepsilon}e^{-ip\cdot(X_x - X_x')} \\ &\quad \times \sum_s U_s(y,N,\mathbf{p}_x)\overline{U}_s(y',N,\mathbf{p}_x), \quad \dots(15) \end{aligned}$$

where E is the positive root obtained from eq.4. The spin sum can be conveniently written by introducing the following notation. Given any vector a^μ, we will define the following 4-vectors whose components are given by

$$\begin{aligned} a_\parallel^\mu &= (a_0, 0, 0, a_z) \\ \widetilde{a}_\parallel^\mu &= (a_z, 0, 0, a_0) \\ a_\perp^\mu &= (0, a_x, a_y, 0) \quad \dots(16) \end{aligned}$$

in the frame in which the background field is purely magnetic. Then, for any two 4-vectors a and b, we will write

$$\begin{aligned} a\cdot b_\parallel &= a_\alpha b_\parallel^\alpha, \\ a\cdot b_\perp &= a_\alpha b_\perp^\alpha. \quad \dots(17) \end{aligned}$$

In this notation, the spin sum appearing in eq.15 can be written as

$$\begin{aligned} &\sum_s U_s(y,N,\mathbf{p}_x)\overline{U}_s(y',N,\mathbf{p}_x) \\ &= \frac{1}{2(E_N+m)} \\ &\quad \times \Big[\big\{m(1+\sigma_z) + \not{p}_\parallel - \widetilde{\not{p}}_\parallel\gamma_5\big\}I_N(\xi)I_N(\xi') \\ &\quad + \big\{m(1-\sigma_z) + \not{p}_\parallel + \widetilde{\not{p}}_\parallel\gamma_5\big\}I_{N-1}(\xi)I_{N-1}(\xi') \\ &\quad + M_N(\gamma_1 + i\gamma_2)I_N(\xi)I_{N-1}(\xi') \\ &\quad + M_N(\gamma_1 - i\gamma_2)I_{N-1}(\xi)I_N(\xi')\Big], \quad \dots(18) \end{aligned}$$

where

$$\sigma_z \equiv i\gamma_1\gamma_2 = -\gamma_0\gamma_3\gamma_5. \quad \dots(19)$$

The resulting propagator is called the propagator in the Furry picture.

Alternatively, one uses a functional procedure introduced by Schwinger[10] where the propagator is written in the form

$$iS_B(X,X')$$
$$= \Psi(X,X') \int \frac{d^4p}{(2\pi)^4} e^{-ip \cdot (X-X')} iS_B(p),\dots(20)$$

where $S_B(p)$ is expressed as an integral over a variable s, usually (though confusingly) called the 'proper time':

$$iS_B(p) = \int_0^\infty ds\, e^{\Phi(p,s)} G(p,s). \qquad \dots(21)$$

The quantities $\Phi(p,s)$ and $G(p,s)$ can be written in the following way, using the notation of eq.17:

$$\Phi(p,s) \equiv is\left(p_\parallel^2 - \frac{\tan eBs}{eBs} p_\perp^2 - m^2\right) - \varepsilon|s|,$$
$$\dots(22)$$

$$G(p,s) \equiv \frac{e^{ieBs\sigma_z}}{\cos eBs}\left(\not p_\parallel - \frac{e^{-ieBs\sigma_z}}{\cos eBs}\not p_\perp + m\right)$$
$$= (1 + i\sigma_z \tan eBs)(\not p_\parallel + m)$$
$$- (\sec^2 eBs)\,\not p_\perp. \qquad \dots(23)$$

In a typical loop diagram, one therefore will have to perform not only integrations over the loop momenta, but also over the proper time variables.

The other factor $\Psi(X,X')$ appearing in eq.20 is a phase factor which breaks translation invariance and is given by[10]

$$\Psi(X,X')$$
$$= \exp\left(ie \int_{X'}^X d\xi^\mu \left[A_\mu(\xi) + \frac{1}{2}B_{\mu\nu}(\xi^\nu - X'^\nu)\right]\right).$$
$$\dots(24)$$

The integral is path-independent. The second term does not contribute if one chooses a straight line path characterized by

$$\xi^\mu = (1-\lambda)X'^\mu + \lambda X^\mu, \qquad 0 \le \lambda \le 1. \dots(25)$$

Further, the vector potential for a constant field $B_{\mu\nu}$ can be written as

$$A_\mu(\xi) = -\frac{1}{2}B_{\mu\nu}\xi^\nu. \qquad \dots(26)$$

The integration in eq.24 can then be performed easily and one obtains

$$\Psi(X,X') = \exp\left(-\frac{1}{2}ieX^\mu B_{\mu\nu}X'^\nu\right). \qquad \dots(27)$$

In what follows, we will indicate where this phase factor cancels between different propagators, and where it does not.

It is not difficult to obtain the modification of the propagator if the charged particle is in a background magnetized plasma. In this case, the background contains both matter and magnetic field. The clue can now be obtained from propagator in thermal matter without any magnetic field. In the real-time formalism, the propagator $iS'(p)$ involving the time-ordered product[e] can be written in terms of the free propagator $iS_0(p)$:

$$iS'(p) = iS_0(p) - \eta_F(p)\left[iS_0(p) - i\overline{S}_0(p)\right], \dots(28)$$

where

$$\overline{S}_0(p) = \gamma_0 S_0^\dagger(p)\gamma_0, \qquad \dots(29)$$

and $\eta_F(p)$ contains the distribution function for particles and antiparticles:

$$\eta_F(p) = \Theta(p \cdot u)f_F(p,\mu,\beta)$$
$$+ \Theta(-p \cdot u)f_F(-p,-\mu,\beta). \qquad \dots(30)$$

Here, Θ is the step function which takes the value $+1$ for positive values of its argument and vanishes for negative values of the argument, u^μ is the 4-vector denoting the center-of-mass velocity of the background plasma, and f_F denotes the Fermi-Dirac distribution function:

$$f_F(p,\mu,\beta) = \frac{1}{e^{\beta(p \cdot u - \mu)} + 1}. \qquad \dots(31)$$

In a similar manner, the propagator in a magnetized plasma is given by[11]

$$iS'_B(p) = iS_B(p) - \eta_F(p)\left[iS_B(p) - i\overline{S}_B(p)\right]\dots(32)$$

In the Schwinger proper-time representation, this can also be written as an integral over the proper-time variable s:

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[e] It should be mentioned here that other orderings also appear in the evaluation of general Green's functions. We will not talk about these other propagators.

$$iS'_B(p) = \int_0^\infty ds \, e^{\Phi(p,s)} G(p,s)$$
$$- \eta_F(p) \int_{-\infty}^\infty ds \, e^{\Phi(p,s)} G(p,s), \qquad \ldots(33)$$

where $\Phi(p,s)$ and $G(p,s)$ are given by the expressions in eqs.22 and 23.

3   Magnetic Field Effects on Neutrinos from Virtual Charged Particles

Neutrino Self-Energy

We have already mentioned in Sec. 1 that the simplest physical quantity where background magnetic field effects appear through virtual lines of charged particles is the self energy of the neutrino. The 1-loop diagram for the self energy is given in Fig. 1.

It is easy to see how the self-energy might be modified within a magnetized plasma. In the vacuum, the self-energy of a fermion has the general structure

$$\Sigma(p) = a\gamma^\mu k_\mu + b, \qquad \ldots(34)$$

which is the most general form dictated by Lorentz covariance. Here, a and b are Lorentz invariant, and can therefore depend only on k^2. In the presence of a homogeneous medium, the self-energy will involve the 4-vector u^μ introduced in eq.30. Further, if the medium contains a background magnetic field, the background field $B_{\mu\nu}$ also enters the general expression for the self-energy. These new objects, u^μ and $B_{\mu\nu}$, enter in two different ways. First, any form factor now can depend on more Lorentz invariants which are present in the problem. Second, the number of form factors also increases, since it is possible to write some more Lorentz covariant terms using u^μ and $B_{\mu\nu}$. There will in fact be a lot of form factors in the most general case. However, if we have chiral neutrinos as in the standard electroweak theory, the expression is not very complicated:

$$\Sigma_B(p) = \Big(a_1 k_\mu + b_1 u_\mu + a_2 k^\nu B_{\mu\nu} + b_2 u^\nu B_{\mu\nu}$$
$$+ a_3 k^\nu \widetilde{B}_{\mu\nu} + b_3 u^\nu \widetilde{B}_{\mu\nu}\Big) \gamma^\mu L, \qquad \ldots(35)$$

where L is the left-chiral projection operator, and

$$\widetilde{B}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\lambda\rho} B^{\lambda\rho}. \qquad \ldots(36)$$

We first consider the self-energy when the background consists of a pure magnetic field, without any matter. Then all b-type form-factors disappear from the self-energy. The dispersion relation of neutrinos can then be obtained by the zeros of $k - \Sigma_B$, which gives

$$\Big[(1-a_1)k^\mu - a_2 k^\nu B_{\mu\nu} - a_3 k^\nu \widetilde{B}_{\mu\nu}\Big]^2 = 0. \quad \ldots(37)$$

Performing the square is trivial, and one obtains

$$(1-a_1)^2 k^\mu k_\mu + a_2^2 k^\nu k_\lambda B_{\mu\nu} B^{\mu\lambda} + a_3^2 k^\nu k_\lambda \widetilde{B}_{\mu\nu} \widetilde{B}^{\mu\lambda}$$
$$+ 2a_2 a_3 k^\nu k_\lambda B_{\mu\nu} \widetilde{B}^{\mu\lambda} = 0. \qquad \ldots(38)$$

It is interesting to note that the terms linear in the background field all vanish due to the antisymmetry of the field tensor. Moreover, the $a_2 a_3$ term is also zero for a purely magnetic field.

The remaining terms can be most easily understood if we take the z-axis along the direction of the magnetic field. Then the only non-zero components of the tensor $B_{\mu\nu}$ and $\widetilde{B}_{\mu\nu}$ are given by

$$B_{12} = -B_{21} = B, \qquad \widetilde{B}_{03} = -\widetilde{B}_{30} = B, \quad \ldots(39)$$

where we have adopted the convention

$$\varepsilon_{0123} = +1. \qquad \ldots(40)$$

Thus

$$k^\nu k_\lambda B_{\mu\nu} B^{\mu\lambda} = -(k_x^2 + k_y^2)B^2 = -k_\perp^2 B^2,$$
$$k^\nu k_\lambda \widetilde{B}_{\mu\nu} \widetilde{B}^{\mu\lambda} = (\omega^2 - k_z^2)B^2 = k_\parallel^2 B^2, \quad \ldots(41)$$

where the notations for parallel and perpendicular products were introduced in eq.17. The form factor a_1 can be set equal to zero by a choice of the renormalization prescription. So the dispersion relation is now a solution of the equation

$$k^2 - a_2^2 k_\perp^2 B^2 + a_3^2 k_\parallel^2 B^2 = 0, \qquad \ldots(42)$$

which can also be written as

$$\omega^2 = k_z^2 + \frac{1 + a_2^2 B^2}{1 + a_3^2 B^2} k_\perp^2. \qquad \ldots(43)$$

Of course, this should not be taken as the solution for the neutrino energy, because the right hand side contains form factors which, in general, are functions of the energy and other things. But at least it shows that in the limit $B \to 0$, the vacuum dispersion relation is
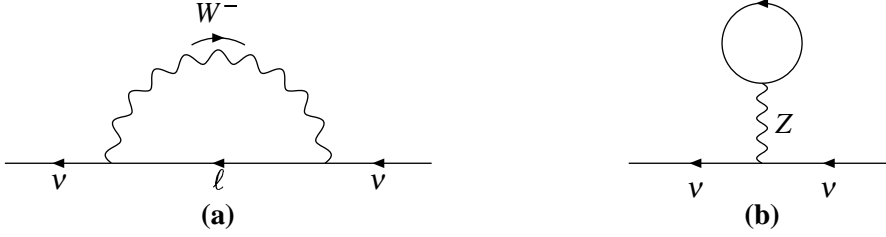
Fig. 2 One-loop diagrams for neutrino self-energy in a magnetized medium. Diagram b is absent if the background contains only a magnetic field but no matter. For legends and related diagrams, see the caption of Fig. 1.

recovered. If we retain the lowest order corrections in B, we can treat the form-factors to be independent of B and write

$$\omega^2 = \mathbf{k}^2 + (a_2^2 - a_3^2)B^2 k_\perp^2. \qquad \ldots (44)$$

Calculation of this self-energy was performed by Erdas and Feldman[12] using the Schwinger propagator, where they also incorporated the modification of the W-propagator due to the magnetic field. Importantly, the W-propagator contains the same phase factor as given in eq.27. Therefore, the phase factors from the charged lepton and the W-lines are of the form $\Psi(X, X')\Psi(X', X)$. From eq.27, it is easy to see that this is equal to unity, and therefore the phase factors do not contribute in the final expression. Detailed calculations show that[12]

$$a_2^2 - a_3^2 = \left(\frac{eg}{2\pi M_W^2}\right)^2 \left(\frac{1}{3}\ln\frac{M_W}{m} + \frac{1}{8}\right), \quad \ldots (45)$$

where m is the mass of the charged lepton in the internal line. For strong magnetic fields, the dispersion relation has been calculated more recently by Elizalde, Ferrer and de la Incera[13].

Let us next concentrate on the terms which can occur only in a magnetized medium. In other words, we select out the terms which cannot occur if the neutrino propagates in a background of pure magnetic field without any material medium. This means that, apart from the term k which occurs also in the vacuum, we look for the terms which contain both u^μ and $B_{\mu\nu}$. Further, if the background field is purely magnetic in the rest frame of the medium, $u^\nu B_{\mu\nu} = 0$ since u has only the time component whereas the only non-zero components of $B_{\mu\nu}$ are spatial. Thus we are left with[14]:

$$\Sigma_B(p) = \left(a_1 k_\mu + b_1 u_\mu + b_3 u^\nu \widetilde{B}_{\mu\nu}\right)\gamma^\mu L. \quad \ldots (46)$$

Once again, setting $a_1 = 0$ through a renormalization prescription, we can find the dispersion relation of the neutrinos in the form[14]:

$$\omega = \left|\mathbf{k} - b_3 \mathbf{B}\right| + b_1 \approx |\mathbf{k}| - b_3 \hat{k}\cdot\mathbf{B} + b_1, \quad \ldots (47)$$

where $\hat{k}$ is the unit vector along $\mathbf{k}$, and we have kept only the linear correction in the magnetic field. This form for the dispersion relation was first arrived at by D'Olivo, Nieves and Pal (DNP)[14] who essentially performed a calculation to the first order in the external field. As for the form factors, b_1 was known previously, obtained from the analysis of neutrino propagation in isotropic matter, i.e., without any magnetic field. The result was[15-18]

$$b_1 = \sqrt{2}G_F(n_e - n_{\bar{e}}) \times (y_e + \rho c_V), \ldots (48)$$

where

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W}, \qquad \ldots (49)$$

n_e, $n_{\bar{e}}$ are the densities of electrons and positrons in the medium,

$$y_e = \begin{cases} 1 & \text{for } \nu_e, \\ 0 & \text{for } \nu \neq \nu_e, \end{cases} \qquad \ldots (50)$$

and c_V is defined through the coupling of the electron to the Z-boson, whose Feynman rule is

$$-\frac{ig}{2\cos\theta_W}\gamma_\mu(c_V - c_A\gamma_5). \qquad \ldots (51)$$

In other words, in the Standard Model

$$c_V = -\frac{1}{2} + 2\sin^2\theta_W, \qquad c_A = -\frac{1}{2}. \qquad \ldots (52)$$

The contribution to b_3 from background electrons and positrons was calculated by DNP[14]. They obtained[f]

$$
\begin{aligned}
b_3 &= -2\sqrt{2}eG_F \int \frac{d^3p}{(2\pi)^3 2E} \frac{d}{dE}(f_e - f_{\bar{e}}) \\
&\quad \times (y_e + \rho c_A)\,, \qquad \ldots(53)
\end{aligned}
$$

where f_e and $f_{\bar{e}}$ are the Fermi distribution functions for electrons and positrons, and

$$
E = \sqrt{\mathbf{p}^2 + m_e^2}\,. \qquad \ldots(54)
$$

Later authors have improved on this result in two different ways. Some authors[19] have included the contributions coming from nucleons in the background. Some others[11, 20] have used the Schwinger propagator and extended the results to all orders in the magnetic field.

Neutrino Mixing and Oscillation

Calculation of neutrino self-energy has a direct consequence on neutrino mixing and oscillations. Of course neutrino oscillations require neutrino mixing and therefore neutrino mass. For the sake of simplicity, we discuss mixing between two neutrinos which we will call ν_e and ν_μ. The eigenstates will in general be called ν_1 and ν_2, which are given by

$$
\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}. \quad \ldots(55)
$$

We will denote the masses of the eigenstates by m_1 and m_2, and assume that the neutrinos are ultra-relativistic. Then in the vacuum, the evolution equation for a beam of neutrinos will be given by

$$
i\frac{d}{dt}\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \frac{1}{2\omega}M^2 \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}. \qquad \ldots(56)
$$

where the matrix M^2 is given by

$$
M^2 = \begin{pmatrix} -\frac{1}{2}\Delta m^2 \cos 2\theta & \frac{1}{2}\Delta m^2 \sin 2\theta \\ \frac{1}{2}\Delta m^2 \sin 2\theta & \frac{1}{2}\Delta m^2 \cos 2\theta \end{pmatrix}, \ldots(57)
$$

where $\Delta m^2 = m_2^2 - m_1^2$. In writing this matrix, we have ignored all terms which are multiples of the unit matrix, which affect the propagation only by a phase which is common for all the states.

In a non-trivial background, the dispersion relations of the neutrinos change, as discussed in Sec. 3. This adds new terms to the diagonal elements of the effective Hamiltonian in the flavor basis, which we denote by the symbol A. As a result, the matrix M^2 should now be replaced by

$$
\widetilde{M}^2 = \begin{pmatrix} -\frac{1}{2}\Delta m^2 \cos 2\theta + A_{\nu_e} & \frac{1}{2}\Delta m^2 \sin 2\theta \\ \frac{1}{2}\Delta m^2 \sin 2\theta & \frac{1}{2}\Delta m^2 \cos 2\theta + A_{\nu_\mu} \end{pmatrix},
$$
$$\ldots(58)$$

where the extra contributions are in general different for ν_e and ν_μ. The eigenstates and eigenvalues change because of these new contributions. For example, the mixing angle now becomes $\widetilde{\theta}$, given by

$$
\tan 2\widetilde{\theta} = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta + A_{\nu_\mu} - A_{\nu_e}}. \qquad \ldots(59)
$$

In a pure magnetic field, the self-energies were shown in eq.44 and eq.45. The quantity m appearing in eq.45 is the mass of the charged lepton in the loop. Thus, for ν_e, it is the electron mass whereas for ν_μ, it is the muon mass. Thus $A_{\nu_\mu} \neq A_{\nu_e}$. However, the difference appears in logarithmic form, and is presumably not very significant.

In a magnetized medium, however, the situation changes. The reason is that the medium contains electrons but not muons. Accordingly, the quantities A_{ν_e} and A_{ν_μ} can be very different, as seen by the presence of the term y_e in eq.53. If we take self-energy corrections only up to linear order in B, as done in eq.47, we obtain

$$
\begin{aligned}
A_{\nu_\mu} - A_{\nu_e} &= -\sqrt{2}G_F(n_e - n_{\bar{e}}) \\
&\quad -2\sqrt{2}eG_F\hat{k}\cdot\mathbf{B}\int \frac{d^3p}{(2\pi)^3 2E}\frac{d}{dE}(f_e - f_{\bar{e}}). \ \ldots(60)
\end{aligned}
$$

The first term on the right side comes just from the background density of matter, and the second term is the magnetic field dependent correction. This quantity has been calculated for various combinations of temperature and chemical potential of the background electrons[11, 21, 22].

If the denominator of the right side of eq.59 becomes zero for some value of $A_{\nu_\mu} - A_{\nu_e}$, the value of $\tan 2\widetilde{\theta}$ will become infinite. This is the resonant level crossing condition. This was first discussed in the con-

---

[f]   The authors of ref.[14] used a convention in which $e < 0$. Here we present the result in the convention $e > 0$.

text of neutrino oscillation in a matter background by Mikheev and Smirnov[23], where a particular value of density would ensure resonance. Presence of a magnetic field will modify this resonant density, as seen from eq.60. The modification will be direction dependent because of the factor $\hat{k} \cdot \mathbf{B}$. Some early authors[21, 22] contemplated that, for large B, the magnetic term might even drive the resonance. However, later it was shown[24] that the magnetic correction would always be smaller than the other term. So, if one considers values of B which are so large that the last term in eq.60 is larger than the first term on the right hand side, it means that one must take higher order corrections in B into account.

It should be noted that the type of corrections to the dispersion relation discussed in Sec. 3 appear from chiral neutrinos. Thus, they produce chirality-preserving modifications to neutrino oscillations. In addition, if the neutrino has a magnetic moment, there will be chirality-flipping modifications as well. Many of these modifications were analyzed in the context of the solar neutrino problem, and we do not discuss them here. [g]As pointed out in Sec. 1, they are not important for solar neutrinos, although may be important in other stellar objects like the neutron star where the magnetic fields are much larger.

Electromagnetic Vertex of Neutrinos

A lot of work has also gone into evaluating the neutrino-neutrino-photon vertex in the presence of a background magnetic field. The vertex arises from the diagrams of Fig. 1, which contain internal W-lines. In addition, there is a diagram mediated by the Z-boson, as shown in Fig. 3. For phenomenological purposes, we require the electromagnetic vertex of neutrinos only in the leading order in Fermi constant. It should be realized that in this order, the diagram of Fig. 1b does not contribute at all, since it has two W-propagators. The remaining diagrams, shown in Fig. 1a and Fig. 3, can both be represented in the form shown in Fig. 4, where an effective 4-fermi vertex has been used. The effective 4-fermi interaction can be written as

$$\mathcal{L}_{\text{eff}} = -\sqrt{2} G_F \left[\overline{v}_L \gamma^\lambda v_L \right] \left[\overline{\ell} \gamma_\lambda (g_V - g_A \gamma_5) \ell \right] \dots (61)$$

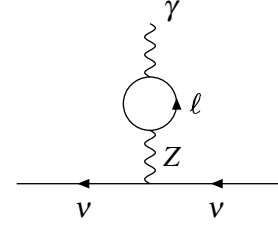If the neutrino and the charged lepton belong to

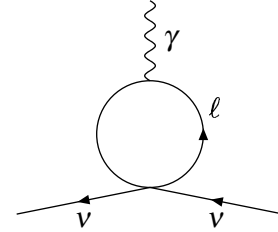Fig. 3 Z-photon mixing diagram contributing to the neutrino electromagnetic vertex.

Fig. 4 The neutrino electromagnetic vertex in the leading order in the Fermi constant.

different generations of fermions, this effective Lagrangian contains only the neutral current interactions, and in that case g_V and g_A are identical to c_V and c_A defined in eq.52. On the other hand, if both v and ℓ belong to the same generation, we should add the charged current contribution as well, and use

$$g_V = c_V + 1, \qquad g_A = c_A + 1. \qquad \dots (62)$$

Many processes involving neutrinos and photons have been calculated using the 4-fermi Lagrangian of eq.61. The calculations simplify in this limit for various reasons. First, we do not have to use the momentum dependence of the gauge boson propagators. Second, since two charged lepton lines form a loop in Fig. 4, the phase factor of eq.27 appearing in their propagators cancel each other.

The background magnetic field can give rise to many physical processes which are impossible to occur in the vacuum. One such process is the decay of a photon into a neutrino-antineutrino pair:

$$\gamma \rightarrow v + \overline{v}. \qquad \dots (63)$$

---

[g] A recent paper on chirality-flipping oscillations is ref.[25],where one can obtain references to earlier literature. Some early references are also found in refs.[1] and [7]

This was calculated using the Schwinger propagator in some very early papers[26, 27]. Assuming two generations of fermions, the decay rate was found to be

$$\Gamma = \frac{\alpha^2 G_F^2}{48\pi^3\Omega}\left|\varepsilon^\mu q^\nu \widetilde{B}_{\mu\nu}\right|^2 \left|\mathcal{M}_e - \mathcal{M}_\mu\right|^2, \quad \dots(64)$$

where ε^μ, q^μ and Ω are the polarization vector, the momentum 4-vector and the energy of the initial photon, and the quantity $\mathcal{M}_\ell$ was evaluated in various limits by these authors. For example, if $\Omega \ll m_\ell$, they found

$$\mathcal{M}_\ell = \frac{\Omega^2}{eB}\sin^2\theta \times \begin{cases} \frac{2}{15}\left(\frac{eB}{m_\ell^2}\right)^3 & \text{for } eB \ll m_\ell^2, \\ \frac{1}{3}\left(\frac{eB}{m_\ell^2}\right) & \text{for } eB \gg m_\ell^2, \end{cases} \dots(65)$$

where θ is the angle between the photon momentum and the magnetic field. No matter which neutrino pair the photon decays to, both charged leptons appear in the decay rate because of the loop in Fig. 3. The calculation has been carried out in the leading order in Fermi constant, where only the axial couplings of the charged leptons contribute to the amplitude.

A related process is Cherenkov radiation from neutrinos:

$$\nu \to \nu + \gamma. \qquad \dots(66)$$

Again, this is a process forbidden in the vacuum. But a background magnetic field modifies the photon dispersion relation, and so this process becomes feasible. The rate of this process has been calculated by many authors[26-30], and all of them do not get the same result. According to ref.[30], the rate for the process is given by

$$\Gamma = \frac{\alpha G_F^2}{8\pi^2}(g_V^2 + g_A^2)(eB)^2$$
$$\times \omega \sin^2\theta F(\omega^2 \sin^2\theta/eB), \qquad \dots(67)$$

where ω is the initial neutrino energy and θ is the angle between the initial neutrino momentum and the background field. For large magnetic fields satisfying the condition $eB \gg \omega^2 \sin^2\theta$, the function F is given by

$$F(x) = 1 - \frac{x}{2} + \frac{x^2}{3} - \frac{5x^3}{24} + \frac{7x^4}{60} + \cdots \quad \dots(68)$$

The modification of this process in the presence of background matter has also been calculated[31].

Another process that has been discussed is the radiative neutrino decay

$$\nu_a \to \nu_b + \gamma. \qquad \dots(69)$$

Unlike the previous processes, this can occur in the vacuum as well when the neutrinos have mass and mixing. However, a background magnetic field adds new contributions to the amplitude, and the rate can be enhanced. Gvozdev, Mikheev and Vasilevskaya[32] calculated the rate of this decay in a variety of situations depending on the field strength and the energy of the initial neutrino. For a strong magnetic field ($B \gg B_e$), they found the decay rate of an ultra-relativistic neutrino of energy ω to be

$$\Gamma = \frac{2\alpha G_F^2}{\pi^4}\frac{m_e^6}{\omega}\left(\frac{B}{B_e}\right)^2 |K_{ae}K_{be}^*|^2$$
$$\times J(\omega \sin\theta/2m_e), \qquad \dots(70)$$

where K is the leptonic mixing matrix, θ is the angle between the magnetic field and the neutrino momentum, and

$$J(z) = \int_0^z dy\,(z-y)$$
$$\times \left(\frac{1}{y\sqrt{1-y^2}}\tan^{-1}\frac{y}{\sqrt{1-y^2}} - 1\right)^2. \quad \dots(71)$$

The curious feature of this result is that this is independent of the initial and the final neutrino masses.

The form for the 4-fermi interaction in eq.61 suggests that the neutrino electromagnetic vertex function Γ_λ can be written as[33]:

$$\Gamma_\lambda = -\frac{\sqrt{2}G_F}{e}\gamma^\rho L\left(g_V \Pi_{\lambda\rho} - g_A \Pi_{\lambda\rho}^5\right). \quad \dots(72)$$

Here, the term $\Pi_{\lambda\rho}$ is exactly the expression for the vacuum polarization of the photon, and appears from the vector interaction in the effective Lagrangian. The other term, $\Pi_{\lambda\rho}^5$ differs from $\Pi_{\lambda\rho}$ in that it contains an axial coupling from the effective Lagrangian.

This equality is valid even when one has a magnetic field and a material medium as the background, as long as one restricts oneself to the leading order in Fermi constant. Thus, the calculation of the photon self-energy in a background magnetic field in matter can give us information about the neutrino electromagnetic vertex in the same situation. The calculation of the photon self-energy was done to the first order

in B by Ganguly, Konar and Pal[34]. Later, it was extended to all orders by D'Olivo, Nieves and Sahu[35]. The calculation of $\Pi^5_{\lambda\rho}$, on the other hand, was undertaken in a series of papers by Bhattacharya, Ganguly, Konar and Das[36−38]. In particular, it was shown[37] that the terms which are odd in B contribute to the vertex function even at zero momentum transfer, which means that they contribute to an effective charge of the neutrino.

Neutrino-Photon Scattering

Gell-Mann showed[39] that the amplitude of the reaction

$$\gamma + \nu \rightarrow \gamma + \nu \qquad \dots (73)$$

is exactly zero to order G_F because by Yang's theorem[40, 41] two photons cannot couple to a $J = 1$ state. In the standard model, therefore, amplitude of the above process appears only at the level of $1/M_W^4$ and as a result the cross-section is exceedingly small[42]. But there is no such restriction on the coupling of three photons with neutrinos as,

$$\gamma + \nu \rightarrow \gamma + \gamma + \nu. \qquad \dots (74)$$

The cross-section of the above process can be calculated from the effective Lagrangian proposed by Dicus and Repko[43]. The diagrams for the two neutrino three photon interaction are shown in Fig. 5 where Fig. 5a shows the contribution from the W exchange diagram and Fig. 5b shows the contribution from Z exchange. Denoting the photon field tensor as $F_{\mu\nu}$ and the neutrino fields by ψ, and integrating out the particles in the loop the effective Lagrangian comes out as

$$\mathscr{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \frac{e^3(c_V + 1)}{360\pi^2 m^4} \Big[5(N_{\mu\nu}F^{\mu\nu})(F_{\lambda\rho}F^{\lambda\rho})$$
$$- 14 N_{\mu\nu}F^{\nu\lambda}F_{\lambda\rho}F^{\rho\mu} \Big], \dots (75)$$

where c_V was defined in eq.52, and

$$N_{\mu\nu} = \partial_\mu(\overline{\psi}\gamma_\nu L\psi) - \partial_\nu(\overline{\psi}\gamma_\mu L\psi). \qquad \dots (76)$$

For energies much smaller than the electron mass, this can be used as an effective Lagrangian to calculate various processes involving photons and neutrinos in the presence of a background magnetic field $B_{\mu\nu}$. For this, we simply have to write

$$F_{\mu\nu} = f_{\mu\nu} + B_{\mu\nu}, \qquad \dots (77)$$

where now $f_{\mu\nu}$ is the dynamical photon field, and look for the terms involving $B_{\mu\nu}$. For example, Shaisultanov[44] calculated the rate of $\gamma\gamma \rightarrow \nu\overline{\nu}$ in a background field. Eq.75 shows that in the lowest order, the amplitude for involving ν_e's would be proportional to

$$\frac{G_F B}{m_e^4} \sim \frac{B}{M_W^2 m_e^2 B_e}, \qquad \dots (78)$$

where B_e is the value of the magnetic field defined in eq.5. Since the amplitude without any magnetic field[42] is of order $1/M_W^4$, it follows that the background field increases the amplitude by a factor of order $(M_W/m_e)^2 B/B_e$, or the rate by a factor $(M_W/m_e)^4 (B/B_e)^2$. Later calculations[45] have extended these results by including other processes obtained by crossing, like $\nu\overline{\nu} \rightarrow \gamma\gamma$ and $\nu\gamma \rightarrow \nu\gamma$. To obtain higher B terms in these cross sections, one needs the effective Lagrangian containing higher order terms in the electromagnetic field strength. Such an effective Lagrangian has been derived by Gies and Shaisultanov[46].

Alternatively, the amplitudes can be calculated using the Schwinger propagator for charged leptons. Such calculations for $\gamma\gamma \rightarrow \nu\overline{\nu}$ were done some time ago[47, 48]. One of the important features of this calculation is that in the 4-fermi limit, the diagram contains three electron propagators. In such situations, the phase factor $\Psi(x, x')$ appearing in the Schwinger propagator of eq.20 cannot be disregarded.

In the calculation, only the linear term in B was retained in the amplitude so that the results are valid only for small magnetic fields. However, since no effective Lagrangian was used, the results are valid even when the energies of the neutrinos and/or the photons are comparable to, or greater than, the electron mass. Later authors[45] reported some mistakes in this calculation and corrected them.

4 Magnetic Field Effects on Neutrino Processes from External Charged Particles

As discussed in Sec. 1, rates for neutrino processes are modified in a background magnetic field because of the presence of charged particles in the initial and/or final states. Some such processes might have very important astrophysical implications, some of which will be discussed in Sec. 5.
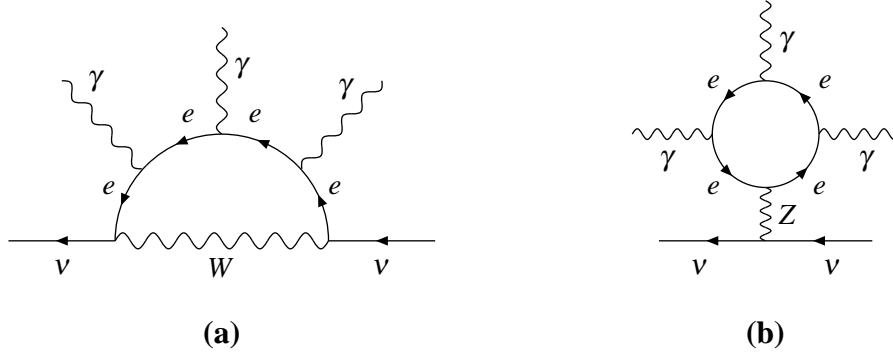
(a)   **(b)**

Fig. 5 The 1-loop effective vertex for two neutrinos and three photons.

Processes Involving Nucleons

The charged current interaction Lagrangian involving neutrinos and nucleons is given by

$$\mathscr{L}_{\text{int}} = \sqrt{2} \left[\overline{\psi}_{(e)} \gamma^{\mu} L \psi_{(\nu_e)} \right] \\ \times \left[\overline{\psi}_{(p)} \gamma_{\mu} \left(G_V + G_A \gamma_5 \right) \psi_{(n)} \right], \qquad \dots (79)$$

where $G_V = G_F \cos \theta_C$, θ_C being the Cabibbo angle, and $G_A/G_V = -1.26$. This can be used to find the cross section for various neutrino-nucleon scattering processes, as we describe now.

First we consider some processes in which a neutrino or an antineutrino appears only in the final state. In a star, when such reactions occur, the final neutrino or the antineutrino escapes and the star loses energy. Such processes are collectively known as Urca processes, named after a casino in Rio de Janeiro where customers lose money little by little[49]. One such process is the neutron beta-decay,

$$n \quad \rightarrow \quad p + e^- + \overline{\nu}. \qquad \dots (80)$$

The rate of this process in a magnetic field was calculated by various authors. An early paper by Fassio-Canuto[50] derived the rate in a background of degenerate electrons. Contemporary papers by Matese and O'Connell[51, 52] derived the rate where the background did not contain any matter, but included the effects of the polarization of neutrons due to the magnetic field. Protons and neutrons were assumed to be non-relativistic in the calculations. Further, the magnetic field was assumed to be much smaller than m_p^2/e so that its effect on the proton wave function could be neglected. The dispersion relation of eq.4 then suggests that the Landau level of the electron is bounded

by the relation

$$N < \frac{Q^2 - m_e^2}{2eB}, \qquad \dots (81)$$

where

$$Q \equiv m_n - m_p. \qquad \dots (82)$$

The rate of the process should then include contribution from all possible Landau levels in this range, and the general expression for this rate was obtained[51].

Other examples of Urca processes are

$$p + e^- \quad \rightarrow \quad n + \nu, \qquad \dots (83)$$
$$n + e^+ \quad \rightarrow \quad p + \overline{\nu}. \qquad \dots (84)$$

The first reaction requires a threshold energy. The second one is possible at any energy. Various calculations of these processes exist in the literature. Some calculations take the background matter density into account[53–57], some include magnetic effects on the proton wavefunction as well[57].

We now consider processes where neutrinos or antineutrinos appear in the initial state only. In a star, such processes contribute to the opacity of neutrinos and antineutrinos. One example of such process is the inverse beta-decay process

$$n + \nu \quad \rightarrow \quad p + e^-. \qquad \dots (85)$$

The cross section of this process has been calculated by several authors. In the early calculation by Roulet[58] and by Lai and Qian[59], the modification of the electron wave function due to the magnetic field was not taken into account. The magnetic field effects entered only through the following modification of the phase space integral and the spin factor of the electron:

$$2 \int \frac{d^3 p}{(2\pi)^3} \longrightarrow \frac{eB}{(2\pi)^2} \sum_N g_N \int dp_z, \qquad \dots (86)$$

where g_N is the degeneracy of the N-th Landau level, which is 1 for $N = 0$ and 2 for all other levels. The sum over N is restricted to the region

$$N < \frac{(Q+\omega)^2 - m_e^2}{2eB}, \qquad \ldots (87)$$

where ω is the neutrino energy.

Subsequent calculations incorporated the modification of wave functions. Arras and Lai[60], while still treating the nucleons as non-relativistic, used the non-relativistic Landau levels as well as the finiteness of the recoil energy for the proton. They found the cross section and went on to derive expressions for the neutrino opacity. From the final expressions, one can only recognize the terms linear in B. The opacity was calculated also by Chandra, Goyal and Goswami[61]. Like the previous authors, they also considered the contribution to the opacity from other reactions like neutrino-nucleon elastic scattering. In the work of Bhattacharya and Pal[62, 63], the cross section has been calculated ignoring nucleon recoil, but the results are correct to all orders in B.

Incorporation of the proper wave functions of eq.12 reveal a property of the cross section that is not obtained by a mere modification of the phase space. The cross section is sensitive to the angle θ between the neutrino momentum and the magnetic field even if the neutron is unpolarized. This is also true for the Urca processes discussed above. In addition, when one includes neutron polarization, there are extra terms which depend on θ.

Neutrino-Electron Scattering and Related Processes

The cross-section for the elastic neutrino-electron scattering

$$\nu + e \to \nu + e \qquad \ldots (88)$$

was calculated by Bezchastnov and Haensel[64] using the exact wave functions given in eq.12. They considered the reaction taking place in a background of electrons.

There are related processes, obtained by crossing, which contain a neutrino-antineutrino pair in the final state. For example, one can have the pair annihilation of electron and positron into neutrino-antineutrino:

$$e^- + e^+ \to \nu + \overline{\nu}. \qquad \ldots (89)$$

In addition, one can consider the process

$$e^- \to e^- + \nu + \overline{\nu}. \qquad \ldots (90)$$

This is similar to a synchrotron radiation reaction, the difference being that a neutrino-antineutrino pair is produced instead of a photon. The process is usually called neutrino synchrotron radiation. It should be noted that this process cannot occur in the vacuum. However, in the presence of a background magnetic field and background matter, the dispersion relation of the electron changes so that it becomes kinematically feasible. These processes provide important mechanism for stellar energy loss, and the rates of these processes have been calculated[65–67]. The reverse of these processes are important for neutrino absorption, and have also been studied [68].

5   Possible Implications

A magnetic field, as commented in Sec. 1, has sizeable effect on neutrino properties if its magnitude is comparable to, or larger than, 10^{13} G. We also mentioned in Sec. 1 that there are stellar objects where such high fields presumably exist. One can therefore speculate the effects of such high fields on various processes involving neutrinos on the equilibrium, dynamics and evolution of these stars.

One of the effects of a strong magnetic field is to enhance the rate of creation of neutrinos. Various such processes were discussed in Sec. 4, such as the Urca processes, e^+-e^- pair annihilation and neutrino synchrotron radiation. We also commented that, once produced, the neutrinos can easily go out of the star because their mean free path is large. Neutrino emission from young neutron stars is the most important mechanism through which these stars lose energy and become colder.

Magnetic field enhances stellar energy loss in two ways. First, as the calculations show, the rates of different neutrino-producing processes increase in the presence of a magnetic field. Second, processes such as the neutrino synchrotron radiation, eq.90, which cannot take place in the vacuum, become possible due to the presence of a magnetic field and provide new channels for energy loss.

We now discuss processes like the neutrino-electron scattering and the inverse beta-decay where neutrinos are not produced. The first of these processes, eq.88, controls the propagation of the neutrinos from the core of the star to the boundary. The second process, eq.85, is directly related with the opacity of neutrinos inside the star. The cross section of both these reactions are

enhanced in strong magnetic fields, implying that high magnetic fields enhance not only the emissivity but also the opacity of neutrinos.

A background magnetic field provides a preferred direction to a given problem, and this shows up in the scattering cross-section of neutrinos. Enhancement and anisotropy of the cross-sections are interrelated in a constant background magnetic field, but to make things simpler we will discuss the two effects separately. The anisotropic effects on the Urca processes have been calculated and also we know how the reactions responsible for the opacity of neutrinos respond to a unique direction of the magnetic field. There is a specific aim for the calculations of anisotropic effects, viz., finding an explanation for the high velocities of the pulsars, of the order of 450 ± 90 Km s^{-1}. Typical pulsars have masses between $1.0 M_\odot$ and $1.5 M_\odot$, i.e., about 2×10^{33} g. The momentum associated with the proper motion of a pulsar would therefore be of order 10^{41} g cm/s. On the other hand the energy carried off by neutrinos in a supernova explosion is about 3×10^{53} erg, which corresponds to a sum of magnitudes of neutrino momenta of 10^{43} g cm/s. Thus an asymmetry of order of 1% in the distribution of the outgoing neutrinos would explain the kick of the pulsars. It has been argued that an asymmetry of this order in the distribution of outgoing neutrinos can be generated by the anisotropic cross-sections of the various neutrino related processes in presence of a constant magnetic field[53, 62, 69, 70]. If, on the other hand, the magnetic field is toroidal, anisotropic neutrino emission can also produce a torque on the star and help regenerate the magnetic field[71].

We next discuss some possible applications of the electromagnetic interactions of neutrinos in a magnetic field background. The photons present in a neutron star are trapped due to their large cross-sections with electrons or positrons. Now if due to the existence of a medium or of the magnetic field or of both the dispersion relation of the photon is changed, real photons can decay into neutrino-antineutrino pair, as discussed in Sec. 3. This provides new ways of energy emission from the star. Other related processes involving photons and neutrinos, such as

$$\gamma + e^- \quad \rightarrow \quad e^- + \nu + \overline{\nu}, \qquad \dots(91)$$
$$\gamma + \gamma \quad \rightarrow \quad \nu + \overline{\nu}, \qquad \dots(92)$$

are also responsible for emission of neutrinos from the star.

Studies on propagation of neutrinos in a magnetic field has led to investigations concerning their dispersion relation, as has been discussed in Sec. 3. Calculations of dispersion relation of neutrinos has been done in vacuum and in a medium, for strong fields and for weak fields. An interesting cosmological consequence of these dispersion relations has been discussed in the literature[13]. It is based upon the assumption that in the time between the QCD phase transition epoch and the end of nucleosynthesis, a cosmic magnetic field in the magnitude range

$$m_e^2 \leq eB \leq M_W^2 \qquad \dots(93)$$

could have existed. We have seen in eq.47 that in presence of a magnetic field, the neutrino dispersion relation acquires a direction dependent term. This would be reflected in the propagation of neutrinos, and must leave its footprints in the neutrino relic background. Of course the effect would be appreciable only if $eB \sim T^2$ where T is the temperature during the neutrino decoupling era, so that thermal fluctuations do not wash out these anisotropies.

Another interesting possible consequence of neutrino oscillations have also been discussed[72] in the context of high velocities of neutron stars. In a material background containing electrons but not any other charged leptons, the cross section of ν_e's is greater than that of any other flavor of neutrino. If ν_e's can oscillate resonantly to any other flavor, they can escape more easily from a star. In a proto-neutron star, the resonant density at an angle θ with the magnetic field occurs at a distance $R_0 + \delta \cos\theta$ from the center, where δ is a function of the magnetic field and specifies the deformation from a spherical surface. So this distance is direction dependent, as we have discussed in Sec. 3. Therefore the escape of neutrinos is also direction dependent, and the momentum carried away by them is not isotropic. The star would get a kick in the direction opposite to the net momentum of escaped neutrinos. This was suggested by Kusenko and Segrè[72], who estimated that the momentum imbalance is proportional to δ and can have a magnitude of around 1% for reasonable values of B. Later authors[73] criticized their analysis and argued that the effect was overestimated by them, because the kick momentum vanishes in the lowest order in δ. A recent and detailed study[74] indicates that these criticisms may not be well-placed, and the kick momentum might indeed be proportional to the surface deformation parameter δ.

6   Concluding Remarks

Unfortunately, there is no conclusive remark on this subject. Many calculations have been done, but there is no consensus about whether any of them explains any observational or experimental data. If the magnetic field is small, the effect is small and cannot be disentangled from the background. Very large magnetic fields, $B > B_e$, are obtained only at astronomical distances, presumably in neutron stars and magnetars. For such distant objects, observational data are not clean enough to resolve the effects of the magnetic field. There are theses and anti-theses, but no synthesis so far. The calculations are in search of a physical effect to be explained by them, much like the six characters in search of an author in the great Italian dramatist Luigi Pirandello's play *Sei personaggi in cerca d'autore*.

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