

Fermi and the Theory of Weak Interactions

G Rajasekaran

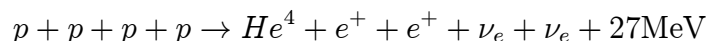


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The history of weak interactions starting with Fermi's creation of the beta decay theory and culminating in its modern avatar in the form of the electroweak gauge theory is described. Discoveries of parity violation, matter–antimatter asymmetry, W and Z bosons and neutrino mass are highlighted.

Introduction

Sun gives us light and heat that makes life possible on the Earth. How do the Sun and stars produce energy and continue to shine for billions of years? Thermonuclear fusion is the answer as Eddington proposed in 1920 and Bethe demonstrated explicitly in 1939. Through a series of nuclear reactions, four protons (which are hydrogen nuclei) in the core of the Sun combine to form a helium nucleus emitting two positrons and two neutrinos and releasing 27 MeV of energy:



This can be regarded as the most important reaction for all life, for without it there can be no life on Earth!

The above reaction is caused by one of the basic forces of Nature, called weak interaction. Beta decays of nuclei and in fact the decays of most of the elementary particles are now known to be due to weak interaction.

Enrico Fermi formulated the theory of weak interactions in 1934 and his theory has stood the ground very successfully with appropriate amendments and generalizations and finally served as a core part of the Standard Model of High Energy Physics, which is now known as the basis of almost *all* of physics, except for gravitation.

Keywords

Fermi, beta decay, parity violation, electroweak theory, neutral current, quarks and leptons, neutrino mass.



In this article we trace the historical evolution of the theoretical ideas punctuated by the landmark experimental discoveries. This history can be divided into two parts separated by the year 1972 which marks the watershed year since the gauge theoretic revolution that converted Fermi's theory into the modern electroweak theory occurred roughly around that year. Discovery of P and CP violation as well as the discovery of neutrino oscillation and neutrino mass are woven into this tapestry as integral parts of weak interaction physics and its history.

Early History: Weak Interactions Upto 1972

The story of weak interactions starts with Henri Becquerel's discovery of radioactivity in 1896 and its subsequent classification into alpha, beta and gamma decays of the nucleus by Ernest Rutherford and others. But the real understanding of beta decay in the sense we know it now came only after Enrico Fermi invented a physical mechanism for the beta decay process in 1934.

The basic ingredient for Fermi's theory had been provided by Wolfgang Pauli. To solve the puzzle of the continuous energy spectrum of the electrons emitted in the beta decay of the nuclei, Pauli had suggested that along with the electron, an almost massless neutral particle also was emitted. Fermi succeeded in incorporating Pauli's suggestion and thus was born the theory of weak interactions. Fermi also named the particle as neutrino.

Drawing an analogy with electromagnetic interaction which at the quantum level is the emission of a photon by an electron, Fermi pictured the weak interaction responsible for the beta decay of the neutron as the emission of an electron-neutrino pair, the neutron converting itself into a proton in the process (*Figure 1*).

By initiating Quantum Electrodynamics, Dirac had laid the foundation for Quantum Field Theory (QFT) in 1927. Within a few years Fermi made the first non-

Pauli proposed the neutrino and Fermi created his theory of weak interactions incorporating the neutrino.



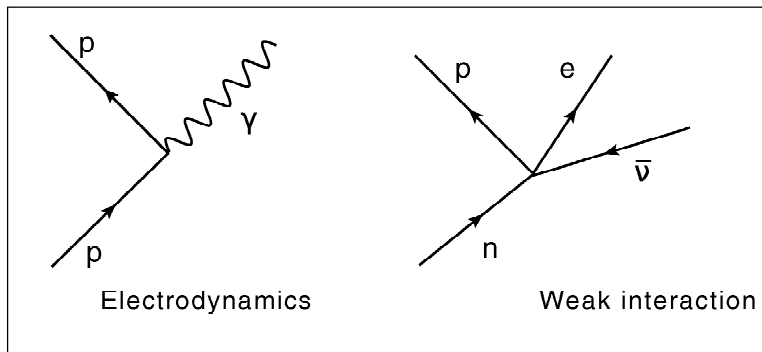


Figure 1. Fermi's analogy.

trivial application of QFT to weak interactions in which material particles are created.

Either because of the neutrino which most people at that time did not believe in, or because of QFT which most people did not understand at that time or because of both, Fermi's note on beta decay theory was rejected by *Nature* with the comment "it contained speculations too remote from reality to be of interest to the reader". Fermi then sent the paper to *Nuovo Cimento* which accepted it; another version was published in *Zeitschrift* [1,2]. (A vivid picture of those times is given in [3]).

In Quantum Electrodynamics (QED) the electromagnetic current J_E of the charged particle like the electron interacts with A , the electromagnetic vector potential which becomes the field operator for the photon:

$$L_E = eJ_E A.$$

The symbol e is the numerical value of the electrical charge of the electron and characterizes the strength of the electromagnetic interaction. In Fermi's theory of weak interactions, the weak current of the proton-neutron pair written as $\bar{p}n$ interacts with the weak current of the electron-neutrino pair denoted by $\bar{e}\nu$:

$$L_F = \frac{G_F}{\sqrt{2}}(\bar{p}n \bar{e}\nu + \bar{n}p \bar{\nu}e) ,$$

Within a few years of the founding of Quantum Field Theory by Dirac, Fermi made its first nontrivial application to beta decay in which material particles are created.



where the particle symbols p, n, e, ν , etc., represent the corresponding field operators. Detailed explanation of the language of QFT needed to understand QED and Fermi's theory is given in *Box 1*. The strength of the weak interaction is characterized by the Fermi coupling constant G_F whose value is

$$G_F = \frac{10^{-5}}{m_p^2}$$

where m_p is the proton mass. It is because of the smallness of this number that this interaction is called 'weak' in contrast to the nuclear force which is 'strong'.

Box 1. Quantum Field Theory (QFT)

Quantum Field Theory created by Dirac and used by Fermi to describe weak interactions remains to this day as the correct basic language to understand *all* high energy physics. Here we give an elementary account of its symbolism and interpretation.

We start with Quantum Electrodynamics. The electromagnetic force between two charged particles such as proton and electron can be represented by the diagram in *Figure A*. The wavy line denotes the photon which is the quantum of the electromagnetic field and it is exchanged between the proton and the electron. It is this exchange that leads to the electromagnetic force between the charged particles. This is the quantum version of the classical picture where the proton is considered to produce the electromagnetic field which then influences the electron placed in the field.

In QFT, the range of a force is inversely proportional to the mass of the quantum that is exchanged. Since photon mass is zero, the electromagnetic force mediated by the exchange of photons is of infinite range.

We shall make a rather liberal use of pictorial representations of interactions and processes such as in *Figure A*. These are called Feynman diagrams, after Feynman whose use of such diagrams in an intuitive interpretation of complex calculations in QFT was an important step in the elucidation of the fundamental processes of Nature.

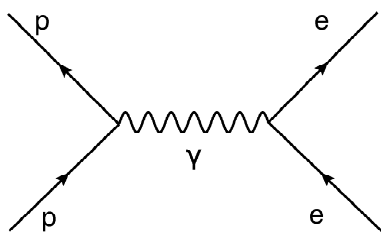


Figure A. Exchange of photon.

Box 1. Continued...

One can split up the above diagram into two parts, one describing the emission of the photon by the proton and the other describing the absorption of the photon by the electron. So the basic QED interaction in symbolic form is written as $eJ_E A$. Here A denotes the quantum field operator for the photon and J_E denotes the electromagnetic current of the charged particles.

The photon field operator A is actually the quantum version of the vector potential of classical electrodynamics (from which the electric field and magnetic field can be obtained by taking suitable space and time derivatives). So all the fields become quantum operators whose main property is that they can either create or annihilate particles. In this case, A can create or annihilate a photon, thus explaining the emission or absorption of a photon.

Further the current J_E also is composed of field operators, but field operators of the charged particles like proton and electron. In QFT every particle is a quantum of the corresponding field; electron is a quantum of the electron field. Denoting the proton and electron field operators by p and e respectively, we write J_E symbolically as a sum of terms $\bar{p}p, \bar{e}e$, etc.

In general, a field operator such as p can either annihilate a particle or create the corresponding antiparticle while a field operator with a bar above such as \bar{p} can either annihilate an antiparticle or create a particle. The antielectron is the positron. In general the antiparticle is different from particle. But for photon antiparticle is not different; antiphoton is the same as photon and so A annihilates or creates the photon only. Hence the interaction described by the combined operator $\bar{e}eA$ actually describes the 8 processes given in *Figure B*. These describe the emission or absorption of a photon by an electron or positron and also the annihilation or creation of an electron–positron pair. Similar processes exist with proton and antiproton.

In all Feynman diagrams time increases vertically upwards. Particles and antiparticles are distinguished by having their arrows in the upward and downward directions respectively.

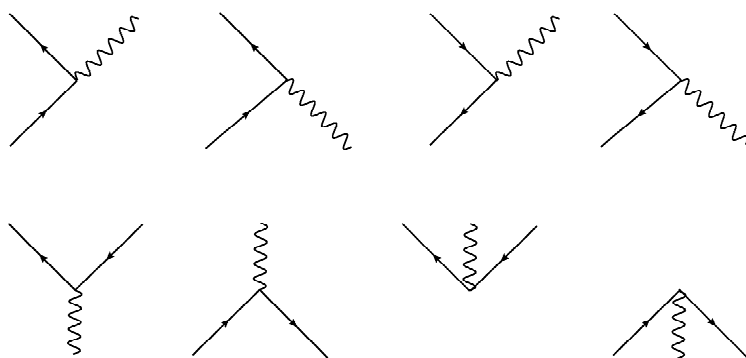


Figure B. The basic QED processes.

Box 1. Continued...



Box 1. Continued...

All the processes in *Figure B* are virtual processes to be used as basic elements in building the diagrams of real physical processes, the simplest of them being the scattering of an electron by a proton depicted in *Figure A*.

This is the basic symbolism of QFT that has to be kept in mind when we describe Fermi's theory and its subsequent development.

In Fermi's interaction L_F (given in the main text) the terms $\bar{p}n \bar{e}\nu$ and $\bar{n}p \bar{\nu}e$ contain the field operators p, n, ν and e and their barred counterparts. These create or annihilate the corresponding particle or antiparticle as explained above. Hence L_F leads to many related weak processes apart from the decay of proton and neutron.

Particles or quanta of fields come in two varieties, bosons and fermions. Bosons are particles with integral spins, photon with spin 1 being their main representative and they follow Bose–Einstein statistics. Fermions have half-integral spins, electron with spin $\frac{1}{2}$ being a fermionic example and they follow Fermi–Dirac statistics¹. Since in Fermi's L_F four fermionic lines meet at a point, it is also called the four-fermion interaction.

¹ S Chaturvedi, Fermi and his Statistics, *Resonance*, this issue,

The two terms in Fermi's L_F give rise to the decays of the neutron and proton:

$$n \rightarrow p + e^- + \bar{\nu},$$

$$p \rightarrow n + e^+ + \nu.$$

Although proton does not decay in free space since it is lighter than neutron, such decays can occur when nuclei are involved. These two terms lead to all nuclear beta decays.

The electric current J_E and the vector potential A are vectors and so Fermi adopted the vector form for the weak currents also. We suppress the vector indices in J_E and A and further we do not write Fermi's weak currents in the vector form, for simplicity of presentation.

This theory of weak interactions proposed by Fermi almost 80 years ago purely on an intuitive basis stood the test of time inspite of many amendments that were incorporated into Fermi's theory successfully. One important amendment came after the discovery of parity



violation in weak interaction by T D Lee, C N Yang and C S Wu in 1956. (See *Box 2*.) But Fermi's theory survived even this fundamental revolution and the only modification was to replace the vector current of Fermi by an equal mixture of vector (V) and axial vector (A) currents. Vectors and axial vectors behave differently when we go from left to right-handed coordinate systems and hence the parity violation. This is the V-A form discovered by Sudarshan and Marshak, Feynman and Gell-Mann and Sakurai in 1957.

Box 2. Violation of Left-Right Symmetry and CP

Left-right symmetry is also called reflection symmetry or parity symmetry and is denoted by P. Madam Wu's famous experiment which established P violation was done using the beta decay of Co^{60} nuclei. She aligned the spins of cobalt nuclei by an external magnetic field produced by a circulating current and counted the number of beta electrons emitted in all directions. She found more beta electrons emitted in the direction of the magnetic field as compared to the opposite direction. This was the discovery of P violation.

In the right-handed coordinate system, the directions of the x , y and z axes are such that, if we imagine a screw (actually a right-handed screw which is what we normally use) being rotated from x to y , the screw will advance along z . The left-handed coordinate system is obtained by mirror-reflection. (See *Figure A*).

Can the laws of physics distinguish between the two coordinate systems? Except for the weak interaction, all other laws of physics are symmetric under mirror reflection and hence cannot be used to distinguish between the left and right coordinate systems.

The significance of this left-right symmetry, as well as its violation can be appreciated better, if we think of the following attempt at intergalactic communication.

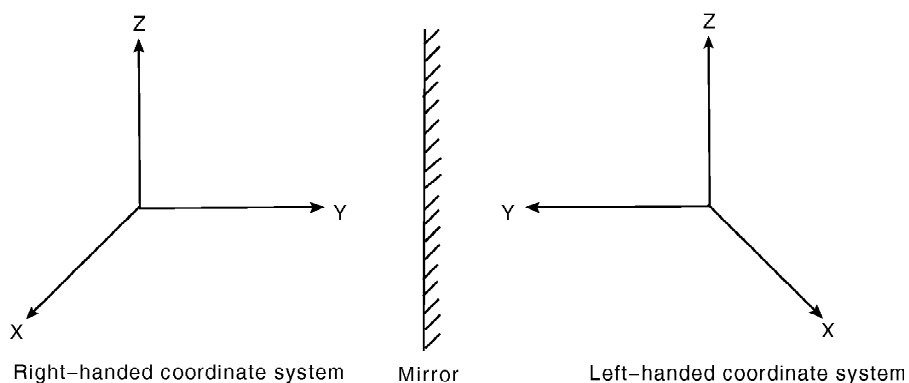


Figure A. Reflection in a mirror.

Box 2. Continued...

Box 2. Continued...

Suppose we want to communicate with somebody in a distant galaxy through radio waves. How do we define a right-handed coordinate system for him? Screws will not help here since we do not know whether they use a right-handed screw or a left-handed one in that galaxy! We can use any of the laws of physics for this purpose. If none of the laws distinguishes between the two coordinate systems, we will never be able to convey a definition of the right-handed coordinate system to a being in a distant part of the Universe.

However, thanks to weak interactions, this can be done. The following instruction can be conveyed: "Take Co^{60} nuclei and arrange a sufficient number of electrons to go around these nuclei, thus forming an electric current. If a rotation from the x axis to the y axis is in the direction of the circulating electrons, then the z axis is the direction in which more electrons are emitted." This would define the right-handed coordinate system for our friend in the distant galaxy. (See *Figure B*). Thus weak interaction allows us to define a right-handed coordinate system by using natural physical laws.

A word of caution, however. We have to make sure that the planet inhabited by our friend is made of matter and not of antimatter. If it is made of antimatter, he would really take nuclei of anti-Co nuclei and electric current made of positrons and would end up with a left-handed coordinate system by following our instructions!

What is stated in the last para above is the result of CP symmetry, C standing for particle-antiparticle conjugation. In other words, weak interactions violate P symmetry and C symmetry. But if both C and P are applied together weak interactions remain invariant. It was thought until 1964 that CP symmetry remains intact in weak interactions. We now know that even this is not correct, as a consequence of the discovery of CP violation by Cronin and Fitch.

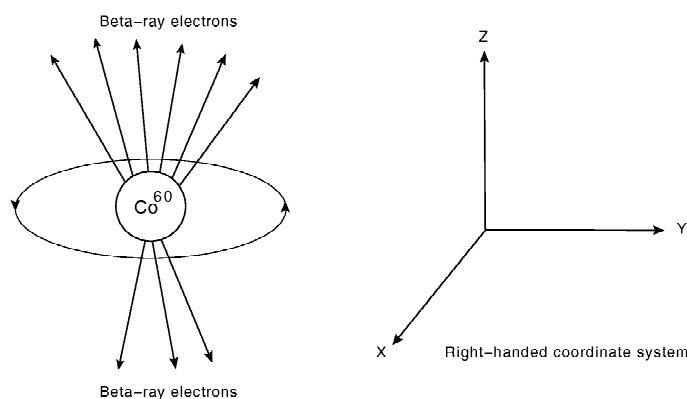


Figure B. Parity violation in the beta decay of Co^{60} .

However, there was a period of utter confusion before the above correct form of the interaction was found. As we saw earlier, Fermi based his intuition on electromagnetism which involves a vector current and we shall see in the context of later developments how sound this intuition proved to be. In fact this was a master stroke of Fermi. However, subsequently, in an attempt at generalization, Fermi's vector form was replaced by an arbitrary combination of scalar, vector, tensor, axial vector and pseudoscalar (S, V, T, A, P) interactions and it led to enormous complication and confusion in the confrontation of experiments with theory. The confusion was resolved and the correct V–A form could be found only because of the additional experimental clues provided by parity violation.

Fermi used the analogy with electrodynamics with great success.

During 1937–1955, many new particles such as muons, pions, kaons and hyperons were discovered and all of them were found to decay by weak interactions. In fact parity violation itself was triggered by the famous tau-theta puzzle in the decays of the kaons which was the culmination of the masterly phase-space plot analysis of the three-pion decay mode of the kaon by Richard Dalitz. The field of weak interactions thus got enriched by a multitude of phenomena, of which nuclear beta decay is just one. Weak interaction is indeed a universal property of all fundamental particles.

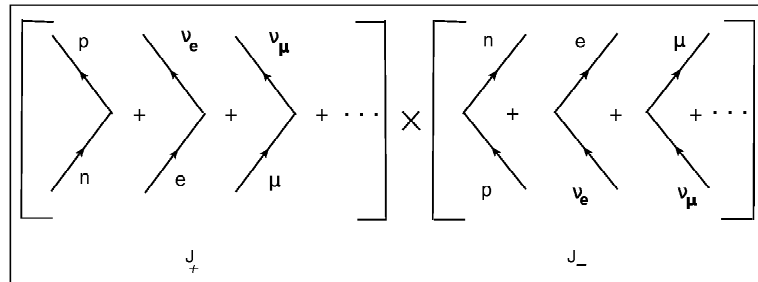
Remarkably enough, all the weak phenomena, namely the weak decays of all the particles could be incorporated in a straight-forward generalization of the original Fermi interaction. This was achieved by Feynman and Gell-Mann (1957) in the form of the current \times current interaction:

$$L_{FG} = \frac{G_F}{2\sqrt{2}}(J_+ J_- + J_- J_+),$$

Weak interaction is a universal property of all elementary particles.



Figure 2. Current \times current interaction.



$$J_+ = \bar{p}n + \bar{\nu}_e e + \bar{\nu}_\mu \mu + \dots,$$

$$J_- = \bar{n}p + \bar{e}\nu_e + \bar{\mu}\nu_\mu + \dots$$

The dots at the end refer to other terms that can be added in order to incorporate the weak decays of other particles such as the strange particles Λ , Σ , and K . A diagrammatic representation of this is given in *Figure 2*. The current J_+ represents a neutron turning itself into a proton, an electron turning itself into a neutrino or a muon turning itself into a neutrino – all these transitions result in an increase of electrical charge by one unit. The weak current J_+ is called the charge-raising current; J_- describes the opposite transition and is called the charge-lowering current.

One can see that Fermi’s original form of the interaction describing the beta decay of the neutron is just one term $\bar{p}n \bar{e}\nu_e$ in the product J_+J_- . The decay of the muon and the absorption of the muon by the proton are described by the terms $\bar{\nu}_\mu \mu \bar{e}\nu_e$ and $\bar{\nu}_\mu \mu \bar{n}p$ respectively. These are illustrated in *Figure 3*. By turning around the line, a particle in the initial state can become an antiparticle in the final state. This can be understood from *Box 1* where it is explained that a field operator can annihilate a particle or create an antiparticle. This happens for the neutrino in n decay and μ decay depicted in *Figure 3*.

A fundamental experimental discovery – the discovery of CP violation was made by Cronin and Fitch in 1964, in the weak decays of neutral kaons. It is this asymmetry in

Discovery of matter-antimatter asymmetry by Cronin and Fitch in the weak decays of neutral kaons is of fundamental importance since it can explain why the Universe contains only matter.



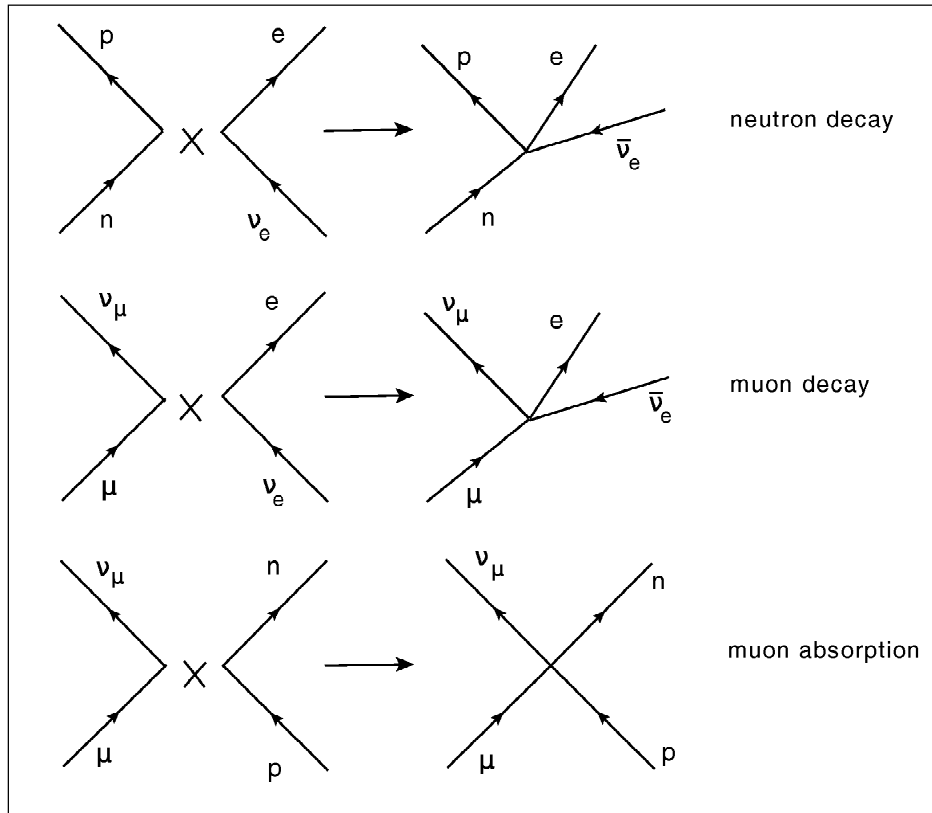


Figure 3. Consequences of current \times current theory.

the basic laws of Nature that is presumed to be responsible for the evolution of the original matter–antimatter symmetric Universe into the present-day asymmetric Universe that contains only matter.

The story of weak interactions is not complete without due recognition of the neutrino, especially because of more recent developments to be described later.

Pauli proposed the neutrino in 1930. Although because of the success of Fermi’s theory based on neutrino emission in explaining quantitatively all the experimental data on nuclear beta decays, there was hardly any doubt (at least in theorists’ minds) that neutrinos existed, a direct detection of the neutrino came only in 1956. This achievement was due to Cowan and Reines who succeeded in detecting the antineutrinos produced from fission fragments in nuclear reactors.



Cosmic-ray produced neutrinos were detected in the underground laboratory at Kolar Gold Fields in 1965.

Subsequently, it became possible to detect the neutrinos from the decays of pions and kaons produced in high-energy accelerators. It is by using the accelerator-produced neutrinos that the important experiment proving ν_μ not to be the same as ν_e was done.

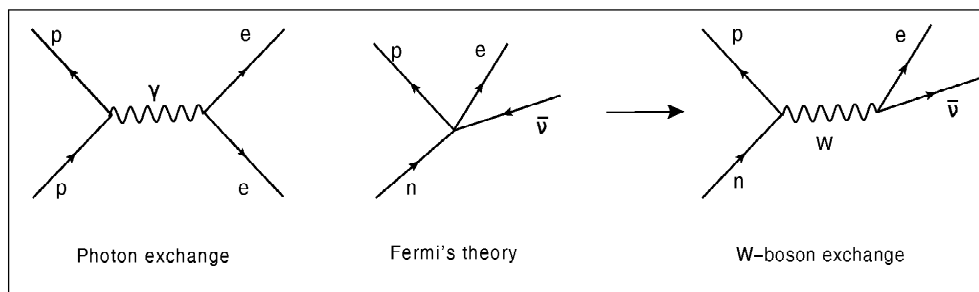
Further, even neutrinos produced by cosmic rays were detected. The underground laboratory at the deep mines of the Kolar Gold Fields in South India was one of the first to detect cosmic-ray produced neutrinos called atmospheric neutrinos. This was in 1965.

Electroweak Theory: Weak Interaction after 1972

We have already drawn attention to the analogy between weak interactions and electrodynamics which Fermi exploited in constructing his theory. One may attribute it to the intuition of Fermi's genius or to just good luck. Whatever it is, the analogy with electrodynamics that he banked upon not only made him to choose the correct form of the weak interaction – the vector form in contrast to the scalar or tensor form that were introduced later but then rejected – but also served as a fruitful analogy in the search for a more complete theory of weak interactions. This is what we shall describe now.

In beta decay, Fermi had imagined the $n - p$ line and the $e - \nu$ line interacting at the same space-time point. But clearly the correspondence with electrodynamics is greatly enhanced if the two pairs of lines are separated and an exchange of a quantum W between the $n - p$ and $e - \nu$ lines is inserted. (See *Figure 4*).

Figure 4. Genesis of W-boson theory.



What are the properties of this new quantum or particle?

(a) W has to be charged, in contrast to the photon, as can be seen by conserving charge at the two vertices of the W -exchange diagram in *Figure 4*. The neutron turns into a proton by emitting a W and so this W should be negatively charged.

(b) Just like the photon, the W particle also has a spin angular momentum of one unit. Both photon and W are bosons.

(c) In contrast to the photon, the W boson has to be a very massive object. For, the weak interaction has a short range unlike the infinite-ranged electromagnetic interaction.

In Fermi's theory the coupling constant was G_F . In the W -boson theory we have a coupling constant g at each vertex and so for the same process G_F is replaced by a factor g^2 multiplied by the propagation factor for the W boson. This propagation factor is $1/m_W^2$, where m_W is the mass of W , for small energy and momentum transfers relevant in beta decay. Thus we have the important relationship:

$$G_F = \frac{\sqrt{2} g^2}{8 m_W^2}.$$

By introducing the fields W^+ and W^- for the positively and negatively charged W bosons, the current \times current form of the weak interaction can be split into the form

$$g(J_+ W^+ + J_- W^-).$$

This form is very similar to the electrodynamic interaction $eJ_E A$ and so we have achieved a greater degree of symmetry between weak interaction and electrodynamics. (See *Figure 5*.)

The transition from Fermi's theory to its modern avatar, the electroweak theory, took place around 1972.

The W boson has to be massive since weak interaction is short-ranged.



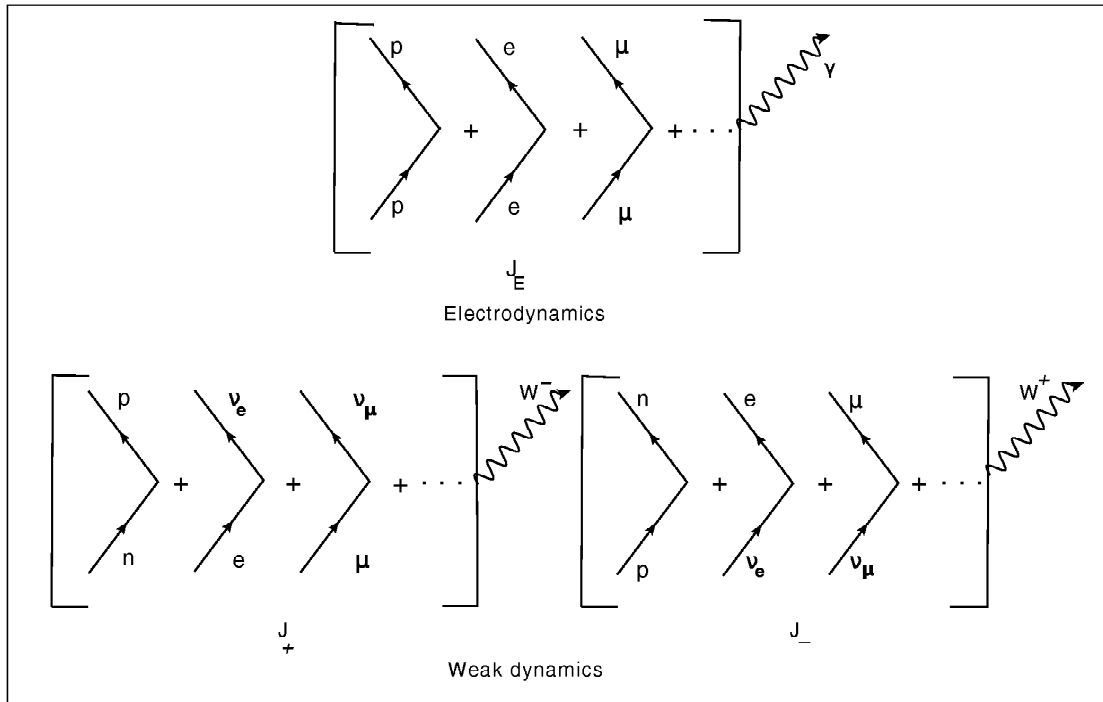


Figure 5. Symmetry between electrodynamics and weak dynamics.

The next step of the argument is to realize that the symmetry between the W -boson form of weak interaction and electrodynamics noted above is only apparent and does not hold at a deeper level.

Conservation of electric charge is a cornerstone of electrodynamics. The total charge of an isolated system can neither be increased nor decreased and remains constant. A related question concerns gauge invariance which simply means that different electromagnetic potentials A lead to the same physical effects as long as the electric field and the magnetic field that are obtained by taking space and time derivatives of A are the same. Are such properties valid for the W -boson theory formulated above? The answer is in the negative.

Certain important structural modifications have to be made in the W -boson theory in order to achieve conservation of the generalized charge involved in weak interaction and the corresponding gauge invariance.



The required basic theoretical structure has been known since 1954 when C N Yang and R L Mills introduced nonabelian gauge theory which is a generalization of electrodynamics. The gauge invariance of electrodynamics is known as abelian gauge invariance and Yang–Mills theory has nonabelian gauge invariance based on a nonabelian Lie group. But many other ideas had to be discovered before this theory could be tailored to meet the experimental facts of weak interactions. The final outcome is the electroweak gauge theory of Glashow, Salam and Weinberg which is the successor to Fermi’s theory.

Yang-Mills theory which is a generalization of Maxwell’s electrodynamics provided the framework for the electroweak theory.

This theory generalizes the concept of charge. The single electric charge of electrodynamics is replaced in the new theory by four generalized charges. The current corresponding to each charge interacts with its own boson, called gauge boson. An essential point of electroweak theory is that the twin requirements of generalized charge conservation and gauge invariance force us to combine weak and electromagnetic interactions dynamically into a single framework. As a consequence of this unification of weak and electromagnetic interactions, a new kind of weak interactions is also generated.

The combined interaction in the electroweak theory is

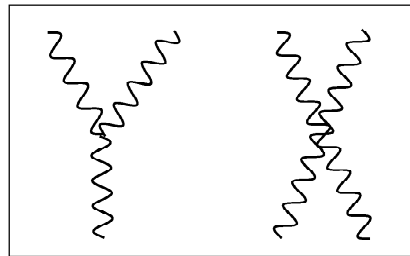
$$eJ_E A + g(J_+ W^+ + J_- W^-) + g_N J_N Z,$$

where the last term is a new interaction. There are four generalized charges whose currents J_E , J_+ , J_- and J_N interact with the four gauge bosons: photon, W^+ , W^- and Z respectively. Thus electroweak theory introduces a symmetry between the photons and the massive weak bosons W^+ , W^- and Z . Photon becomes a member of a family of four electroweak gauge bosons.

The generalization of the concept of charge leads to self-interactions among the gauge bosons, which are shown in *Figure 6*. Both a cubic and a quartic coupling are



Figure 6. Self-interactions among the gauge bosons.



present. This is a new feature not present in electrodynamics. The photon interacts with every electrically charged particle. But the photon itself being uncharged, does not interact with itself. On the other hand the gauge bosons of Yang–Mills theory themselves carry the generalized charges and hence they have to interact with themselves.

So we have completed a full circle. We started with Fermi who made his theory of beta decay by mimicking electrodynamics. We tried to make that copying more and more perfect. We end up by unifying electrodynamics and beta decay into the same framework. The myriad electrodynamic and weak decay phenomena are manifestations of one electroweak force. We now discuss some of the simple consequences of electroweak theory.

Neutral Current Weak Interaction

Electroweak theory encompasses not only the known electromagnetic and weak interactions, but also a new type of weak interaction $g_N J_N Z$. The current J_N consists of terms such as $\bar{p}p, \bar{n}n, \bar{\nu}\nu$ and $\bar{e}e$ illustrated in *Figure 7*. In contrast to J_+ and J_- which change electrical charge, the new current describes transitions in

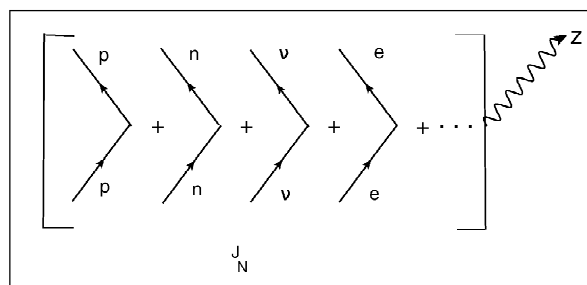


Figure 7. Neutral current interaction.

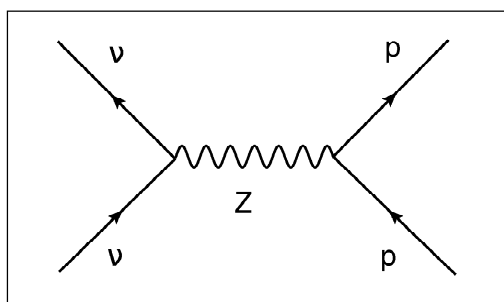


Figure 8. Neutrino scattering on proton by neutral current interaction.

which charge does not change and is called the neutral current. The neutral current interacts with the neutral weak boson Z with coupling constant g_N . The neutral current weak interaction would lead to neutrino scattering processes in which the neutrino emerges as a neutrino (with change of energy and direction) rather than getting converted into a charged particle such as electron or muon. (See *Figure 8*.)

The discovery of the neutral current weak interaction was made in 1973 at CERN, Geneva. This discovery has its own intrinsic importance because it opened up a whole new class of weak interactions which had remained undetected in all the 70 years' history of weak interactions. From the point of view of electroweak theory, it has an added significance since the neutral-current (NC) interaction acts as a bridge between electrodynamics and the old charged-current (CC) weak interaction. It is neutral like electromagnetic current, but involves a massive boson Z like the W involved in the CC interaction. Hence its discovery with properties identical to those predicted by the electroweak theory was the first great triumph of the theory.

In the history of weak interaction physics the discovery of the V–A structure of the charged current was an important milestone. What is the structure of the neutral current? It is not V–A and so not all of weak interaction is described by V–A theory! The relative amount of V and A in neutral current is specified by an important parameter of the electroweak theory called the

Neutral-current weak interaction acts as a bridge between electrodynamics and charged-current weak interaction.



The CERN experiments discovered the W and Z bosons exactly at the masses predicted by electroweak theory.

weak mixing angle θ_W and it has been determined experimentally:

$$\sin \theta_W = 0.23.$$

Discovery of W and Z

An immediate consequence of the dynamical connection between weak and electromagnetic interactions is that their coupling constants are related:

$$e = g \sin \theta_W,$$

$$g_N = \frac{g}{\cos \theta_W}$$

and the masses of W and Z are also related:

$$m_W = m_Z \cos \theta_W.$$

The relationship between G_F and g^2 derived earlier

$$G_F = \frac{\sqrt{2} g^2}{8 m_W^2}$$

now becomes

$$G_F = \frac{\sqrt{2} e^2}{8 \sin^2 \theta_W m_W^2}.$$

This allows us to calculate the masses of W and Z from the known values of G_F , e and $\sin \theta_W$. We get

$$m_W = 80 \text{ GeV},$$

$$m_Z = 91 \text{ GeV}.$$

The discovery of W and Z with these masses at CERN in 1982 was the second great triumph of electroweak theory.

The inverse relationship between G_F and m_W^2 given above helps us to answer the question: why is weak interaction weak? It is because the masses of m_W (and m_Z)



are so large. G_F is the effective weak coupling constant at low energies. Once the energy becomes high enough to produce a real W boson, weak interaction attains its real strength g which is comparable to e , the strength of the electromagnetic interaction.

Discovery of the Higgs boson in 2012 establishes electroweak theory.

Spontaneous Breaking of Symmetry and the Higgs Boson

An essential ingredient of the electroweak theory described so far is spontaneous breakdown of symmetry, also known as Higgs mechanism. The gauge invariance or gauge symmetry of Yang–Mills theory would lead to massless gauge bosons exactly as the gauge invariance of electrodynamics requires massless photon. But we need massive gauge bosons to describe the short-ranged weak interaction. How is this problem solved in electroweak theory? It is solved by the spontaneous breakdown of symmetry engineered by the celebrated Higgs mechanism which keeps photon massless while raising the masses of W and Z to the finite values discussed above. For more on the Higgs mechanism see [4].

An important byproduct of the Higgs mechanism is the existence of a massive spin zero boson, called the Higgs boson. High energy physicists searching for it in all the earlier particle accelerators and colliders could not find it. Finally in 2012, the Higgs boson with a mass of 125 GeV was discovered at the Large Hadron Collider at CERN. Higgs boson remained as the only missing piece in the electroweak theory and with its discovery electroweak theory is fully established.

Electroweak theory and quantum chromodynamics (which is the theory of strong interactions) have become the twin pillars of the Standard Model of High Energy Physics [4].

Renormalizability and Precision Tests

It had been known for a long time that Fermi's original



Electroweak theory
came out with
flying colours in all
the precision
experimental tests.

form of the weak interaction (in which four fermionic fields meet at a single point in a contact interaction) can only be regarded as an effective potential to be used in the lowest order approximation to a perturbative calculation. Any attempt to improve the accuracy of the result by calculating the next order in perturbation leads to infinity which does not make any sense.

Construction of a dynamical theory of weak interactions free from this defect was one of the fundamental problems of high energy physics. Electroweak theory solved this problem. Stated in technical language electroweak theory is renormalizable in the same sense as Quantum Electrodynamics (QED) is. The renormalizability of electroweak theory was proved by Gerard t'Hooft and Martinus Veltman in 1972.

Renormalizability of the electroweak theory elevated it to a theory whose precision now rivals that of QED, which is considered as the most precise theory constructed by man. Electroweak theory came out with flying colours in all the precision tests performed through a series of experiments at the Large Electron Positron Collider (LEP) at CERN in the 1990's.

Transition to the Quark Era

So far in the article, we have used proton and neutron to describe the weak interaction. But we now know that proton and neutron are composed of quarks u and d . Proton is made up of uud and neutron is made up of ddu . At the fundamental level weak interaction acts on the quarks. The currents therefore must be rewritten in terms of the quark fields. In the beta decay of the neutron it is one of the two d quarks that decays into u as shown in *Figure 9*. The other quarks play only spectator role.

There exist 6 types of quarks arranged in the form of 3



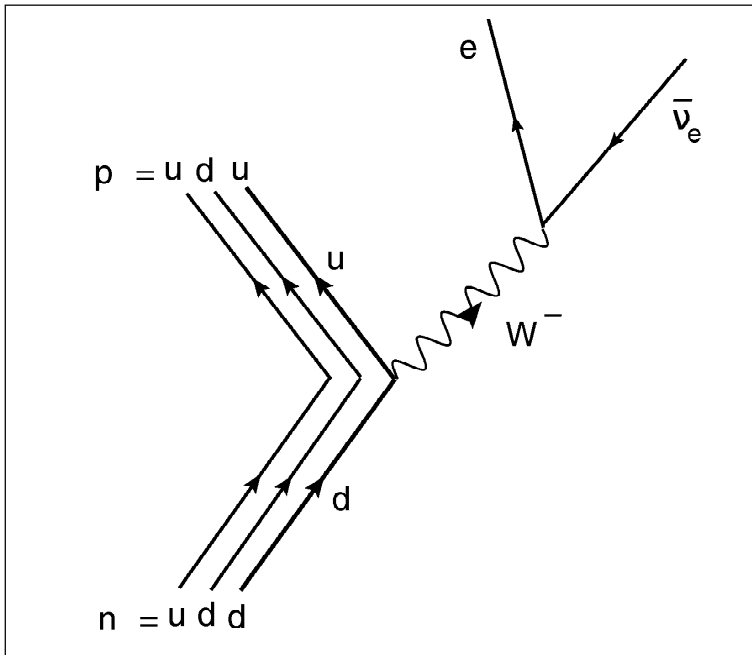


Figure 9. Beta decay of the neutron in terms of quarks.

doublets

$$(u, d) \quad (c, s) \quad (t, b).$$

All composite particles formed out of these quarks are called hadrons. Our familiar proton and neutron are hadrons and many more hadrons are known. Electron and neutrino have remained elementary on par with quarks upto the present. These are called leptons and again 6 types of leptons are known to exist:

$$(\nu_1, e) \quad (\nu_2, \mu) \quad (\nu_3, \tau).$$

The electroweak interaction in terms of the quarks and leptons is given by

$$L_{EW} = eJ_E A + g(J_+ W^+ + J_- W^-) + g_N J_N Z.$$

The electromagnetic and neutral currents will contain terms like

$$\bar{u}u, \bar{d}d, \bar{e}e \dots,$$

At the fundamental level, weak interaction acts on the quarks and leptons.



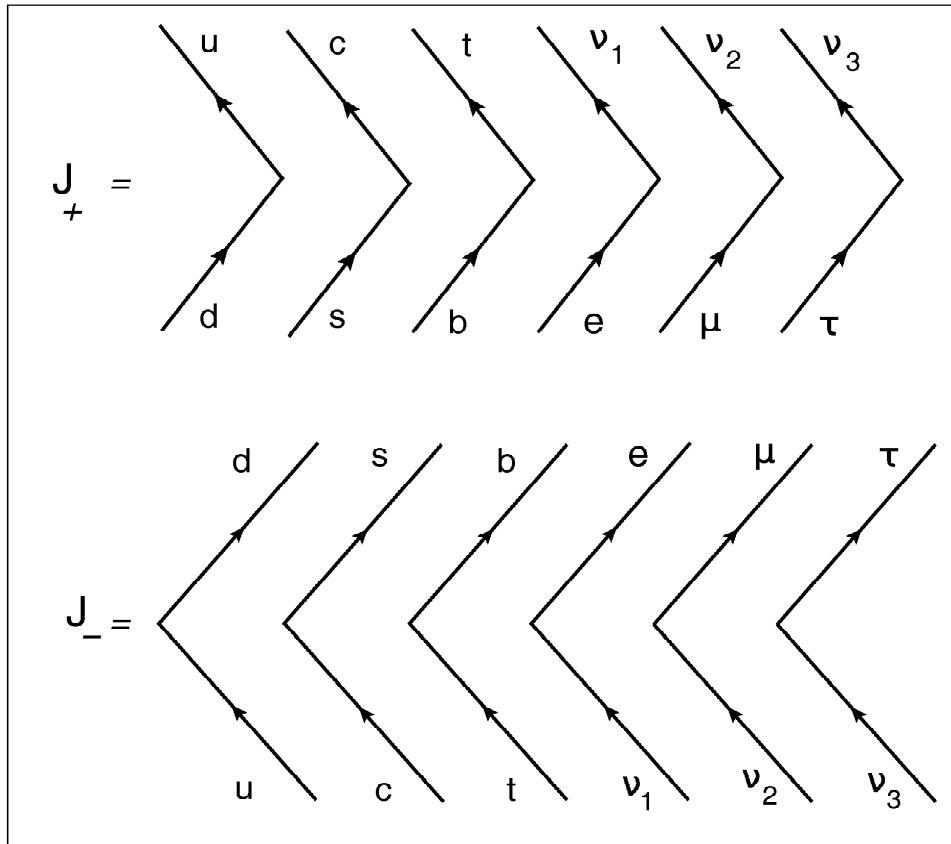


Figure 10. The currents in terms of quarks and leptons.

while the charged currents that describe the transitions from one type of quark to another or from one type of lepton to another (as illustrated in *Figure 10*) are given by

$$J_+ = \bar{u}d + \bar{c}s + \bar{t}b + \bar{\nu}_1e + \bar{\nu}_2\mu + \bar{\nu}_3\tau ,$$

$$J_- = \bar{d}u + \bar{s}c + \bar{b}t + \bar{e}\nu_1 + \bar{\mu}\nu_2 + \bar{\tau}\nu_3 .$$

The electric charge of the ‘up-type’ quarks (u, c, t) is $+\frac{2}{3}$ while that of the ‘down-type’ (d, s, b) is $-\frac{1}{3}$. All the transitions between the up and down type of quarks indicated by the above expression or *Figure 10* have a change of charge by one unit and are of the same sign, exactly like the transitions between the charged leptons (e, μ, τ) and the neutrinos.



Actually we have to change the down quarks (d, s, b) occurring in J_+ and J_- by their linear superpositions (d', s', b') defined as follows. Introducing the notation $q_i (i = 1, 2, 3)$ for (d, s, b), the superposed quarks are given by

$$q'_i = \sum_j U_{ij} q_j ,$$

where U is a 3×3 unitary matrix:

$$U^\dagger U = 1.$$

This U is called the CKM matrix and its discovery by Cabibbo, Kobayashi and Maskawa is an important chapter in the history of weak interactions. (For more details on this part of history, see [5]). These superpositions are natural consequences in electroweak theory and allow the heavier quarks to decay into all the lighter quarks. The matrix U is parametrized by three angles and one phase that is responsible for the CP violation discovered by Cronin and Fitch.

A similar unitary mixing matrix V called PMNS matrix (named after Pontecorvo, Maki, Nakagawa and Sakata) is used in the leptonic part of the currents also and this is what leads to neutrino oscillations and the discovery of neutrino mass. (For more on this, see [6].)

It is important to note that all the 6 quarks and 6 leptons are equally fundamental and all were presumably created in equal numbers in the Big Bang and it is the weak interaction that caused all the heavier particles to decay into the lighter ones u, d, e, ν that make up the fermionic or matter-component of the present-day Universe.

Discovery of Neutrino Mass

In his original paper in 1934 Fermi had already come to the conclusion that the mass of the neutrino was either zero or very small as compared to that of the electron

Superpositions of quarks and similar superpositions of leptons make weak interactions a very rich field: decays of all heavier particles to $u, d, e,$ and ν , matter-antimatter asymmetry and oscillations among the three neutrinos.



It is the neutrino oscillations that led to the discovery that neutrinos have non-zero masses, although tiny.

(0.5 MeV), by comparing the energy distribution of the electrons emitted in beta decay near their end-point energy with what was available experimentally. When the electroweak theory was constructed, massless neutrinos had a natural place in the theory. So neutrinos were considered massless.

However neutrino oscillations were discovered in 1998 by the Super Kamioka group in Japan and this implied mass differences among the three neutrinos and hence neutrinos have mass. This discovery was made in the study of atmospheric neutrinos which had been first detected in India in 1965, as already mentioned.

Actually, indications for neutrino oscillations and neutrino mass came first as early as 1970 from the pioneering solar neutrino experiments of Davis *et al* in USA which were later corroborated by many other solar neutrino experiments. But the clinching evidence that solved the solar neutrino problem in terms of neutrino oscillations had to wait until 2002 when the Sudbury Neutrino Observatory (SNO) could detect the solar neutrinos through both the neutral current as well as the charged current weak interactions. In the introduction we had mentioned that it was the thermonuclear fusion reactions caused by weak interactions that powered the Sun and the stars. The experimental proof of this too came from the SNO experiment. (See [7, 8] for a more complete description.)

All the three neutrino masses are clustered around a value lower than 2.2 eV, with tiny differences between them.

Although neutrinos are now known to have mass from the existence of neutrino oscillations, we do not know the values of the masses since only differences in neutrino mass-squares can be determined from oscillation phenomena. The two differences between the mass-squares of the three types of neutrinos have been determined to be

$$m_2^2 - m_1^2 = 7 \times 10^{-5} eV^2,$$



$$|m_3^2 - m_2^2| = 2 \times 10^{-3} eV^2 .$$

Going back to Fermi's original comment, since more accurate measurements on the end-point energy distribution in nuclear beta are possible now compared to 1934, what can be said? Progressively the upper limits on the neutrino mass determined by this method have come down and the present upper limit from tritium beta decay is 2.2 eV, which is indeed very small. All the three neutrino masses are clustered around a value lower than this upper limit, with tiny mass-differences between them.

It is important to point out that even 80 years after its birth, the fundamental nature of the neutrino is still not known, namely whether neutrino is its own antiparticle or not. If it is its own antiparticle it is called Majorana particle; otherwise it is a Dirac particle just like the other fermions such as electron or quark. This question can be answered only by the 'neutrinoless double beta decay experiment' which is therefore the most important experiment in all of neutrino physics (see [9] for an account of the Majorana problem).

Neutrino physics is now recognized as one of the most important frontiers in high energy physics and it is vigorously pursued in many underground laboratories around the world. The India-based Neutrino Observatory (INO) that is coming up in South India will be one such [10].

As already mentioned, electroweak theory implies massless neutrinos in a natural way. How is the theory to be extended to incorporate nonzero neutrino masses? Only future will tell.

Epilogue

We have seen the vast range of phenomena covered by weak interactions: beta decay of nuclei, thermonuclear

We still do not know whether neutrino is a Dirac particle or a Majorana particle.

India-based Neutrino Observatory (INO) is coming up in South India.



Box 3. Milestones in the History of Weak Interactions

- 1896 Discovery of radioactivity (Becquerel)
- 1930 Birth of neutrino (Pauli)
- 1934 Theory of beta decay (Fermi)
- 1939 Theory of thermonuclear fusion in the Sun (Bethe and Wesszacker)
- 1954 Nonabelian gauge theory (Yang and Mills)
- 1956 Discovery of parity violation (Lee, Yang and Wu)
- 1956 Detection of the neutrino (Cowan and Reines)
- 1957 Discovery of V–A (Sudarshan, Marshak and others)
- 1957 Current \times current formulation (Feynman and Gell-mann)
- 1961 $SU(2) \times U(1)$ as the electroweak group (Glashow)
- 1964 Discovery of CP violation (Cronin and Fitch)
- 1964 Abelian Higgs mechanism (Higgs and others)
- 1967 Nonabelian Higgs–Kibble mechanism (Kibble)
- 1967 Electroweak theory (Salam and Weinberg)
- 1972 Renormalizability of EW theory (t’Hooft and Veltman)
- 1973 Discovery of neutral current (55 physicists at CERN)
- 1973 CKM phase for CP violation (Kobayashi and Maskawa)
- 1982 Discovery of W and Z (Rubbia and van der Meer)
- 1992 Precision tests of EW theory (International Collaboration at CERN)
- 1998 Discovery of neutrino mass (Davis, Koshiba and others)
- 2002 Experimental proof of thermonuclear fusion in the Sun (SNO)
- 2007 Verification of CKM theory of CP violation (KEK, Stanford)
- 2012 Discovery of Higgs boson (ATLAS and CMS Collaborations, CERN)

fusion reactions in the Sun and stars, decays of most of the elementary particles of Nature and removal of antiparticles in the Universe through CP violation. We have touched on the brief history of the important theoretical and experimental discoveries. The milestones in this history are listed in *Box 3*.

Fermi regarded beta decay theory as his most important contribution. Subsequent 80 years have amply justified that view.

Fermi created beta decay theory which was the starting point of all that followed, using the nascent Quantum Field Theory which was perhaps understood by very few physicists at that time. He did this at a time when nuclei were not understood and so nuclear physics did not even exist – not to speak of particle physics (now called high energy physics) which was born only much later. No wonder Fermi responded that it is beta decay theory when asked what he regarded as his most important contribution. There is no doubt that it is not only



Fermi's most important contribution but it is one of the most important contributions made by anybody in that Foundational Epoch of Modern Physics.

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Suggested Reading

- [1] E Fermi, Attempt at a Theory of β -rays, *Il Nuovo Cimento*, Vol.11, p.1, 1934; *Zeitschrift fur Physik*, Vol.88, p.161, 1934.
- [2] For Fermi's article in English, see *The Development of Weak Interaction Theory*, Edited by P K Kabir, Gordon and Breach Science Publishers, 1963. This volume contains most of the original papers in weak interaction physics upto 1962.
- [3] A Pais, *Inward Bound*, Oxford University Press, New York, p.417–418, 1986.
- [4] G Rajasekaran, Standard Model, Higgs Boson and What Next?, *Resonance*, Vol.17, No.10, p.956, 2012.
- [5] G Rajasekaran, Cabibbo Angle and Universality of Weak Interactions, *Physics News*, July 2011, p.21.
- [6] G Rajasekaran, An angle to tackle the neutrinos, *Current Science*, Vol.103, No.6, 2012.
- [7] G Rajasekaran, Hans Bethe, the Sun and the Neutrinos, *Resonance*, Vol.10, No.10, p.49, 2005.
- [8] D Indumathi, MVN Murthy and G Rajasekaran, Neutrinos and how to catch them, *Physics Education*, Vol.23, No.2, Aug 2006.
- [9] G Rajasekaran, Are neutrinos Majorana particles? arXiv:0803.4387[physics]
- [10] More information on INO can be obtained from www.ino.tifr.res.in

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