

Characterizations of e

1. The number e is the unique positive real number such that

$$\frac{d}{dt}e^t = e^t .$$

2. The number e is the unique positive real number such that

$$\frac{d}{dt} \log_e t = \frac{1}{t} .$$

3. The number e is the limit:

$$e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n ; \text{ also}$$
$$e = \lim_{x \rightarrow 0} (1 + x)^{1/x} .$$

4. The number e is the sum of the infinite series:

$$e = \sum_{n=0}^{\infty} \frac{1}{n!} = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots .$$

5. The number e is the unique positive real number such that

$$\int_1^e \frac{1}{t} dt = 1 .$$