What is this constant e=2.718281828459045 ...?

We encountered this number denoted as 'e' in connection with the asymptotic formula for integer partitions. It is one of the numbers devised by Euler and hence the notation 'e' in his honour. Why is this number so special and where does it come from?
This number can be obtained in two different ways. One way is to use the properties of an infinite series given by

$$e = 1 + \frac{1}{1} + \frac{1}{1\times 2} + \frac{1}{1\times 2\times 3} + \frac{1}{1\times 2\times 3\times 4} + \cdots = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots \quad (1)$$

Here the dots ($\cdots$) in the right hand side of the equation indicates that the series has infinitely many terms. That is, you can go on adding terms for ever: for instance, the next term will be $(1/5!)$. The notation 5! which is defined as $5! = 5\times 4\times 3\times 2\times 1$ is called the *factorial* of 5.
As you include more terms on the right hand side, the number becomes closer to the true value of 'e'. For example, after 11 terms (that is, including up to $1/10!$), we get $e \approx 2.718281801$. We can continue this including as many terms as we like to generate the required approximation for 'e'.
Another way of generating the value of the constant e is to look at the expression $\left(1+\frac{1}{k}\right)^k$.
When $k = 2$, we have

$$\left(1+\frac{1}{2}\right)^2 = \left(\frac{3}{2}\right)^2 = 2.25. \quad (2)$$

When k=3, we have

$$\left(1+\frac{1}{3}\right)^3 = \left(\frac{4}{3}\right)^3 = 2.3703707 \quad (3)$$

and for k=10, we get

$$\left(1+\frac{1}{10}\right)^{10} = \left(\frac{11}{10}\right)^{10} = 2.59374246. \quad (4)$$

In fact even at k=1,000,000 that is, one million, we get 2.718280469. As k becomes larger and larger the number tends to a limit that is 'e':

$$\lim_{k\to\infty}\left(1+\frac{1}{k}\right)^k = e = 2.718281828459045\ldots\ . \quad (5)$$

Why bother about this constant; why is it important? The answer lies in the fact that it has numerous applications. You may have already noticed that the limiting formula for 'e' bears a resemblance to the compound interest formula that is used in calculating the interest accumulated in bank accounts. The formula for 'e' was used in population biology first by Thomas Malthus and later by Darwin. Indeed we now know that it is an ubiquitous number like π itself. Like π it is also a transcendental number. A transcendental number is a number not a root of a non-zero polynomial equation with integer or rational coefficients.