Transport and spectra in the half-filled Hubbard model

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### Promises



- Hubbard model and half-filling
- Dynamical mean-field theory (DMFT) + iterated perturbation theory (IPT)
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- $_{2}$  Resistivity: Comparison with expt. NiS $_{2-x}$ Se $_x$
- $\bigcirc$  Resistivity: Thermal hysteresis Cr-doped V $_2O_3$
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Minimal model to depict correlation effect: competition between kinetic/hopping  $(t_{ij})$  and on-site (local) Coulomb repulsion U

Single-band Hubbard Hamiltonian:

$$\hat{H} = -\sum_{\langle ij \rangle,\sigma} t_{ij} (c^{\dagger}_{i\sigma} c_{j\sigma} + \text{h.c.}) + \epsilon_d \sum_{i\sigma} c^{\dagger}_{i\sigma} c_{i\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$
(1)

 $\epsilon_d \Rightarrow \text{Orbital energy}$  $c^{\dagger}_{i\sigma}(c_{i\sigma}) \Rightarrow \text{creates (annihilates) an } e^- \text{ of spin } \sigma \text{ at site } i$ 

Half-filled when one electron per site:  $\langle n_i \rangle = 1 \Rightarrow \epsilon_d = -U/2$ 

Exact solution not available except in 1D (Lieb and Wu, 1968).

# Dynamical mean-field theory (DMFT)



• Write an effective action in term of mean-field single-site Green's function *G* at site-0

$$S_{\text{eff}} = -\int_{0}^{\beta} d\tau \int_{0}^{\beta} d\tau' \sum_{\sigma} c_{0\sigma}^{\dagger}(\tau) \mathcal{G}_{0}^{-1}(\tau - \tau') c_{0\sigma}(\tau') + U \int_{0}^{\beta} d\tau \, n_{0\uparrow}(\tau) n_{0\downarrow}(\tau)$$
(2)

- Lattice problem maps on to a self-consistent single-impurity problem ⇒ mean-field since spatial degrees of freedom integrated out.
- Dynamic since temporal information retained through  $\tau, \tau'$ .
- Exact mapping in the limit of  $\infty$  coordination number z (or  $\infty$  dimension) as the hopping amplitude  $t_{ij}$  is scaled as  $t_*/\sqrt{z}$ .
- Self-energy and vertex function becomes momentum k independent.

# **DMFT: Implementation**

- $\begin{array}{c} \textbf{General} \\ \textbf{i)} \text{ guess } \mathcal{G} \text{ or } \Sigma \end{array}$ 
  - ii) impurity problem solver (IPS)  $\Rightarrow \Sigma \& G$
  - iii) Calc.  $\mathcal{G}$  from Dyson's eq.
  - iv) Repeat ii)-iii) until self-consistency achieved.

# Our IPS: Iterated perturbation theory (IPT) approx: (2nd order about Hartree term).

$$\Sigma_2(\omega) = \lim_{i\omega \to \omega^+} \frac{U^2}{\beta^2} \sum_{m,p} \mathcal{G}(i\omega + i\nu_m) \mathcal{G}(i\omega_p + i\nu_m) \mathcal{G}(i\omega_p)$$

Specific

- Real freq. approach in contrast to conventional Matsubara freq. approach
- Major advantages: can capture T = 0 features and avoid rigor of analytical continuation







# Benchmark: Mott transition at T = 0



#### Spectral function= $-\frac{1}{\pi}$ Im $G(\omega)$

#### U increasing

#### U decreasing



Expected Mott metal-insulator transition observed.

Different transition points  $U_{c1}$  and  $U_{c2}$ : Hysteresis

### Resistivity: expt. on $NiS_{2-x}Se_x$



 $\omega_L = Zt_*, Z$ =quasiparticle residue

'Coherent' peak features governed by the associated low-E scale.

### Thermal hysteresis, V<sub>2</sub>O<sub>3</sub>



 $U = 3.6t_*$  gives  $t_* = 0.63$  eV (~ first-principle value)

# Why thermal hysteresis?



#### Phase diagram geometry plays a role.



Hysteresis occurs only when both spinodal lines are crossed.

## Further investigation



#### Avalanches, Barkhausen noise





#### Further investigation Random Resistor Network model to Random-field Ising model (RRNM+RFIM)

Reason for such mapping: Kotliar '99, Kirkpatrick and Belitz '95, Limelette et al. '03, Papanikolaou et al. '08 (Ising university class)



# Beyond IPT: Local moment approach (LMA)



• Based on unrestricted Hartree fock (UHF) host Green's function (*Cf.* restricted-HF for IPT).

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- Fermi liquid metal obtained by restoring back the broken symmetry.
- Can promise to capture the low-energy scale at strong-coupling (*U* close to MIT) when the polarization propagators considered in the self-energy.



### Summary



- Hubbard model is the minimal model to describe the Mott physics and features can be realized through. DMFT+ impurity problem solver (IPS).
- IPT is such an IPS (approximate), yields MIT with 2 transition pt.s  $U_{c1}$  and  $U_{c2}$ .
- Universality in the resistivity peak when scaled by low-energy Fermi liquid scale.
- IPT can mimic the thermal hysteresis observed in real materials, e.g. V<sub>2</sub>O<sub>3</sub>. Can be understood by geometry of the phase diagram.
- However, DMFT misses the avalanches phenomena. Can be realised through RRNM+RFIM based models.
- To capture the strong-coupling behaviour other IPS, e.g. LMA could be a potential candidate.

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