Simplifying again in the same way by taking 1 to the left hand side and dividing by 3 this time, we have,

$$5 = \sqrt{1 + 4\sqrt{1 + 5\sqrt{1 + 6\sqrt{1 + 7\sqrt{1 + 8\sqrt{1 + \cdots}}}}}}\,,$$

and we may continue endlessly for all positive integers so that the general expression for $n \geq 3$ is given by

$$n = \sqrt{1 + (n-1)\sqrt{1 + n\sqrt{1 + (n+1)\sqrt{1 + (n+2)\sqrt{1 + (n+3)\sqrt{1 + \cdots}}}}}}\,.$$

The golden nested radical

Here is another infinite nested square-root that looks nice and simple but the answer is in fact a famous number:

$$a = \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \cdots}}}}}}\,,$$

which involves only the number 1. To find the value of a, we use the same trick. Just square the expression to get,

$$a^2 = 1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \cdots}}}}}\,,$$

or simply $a^2 = 1 + a$, whose solution is straight-forward (try it!):

$$a = \frac{1 + \sqrt{5}}{2}\,.$$

This is the famous **Golden Ratio** that was written about in the Sep-Oct, 2013, issue of JM! If you have such an old copy, you can find out that golden mean may also be written as a continued fraction involving only the number 1.

You can try to construct such examples on your own and have fun with numbers! •