The Ramanujan Machine

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Very recently, on July 23, 2019, a group of 7 researchers from Israel announced something very interesting. They have written an article titled "***The Ramanujan Machine: Automatically Generated Conjectures on Fundamental Constants***", and also built a website where you can go and generate your own mathematical conjectures. Everything about this is interesting and worth explaining, so listen! Or, rather, read on!

What is a conjecture?

The Cambridge dictionary defines the word conjecture as a guess about something based on how it seems and not on proof. That is, just as you would like to guess answers for problems in your exams (and hope to get them right!), the scientists are doing the same thing! See more on Conjectures in the Box.

Of the authors, **Gal Raayoni, George Pisha, Yahel Manor, Doron Haviv, Yaron Hadad** and **Ido Kaminer** are from Technion, the IIT of Israel. (In English it is called Israel Institute of Technology). **Uri Mendlovic** is from Google's Tel Aviv group. Indeed yes, there are people in companies like Google doing mathematical research.

Not a machine

Even though it is called a machine, what the authors offer is not like a machine you might imagine in a factory. Computer scientists call any automated method (algorithm) a machine. This is an algorithm implemented in a computer program. Normally programs take input from someone, do something and produce an output. This program asks you to input a "fundamental constant" *c* and will give you an expression *e*. The conjecture is then defined as "$c = e$" and it is upto you to prove this right or wrong.

For example, you can give the number π as input and the program gives an infinite summation. But you have to check that substituting values on the right gives the correct infinite expansion of π for more and more digits. Invariably, the program is able to assert the equation confidently, and it is hard to prove it wrong.

Fundamental constants

In school we learn that √2 is not rational: it cannot be written in the form p/q where p and q are positive integers with no common factor. But it can be written as the solution to the equation $x^2 - 2 = 0$. In school we encounter π, which is the ratio of circumference to diameter of ANY circle, however large or small. As it turns out, π cannot even be written as the solution to equations like the one above, we need infinite expressions. For instance, $\pi/4 = 1 - 1/3 + 1/5 - 1/7 + ...$ Such numbers are called *transcendental numbers*.

<insert pi_asseries.png and pi_recurring.png here>

There are many famous transcendental numbers like π which are extensively used in mathematics and science. For instance, *e*, the Euler's constant: e = 2.71828 .... For *e*, the Ramanujan machine gives the conjecture as shown.

<insert r-machine.png here>

Is this correct?! It is upto us to find out!!

Continued fractions

The expression on the right hand side of the equation for e is a continued fraction, where you start with a number on top as numerator, and the denominator has another fraction, whose denominator has another fraction, and so on, for ever. Showing that such equations are correct can be very hard, but it is possible and can be learnt (with great joy). See the article in this same issue for some fun with continued fractions.

Srinivasa Ramanujan

The great Indian mathematical genius Srinivasa Ramanujan was fascinated by continued fractions (CFs). It is known that he spent early years of his research exploring many CFs. Not only CFs, he loved all infinitely continuing expressions.

In 1911, Ramanujan posed *Question 289* in *Volume 3* of the **Journal of the Indian Mathematical Society**: does √(1 + 2 √(1 + 3 √(1 + ...))) have a finite number as solution? If yes what is it? *(Six months went by with nobody giving a solution, until Ramanujan provided one himself.)*

In his famous letter to the mathematician **G H Hardy**, Ramanujan gave several CFs that Hardy said: "defeated me completely". Ramanujan famously wrote a number of equations using CFs (which used numbers like *e* all over the place) without proof, and many were only proved several years later by other mathematicians. In fact, it was not until 1987 that all 17 of Ramanujan's series for $1/\pi$ were proved (by the Canadian mathematicians, the **Borwein brothers**).

This is the reason the Israeli authors have called their algorithm "**the Ramanujan machine**". They hope that it reflects the way Ramanujan worked, by deep analysis of patterns in formulas and "seeing" regularities. Their algorithm uses principles of "deep learning", a method that perceives layer upon layer of patterns. It generates CFs based on patterns, then refines them over and over again, testing for how close the CFs get to the known values, making corrections and then looking for new patterns, this process eventually giving the conjecture.

BOX on Conjectures

Mathematicians celebrate not only theorems, statements that are proved to be true, but also conjectures, which are statements not proven either way so far. These are mere statements of belief by somebody, and as the community of mathematicians sharing that belief increases, the conjecture gains importance and more people start thinking about it.

Thus good mathematicians often offer conjectures for others to work on. Some even claim to have results for which there is no understandable proof, and then other mathematicians refer to these statements as conjectures as well. This is merely being cautious, since however great this mathematician might be, there might turn out to be some mistake in it, until it is proven conclusively one way or the other.

For instance, we all learn in school that every positive integer greater than 1 can be written uniquely as a product of prime numbers. This is celebrated as the Fundamental Theorem of Arithmetic, and we not only learn to prove it, we also use it for learning long division. Instead of products, if we think of sums, there is a simple statement: every positive integer greater than 2 can be written as the sum of two prime numbers. This has been verified to be true for integers upto 4,000,000,000,000,000,000 (4×10^{18}), but it is still only a conjecture. It is the famous *Goldbach Conjecture*, named after the German mathematician **Christian Goldbach** who wrote this in a letter dated 7 June 1742 to the mathematical genius **Leonhard Euler**.

END OF BOX

The "paper"

The authors of the Ramanujan machine have written an account of their algorithm and posted it on the "archive". This is a repository (a place where resources can be stored and accessed by other scientists) of scientific articles, but whatever is written there is not considered a "publication" by the scientific community. Their article, called a "paper", has been submitted to a journal where some reviewers (whose names are unknown) are reading it. They will comment on the techniques and results, and if they are convinced, the paper will be published, and only then the "Ramanujan machine" will be accepted by the community and further developed. This process is itself like conjecturing and proving, isn't it?

Whether the conjectures generated by this algorithm are correct or not, it is a fascinating exercise, and may give new ideas to many mathematicians in future.