On the Mellin transforms of powers of Hardy's function

A. Ivic

Hardy's function $Z(t) = \zeta(\frac{1}{2}+it)\chi(\frac{1}{2}+it)^{-1/2}$, where $\zeta(s) = \chi(s)\zeta(1-s)$, plays an important role in the investigation of zeros of the Riemann zeta-function $\zeta(s)$ on the critical line $Res = 1/2$. Several problems and results on $Z(t)$ are discussed. They include the distribution of values of $Z(t)$, its power moments, Mellin transforms of powers of $Z(t)$ and natural boundaries of Dirichlet series. The Mellin transforms

$$\int_1^\infty Z^k(x)x^{-s}dx \qquad (s = \sigma + it)$$

are discussed in detail when $k = 1, 2, 3$, and connections with the power moments of $|\zeta(\frac{1}{2} + it)|$ are established.

Explicit equivalence for the error terms of primes in short intervals and of the pair correlation conjecture

A. Zaccagnini

This is joint work with Alessandro Languasco (Università di Padova) and Alberto Perelli (Università di Genova).

Let

$$J(X,\theta) := \int_X^{2X} \bigl(\psi(x+\theta x) - \psi(x) - \theta x\bigr)^2 \,\mathrm{d}x$$

denote the Selberg integral and

$$F(X,T) := \sum_{0<\gamma_1,\gamma_2\le T} X^{i(\gamma_1-\gamma_2)} w(\gamma_1-\gamma_2),$$

where $w(u) = 4/(4+u^2)$, be the pair-correlation function for the zeros of the Riemann zeta-function.

We will consider quantitative relationships between the formulae

$$F(X,T) = M_F(X,T) + R_F(X,T),$$

where

$$M_F(X,T) := \frac{T}{2\pi}\log\frac{T}{2\pi} - \frac{T}{2\pi},$$

and

$$J(X,\theta) = M_J(X,\theta) + R_J(X,\theta),$$

where

$$M_J(X,\theta) = \frac{3}{2}\theta X^2(\log(1/\theta) + c)$$

and $c = 1 - \gamma - \log(2\pi)$. In particular, we are interested in the following kind of problem: assuming a bound for $R_F(X,T)$ in a suitable range for T, derive bounds for $R_J(X,\theta)$ in a suitable range for θ and vice-versa.

We adapt techniques due to Montgomery, Heath-Brown, Goldston and, more recently, Chan.

On certain exponential sums over primes

A. Sankaranarayanan

Let $f(x)$ be a real valued polynomial in x of degree $k \geq 4$ with leading coefficient α. In this talk, we prove a non-trivial upper bound for the quantity

$$\left|\sum_{p\leq N}(\log p)\ e(f(p))\right|$$

whenever the leading coefficient α of $f(x)$ is of type 1.

It is a joint work with Helmut Maier.

A Probabilistic Technique for Finding Almost Periods in Additive Combinatorics

Ernie Croot

We (E. Croot and Olof Sisask) introduce a new probabilistic technique for finding 'almost periods' of convolutions of subsets of finite groups. This allows us to give: a new probabilistic proof of Roth's theorem; a new way to approach the 2D corners problem; a new result on the existence of long arithmetic progressions in sumsets A+B; a translation-invariance result for "discontinuous sets" (sets A whose convolution function A*A is somewhat discontinuous); and several non-abelian analogues of classical theorems. In many cases, the proofs we give are the shortest known, and in some cases, like the case of long APs in A+B, we obtain the strongest bounds to date.

On the average number of Goldbach representations of an integer

A. Languasco

Let Λ be the von Mangoldt function and $R(n) = \sum_{h+k=n} \Lambda(h)\Lambda(k)$ be the counting function for the Goldbach numbers.

In a joint paper with Alessandro Zaccagnini (Università di Parma) we proved, assuming the Riemann Hypothesis holds, the following statement:

Let $N \geq 2$. Then

$$\sum_{n=1}^{N} R(n) = \frac{N^2}{2} - 2\sum_{\rho} \frac{N^{\rho+1}}{\rho(\rho+1)} + \mathcal{O}\big(N \log^3 N\big), \tag{1}$$

where $\rho = 1/2 + i\gamma$ runs over the non-trivial zeros of the Riemann Zeta-function $\zeta(s)$.

The first result of this kind was proved in 1991 by Fujii who subsequently improved it until reaching the error term $\mathcal{O}\big((N \log N)^{4/3}\big)$. Then in 2007-08 Granville gave an alternative proof of the same result and, in 2010, Bhowmik and Schlage-Puchta were able to reach the error term $\mathcal{O}\big(N \log^5 N\big)$.

The limit of the method seems to be $\mathcal{O}\big(N \log^2 N\big)$; the loss of a factor $\log N$ is due to a dissection argument used in the proof. If one admits the presence of some suitable weight in the average, this loss can be avoided.

Prime chains and applications

Kevin Ford

A sequence of primes $p_1, ..., p_k$ is a "prime chain" if $p_j | (p_{j+1} - 1)$ for each j. For example: $3, 7, 29, 59, 709$. We describe new estimates for counts of prime chains satisfying various properties, e.g. the number of chains with $p_k < x$ (k variable) and the number of chains with $p_1 = p$ and $p_k < x$. We discuss some applications of these estimates, in particular the settling of a 50-year old conjecture of Erdos that $\phi(a) = \sigma(b)$ has infinitely many solutions (ϕ is Euler's function, sigma is the sum of divisors function). We also focus on the distribution of $H(p)$, the length of the longest chain ending at a given prime p. $H(p)$ is also the height of the "Pratt tree" for p, the tree structure of all chains ending at p. We give new, nontrivial bounds for $H(p)$, valid for almost all p, and settle a conjecture of Erdos, Granville, Pomerance and Spiro from 1990.

We introduce and analyze a random model of the Pratt tree, based on branching random walks, which leads to some surprising conjectures about the distribution of $H(p)$. This is joint work with Sergei Konyagin and Florian Luca.

Sums of Kloosterman sums over arithmetic progressions

J. Sengupta

In this talk we will give an estimate of sums of classical Kloosterman sums $S(m, n, c)$ weighted by the inverse of the moduli c over arithmetic progressions. We will in particular bring out the dependence of the estimate in the parameters m and n. We will use the spectral theory of automorphic forms to derive our result.

Density of Automatic sequences and distribution modulo 1 in $\mathbb{F}_q(X)$

Jean-Marc Deshoulliers

Let $k \geq 1$ be an integer. We are interested in the density of sequences $\mathcal{U} = (u_h)_{h>1}$ of integers which are recognized by a finite k-automaton (*). Introducing those sequences in 1972, Cobham proved that they always have a logarithmic density, that is to say that the limit $\lim_{x\to\infty}(1/\log x)\sum_{1\leq u\leq x}(1/u)$ exists. We shall give a sufficient condition for an automatic sequence to have a natural asymptotic density, that is to say that the limit $\lim_{x\to\infty}(1/x)\sharp\{u \in \mathcal{U} | u \leq x\}$ exists. Our interest in those questions has been recently revived by a joint work with Florian Luca on the density of the integers n such that $n!$ is a sum of three squares (cf. Functiones et Approximatio 39 (2008) 11-20). This led us to revisit the question of the existence of the distribution modulo 1, i.e. modulo $\mathbb{F}_q[X]$, of the sequences $(\theta^n)n$, where θ is an element in $\mathbb{F}_q(X)$ (cf. Recent progress in analytic number theory, Acad. Press (1981) vol. 2, 69-72, and the joint paper with Jean-Paul Allouche Pub. Math. Orsay, vol. 88-02 (1988), 37-47). Although some progress has been performed, the main question remains opened. (*) It means that there exists a finite directed graph of order k, where the k edges starting from any vertex are labeled from 1 to k and which admits two special vertices named input and output. An integer $u = \sum_{t=0}^{T}\epsilon_t(u)k^t$ is said to be recognized if and only if, when we start from the input and follow the edges numbered by the digits $\epsilon_t(u)$ in base k, we end up on the output edge.

On the sumset partition problem

Anna Llano Sanchez

A sequence $m_1 \geq m_2 \geq \cdots \geq m_k$ of k positive integers is n-realizable if there is a partition $X_1, \ldots, X_k$ of the set $\{1, 2, \ldots, n\}$ such that the sum of the elements in X_i is m_i for each $i = 1, 2, \ldots, k$.

We consider the modular version of the problem and prove that all sequences in $\mathbb{Z}_p$ of length $k \le (p-1)/2$ are realizable for any prime $p \ge 3$. The bound on k is best possible. The result is obtained trough an application of the Combinatorial Nullstellensatz of Alon. As a corollary we obtain a new proof, for n a prime, of the fact that such a sequence is n–realizable if $m_{k-1} \ge n$, a result of Chen, Fu, Wang and Zhou.

For the integer case, we characterize all n-realizable sequences with $k \le 4$. For general k, it is shown that $n \ge 4k-1$, $m_k > 4k-1$ are sufficient conditions for n–realizability. We conjecture that these sufficient conditions are asymptotically best possible.

We also obtain characterizations for n-realizable sequences whose elements are almost all below n using some results on complete sets of integers. These characterizations complement the one used in connection with the ascending subgraph decomposition conjecture of Alavi et a., which is at the origin of the problem.

Hilbert cubes in multiplicatively defined sets

C. Elsholtz

Let $a_0, a_1, \ldots, a_d$ denote positive integers and let $a_0 + \{0, a_1\} + \cdots + \{0, a_d\} \subset S$ be a so called Hilbert cube, which is inside a given set of positive integers S.

Such cubes have been studied for example by Hilbert, Szemerédi, Hegyvári, Sárközy, Stewart, Wood, Solymosi, and are connected to the Gowers norm.

For various sets S we give an improvement of the maximal dimension of a cube:

1) If S is the set of squares $x^2 \le N$.

2) If S is a set without an arithmetic progression.

3) We also study sets consisting of specified prime factors only.

This is joint work with R. Dietmann (1 and 2), and I. Ruzsa (3).

Random multiplicative functions

K. Soundararajan

Some Ramsey results for equations in groups

O. Serra

Arithmetic Ramsey Theory can be seen as the study of the existence of monochromatic structures, like solutions of linear systems, in every coloring of sets of integers. Analogous problems can be considered for other color patterns, particularly of rainbow structures (the elements receive pairwise distinct colors).

These problems behave in a particular nice way when considered in finite groups, where the counting of monochromatic or rainbow structures can be handled usually in an easier way. For instance, it can be shown that, in a three coloring of a finite group, twice the number of monochromatic solutions minus the number of rainbow solutions of a linear equation depends only on the cardinality of the color classes but not on the distribution of the colors. On the other hand, the three colorings which are free of 3-term rainbow arithmetic progressions can be characterized and, for a fixed number of colors, linear systems which admit monochromatic solutions in every sufficiently large abelian group can also be characterized. The talk will survey these and other recent results in the area.

Geometry of numbers in Additive Number Theory.

G. Bhowmik

We describe how problems on zero-sums in finite abelian groups can be reduced to problems in real combinatoric geometry by identifying the group $\mathbb{Z}_p$ with the one dimensional real torus $\mathbb{R}/\mathbb{Z}$ for a large prime number p.

As applications we give an upper bound for the largest possible size of a sequence without a zero-sum of length p in $\mathbb{Z}_p^2$ as well as inverse theorems for $\mathbb{Z}_p^2$.

On Shimura-Shintani Correspondence

M. Manickam

Prime number races: An asymptotic formula for the densities

Greg Martin

Given two reduced residue classes a and $b \pmod q$, let $\delta(q;a,b)$ be the "probability", when x is "chosen randomly", that more primes up to x are congruent to $a \pmod q$ than are congruent to $b \pmod q$ (Rubinstein and Sarnak defined this quantity precisely as a logarithmic density). In joint work with Daniel Fiorilli, we give an asymptotic series for $\delta(q;a,b)$ that can be used to calculate it for arbitrarily large q. The asymptotic formula has theoretical ramifications as well: for example, it allows us to compare the relative sizes of the $\delta(q;a,b)$ as a and b vary over residue classes $\pmod q$.

Subconvexity for Rankin-Selberg convolution in the level aspect

R. Munshi

We will discuss subconvex bounds for the central value of the Rankin-Selberg L-function of two cusp forms with fixed weights and varying co-prime levels.

Fibonacci integers

F. Luca

Let G be the multiplicative group generated by the Fibonacci numbers. Since $L_m = F_{2m}/F_m$, the Lucas numbers belong to G. In my talk, I will present upper and lower bounds for the counting function of the positive integers $n \leq x$ which belong to G. Based on joint work with Carl Pomerance and Stephan Wagner.

Romanoff Theorem in a Sparse Set

Yong-Gao Chen

The main results are: (a) For any subset A of positive integers, the number of positive integers which are less than x and can be represented as $2^k + p$ is more than $0.030997A(\log_2 x)\pi(x)$ for all sufficiently large x, where $k \in A$ and p is prime; (b) For any subset A of positive integers, the number of positive integers which are less than x and can be represented as $p - 2^k$ is more than $0.030997A(\log_2 x)\pi(x)$ for all sufficiently large x, where $k \in A$ and p is prime. As applications, we have (a) the number of positive integers which are less than x and can be represented as $2^q + p$ is more than $0.0447x/\log\log x$ for all sufficiently large x, where p and q are prime; (b) the number of positive integers which are less than x and can be represented as $p - 2^q$ is more than $0.0447x/\log\log x$ for all sufficiently large x, where p and q are prime. Several related open problems and conjectures are posed.

On long k-tuples with few prime factors

O. Ramare

We present our latest investigation on the weighted sieve. As a corollary, we will present a proof that, when k is large enough, there are infinitely many k-tuples $(n+h_1, \cdots, n+h_k)$ such that $1/k \sum_{1\le i\le k}(\omega(n+h_i)-\log k)^2 \ll \log k$.

Congruences for Jacobi sums of order l^2

S. A. Katre

Jacobi sums and cyclotomic numbers of order e are interrelated. The congruences for Jacobi sums of prime order l are known in the literature, however for order l^2, the congruences are known only for $l^2 = 9$. In this talk we discuss how the congruences for Jacobi sums can be obtained in terms of cyclotomic numbers of order l. These congruences determine Jacobi sums of order l^2, using the absolute value and the prime ideal decomposition.

Secondary Terms in the Davenport-Heilbronn Theorems

Frank Thorne

In a 1971 paper, Davenport and Heilbronn proved asymptotic formulas for the number of cubic fields of bounded discriminant, and the amount of 3-torsion in the class groups of quadratic fields of bounded discriminant. However, numerical experiments revealed that their formulas were a poor match for the data.

In the cubic field case, Roberts conjectured that this poor match is explained by a secondary term of order $X^{5/6}$, and we will prove his conjecture and the analogous statement for 3-torsion in class groups. Our work is independent of another proof of Roberts' conjecture by Bhargava, Shankar, and Tsimerman, and uses the theory of Shintani zeta functions. This is joint work with Takashi Taniguchi.

Growth in solvable groups

Harald Helfgott

Let $G \subset GL_n(K)$ be a solvable group, K a finite field. Let A be a subset of G. We show under which circumstances it may happen that A fails to grow rapidly under the group operation.

Weighted sums in Finite abelian groups

S. D. Adhikari

For an abelian group G, the Davenport constant $D(G)$ is defined to be the smallest natural number k such that any sequence of k elements in G has a non-empty subsequence whose sum is zero (the identity element). Here, after describing some developments around the notion of Davenport constant with weights, we sketch the proof of a theorem (following some idea used by Yu in proving a result of Bollobás and Leader) which will imply a result of Hamidoune which had confirmed a conjecture of Caro in a special case.

Heights on abelian varieties

Sinnou David

we shall present conjectures on lower bounds for the Neron-Tate height of non torsion points of an abelian variety and their higher dimensional counterparts. Usual techniques to tackle these questions, constructive in nature also yield information on torsion points. We shall describe them go through new results.

On the p-adic closure of a subgroup of rational points on a simple Abelian variety

M. Waldschmidt

Let A a simple abelian variety over $\mathbb{Q}$ of dimension g, Γ a subgroup of $A(\mathbb{Q})$ or rank ℓ over $\mathbb{Z}$, p a prime number, $\log : A(\mathbb{Q}_p) \to T_A(\mathbb{Q}_p)$ the canonical map from the p–adic Lie group $A(\mathbb{Q}_p)$ to the p–adic Lie algebra $T_A(\mathbb{Q}_p)$ and r the dimension of the $\mathbb{Z}_p$–space spanned by $\log \Gamma$ in $T_A(\mathbb{Q}_p)$. The upper bound $r \leq \min\{g, \ell\}$ is plain, and one conjectures $r = \min\{g, \ell\}$. The complex analog of this conjecture is related with a conjecture of B. Mazur.

Transcendence methods yield the lower bound

$$r \geq \frac{\ell g}{\ell + 2g}\cdot$$

As a consequence,

$$r \geq \frac{1}{3} \min\{g, \ell\}.$$

Another consequence is that the condition $\ell > 2g(g-1)$ implies $r = g$.

On the Jacobsthal function and a conjecture of Pomerance

N. Saradha

A positive integer k is said to be a $P-$ integer if the first $\phi(k)$ primes which are coprime to k form a reduced residue system. We shall discuss a conjecture of Pomerance on these $P-$ integers and how Jacobsthal function plays a role in this problem.

The divisor problem for binary cubic forms and applications

Tim Browning

A classical problem in analytic number theory is to estimate the average order of the divisor function as it ranges over the values of polynomials. I will discuss this problem when the polynomial is a reducible binary cubic form and show how the resulting estimates provide a new approach to Manin's conjecture on the density of rational points on suitable algebraic varieties.

On a problem of Ramachandra and the zeta-function on the line $\mathrm{Re}(s) = 1$.

Johan Andersson

We show some new results for the Riemann zeta-function in short intervals on the line $\mathrm{Re}(s) = 1$. We prove the following result:

$$\inf_T \int_T^{T+\delta} |\zeta(1+it)|\, dt = \frac{\pi^2 e^{-\gamma}}{24}\delta^2 + O(\delta^4),$$

as well as a corresponding result for the inverse of the Riemann zeta-function. This improves on results presented by the speaker at conferences in Seoul (lower bound of order $\delta^{2+\epsilon}$) and Mumbai (lower bound of order δ^2), and ultimately improves on results of Balasubramanian and Ramachandra. We also discuss a related problem of Ramachandra and show some effective results on this problem.

Poles and zeros of L-functions satisfying Mass's functional equation

Ravi Raghunathan

We study the possible numbers and locations of the poles of Dirchlet series satisfying Maass's functional equation. We give an application of our main result to comparing the zero sets of two different L-functions.

The number of rational numbers determined by large sets of integers.

D. Suryaramana

The talk reports on a paper of J. Cilleruelo, D.S. Ramana and O. Ramare by the same title. In this paper it is shown that when A and B are subsets of the integers in $[1, X]$ and $[1, Y]$ respectively, with $|A| \geq \alpha X$ and $|B| \geq \beta Y$, the number $|A/B|$ of rational numbers expressible as a/b with (a, b) in $A \times B$ satisfies $|A/B| \gg (\alpha\beta)^{1+\epsilon} XY$ for any $\epsilon > 0$, where the implied constant depends on ϵ alone. We then construct examples that show that this bound cannot in general be improved to a bound of the shape $|A/B| \gg \alpha\beta XY$. We also resolve the natural generalisation of our problem to arbitrary subsets C of the integer points in $[1, X] \times [1, Y]$. Finally, we apply our results to answer a question of Sárközy concerning the differences of consecutive terms of the product sequence of a given integer sequence.

Doubling-Critical Sets in Binary Spaces

Vsevolod F. Lev

We say that a subset A of an abelian group is *doubling-critical* if, for any proper subset $B \subset A$, we have $2B \neq 2A$; that is, removing any element from A affects its doubling $2A$. Motivated by applications in finite geometries, we determine the largest possible size of a doubling-critical set in an elementary abelian 2-group, and give a complete classification of all "large" doubling-critical sets.

The talk is based on a joint paper with David Grynkiewicz.

Universality of the Selberg zeta-function for the modular group

Ramunas Garunkstis

In 1975 S.M. Voronin proved that the Riemann zeta-function is universal in the strip $1/2 < \sigma < 1$. Here we, jointly with P. Drungilas and A. Kacenas, prove that the Selberg zeta-function for the modular group is universal in the strip $0.848... < \sigma < 1$. This is the first example of order two universal zeta-function. To prove the above result, we obtain the universality theorem for a wide class of general Dirichlet series.

Some applications of the generating function of harmonic numbers to the theory of the Riemann zeta function

R. Padma

This talk consists of two applications of the generating function of harmonic numbers. In the first part we discuss the analytic continuation of the Dirichlet series related to harmonic numbers and their alternating variants and find their values at the negative integers (or the residues at the poles) in terms of Bernoulli and Euler numbers. In the second part we combine this generating function with an axiomatization of the calculus of the Ramanujan summation of the divergent series which puts Ramanujan's ideas on a rigorous footing. This leads to new identities connecting the divergent Euler sums to the derivatives of the Riemann zeta function at the negative integers. The first part was done in collaboration with K. N. Boyadzhiev and H. G. Gadiyar and the second part with B. Candelpergher and H. G. Gadiyar.

The second moment of Dirichlet twists of Hecke L-functions

Rizwan Khan

We are interested in the central value of the L-function of a cusp form twisted by a primitive Dirichlet character. We find the square of the central value on average over all primitive characters of a large modulus q, for almost all q.

An equi-distribution theorem

R. Thangadurai

Let $\{H_n\}$ denote a sequence of subgroups of $(\mathbb{Z}/n\mathbb{Z})^*$ of cardinality satisfying that $|H_n|/\sqrt{n} \to \infty$ as $n \to \infty$. Then we prove that the sequence $\{H_n\}$ is set equi-distributed modulo 1. As a consequence, we shall see that for large enough n's the elements of any subgroup H_n with $|H_n|/\sqrt{n} \to \infty$ as $n \to \infty$ are well-spread in $(\mathbb{Z}/n\mathbb{Z})^*$. This is a joint work with M. Ram Murty.

Special values of certain Dirichlet L-functions

B. Ramakrishnan

In this talk we report about our work on obtaining expressions for certain Dirichlet L-functions involving Jacobi symbols.
