Do you see this? So

$$1 + 2 \times \sqrt{14} > 2 \times \sqrt{15}\ ,$$

where the right hand side is simplified a little bit.

George Pólya, 1887–1985

Polya was a Hungarian mathematician who worked for some time in Switzerland before moving to the United States, where he lived for 40 years till his death at the age of 97.

He worked on a number of areas of mathematics, but he is best known for his series of books on *"How to solve it, Mathematics and Plausible Reasoning"*. In these books, Polya discussed problem solving for both mathematical and non-mathematical problems. He also included advice for students on how to learn and teach mathematics.

The book has been translated into over 17 languages. Apart from telling the student how to make a plan to solve a problem (and carry it out), Polya stated a very important principle:

"Much can be gained by taking the time to reflect and look back at what you have done, what worked, and what didn't. Doing this will enable you to predict what strategy to use to solve future problems."

Okay so far? Now just look at what you have shown. Since the values of $\sqrt{14}$ and $\sqrt{15}$ are difficult to get without a calculator, it would be hard for you to believe this. Polya has shown you how to convince yourself. He will now do this magic one more time. First he adds 8 on both sides to get,

$$8 + 1 + 2 \times \sqrt{14} > 8 + 2 \times \sqrt{15}\ ,$$

which can be rewritten as

$$2 + 7 + 2 \times \sqrt{14} > 3 + 5 + 2 \times \sqrt{15}\ .$$

Agreed? So using the SRP, we have,

$$\sqrt{2 + 7 + 2 \times \sqrt{14}} > \sqrt{3 + 5 + 2 \times \sqrt{15}}\ .$$

Now look closely at this equation. You can see that the Square Equation can be used both on the left hand side and the right hand side. Doing this, we see that the two sides can be rewritten as

$$\sqrt{2} + \sqrt{7} > \sqrt{3} + \sqrt{5}\ .$$

If you find this sort of problem solving amusing and interesting, you can try to take this a little further. What Polya has done is actually shown you a *method* to answer the four-number question. But this method will not work always.

Try to find four whole numbers a, b, c, d, with $a < c$ and $b > d$ where the method breaks down.

–Compiled from several sources