Depinning transitions in elastic strings with long range interactions

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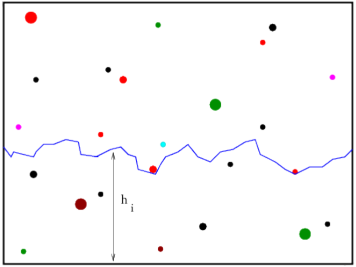
Outline

- Elastic string driven through quenched disordered medium
- Used for modelling of dynamics of various physical phenomena: charge density waves, vortices in superconductors, domain walls in ferromagnents and also in fracture front propagation.
- Pinned to depinned state studied as phase transition: Universality classes
- Our models
- Summary

Dynamics of driven elastic string

- Forces acting on a elastic string driven through a quenched disordered medium



- 'Elastic' force (short, long or infinite range) that tries to restore minimum length
- Pinning forces (correlated or uncorrelated) due to quenched disorder
- Externally applied driving force (uniform).

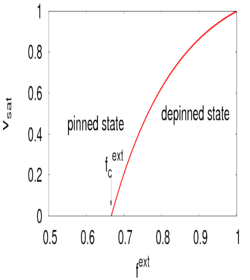
- Equation of motion (discrete)

$$f_i^{tot} = f_i^{elastic} + f_i^{pinning} + f_i^{ext}.$$

$$\begin{aligned} h_i(t+1) &= h_i(t) + 1 \quad \text{if} \quad f_i^{tot} > 0 \\ &= h_i(t) \quad \text{otherwise} \end{aligned}$$

Depinning transition: exponents and scalings

- Depinning is said to have occurred when final velocity of the string is non-zero.



- Velocity is the order parameter
  $$v(t) = \frac{1}{L}\sum_{i=1}^{L}(h_i(t+1) - h_i(t)).$$
- Usual scaling relations are valid for velocity
  - $v(t \to \infty) \sim (f^{ext} - f_c^{ext})^{\theta}$, where θ is order parameter exponent.
  - Also $v(f^{ext}, t) \sim t^{-\delta} F(t|f^{ext} - f_c^{ext}|^{\nu})$, with $\nu = \theta/\delta$.
  - At critical point $v(t) \sim t^{-\delta}$.

- Width of the surface is also of interest
  $$W(t) = \langle \frac{1}{L}\sum_{i=1}^{L}(h_i(t) - \langle h_i(t)\rangle)^2\rangle^{1/2}.$$
  - It satisfies $W(L,t) \sim L^{\alpha} G(t/L^{z})$ (Family-Viscek scaling), with $G(x) = const.$ when $x >> 1$ and $G(x) \sim x^{\beta}$ when $x << 1$, with $\alpha = z\beta$.
  - It is also argued that $v(t) \sim W(t)/t$, giving $\delta + \beta = 1$.

Depinning transition: Universality classes

- Depinning transitions are classified into several universality classes:
- Universality classes depend upon nature of the elastic force
  - $\nabla^2 h \rightarrow$ Edwards-Wilkinson class.
  - $\nabla^2 h + (\nabla h)^2 \rightarrow$ Kardar-Parisi-Zhang class.
  - $\nabla^4 h \rightarrow$ Mullins-Herring class.
  - Long range models $\sum_{i \neq j} \frac{h_i - h_j}{|i-j|^\alpha}$.
  - Model I: $\frac{1}{L} \sum_{j=1}^{L} \left[\sqrt{(h_j(t) - h_{j+1})^2 + 1} - 1 \right] \hat{C}_i$, where $\hat{C}_i = sgn(h_{i+1} + h_{i-1} - 2h_i)$.
  - Model II: $F\hat{C}_i$.

S. Biswas and BKC arxiv:1108.1707

Model I: Critical point

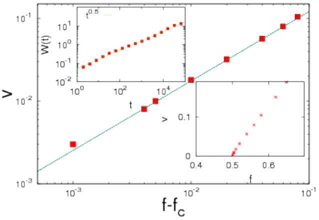
- Height variables are updated following the rule

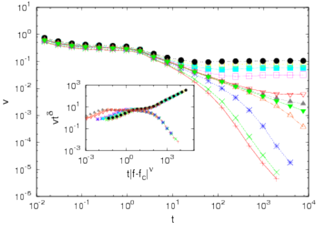
$$h_i(t+\Delta t) = h_i(t) + G_i(t)\Delta t \qquad \text{if} \qquad G_i(t) > 0$$

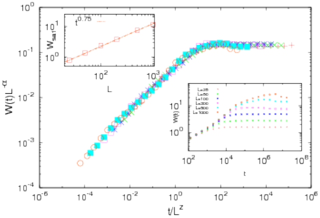
  where $G_i(t) = f_i^{elastic} + f_i^{pinning}(h_i(t)) + f^{ext}$. The pinning force is a random variable distributed uniformly and continuously between [-1:0].
- Depinning would imply moving the strongest pinned site, so depinning condition is $f^{ext} + f^{elastic} = f_{max}^{pinning} = 1$.
- Pinning would imply pinning the weakest pinned site, so pinning condition is $f_{min}^{pinning} + f^{elastic} = f^{ext}$.
- When $f_{min}^{pinning}$ and $f^{elastic}$ both are to minimised for the above equation, one would have $f_{min}^{pinning} = f^{elastic} = \frac{1}{2} f^{ext}$. Implying, $f_c^{ext} = \frac{2}{3} f_{max}^{pinning}$. Verified using numerical simulations.

Model I: Results

- Results of numerical simulations:



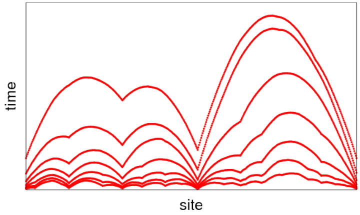


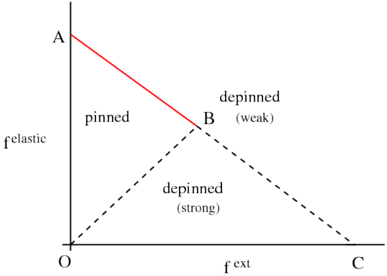


$\beta = 0.50 \pm 0.01$, $\theta = 0.83 \pm 0.01$, $\nu = 1.35 \pm 0.05$, $\delta = 0.60 \pm 0.01$, $\alpha = 0.75 \pm 0.05$, $z = 1.5 \pm 0.1$.
$\delta + \beta \approx 1.1$, $\theta/\delta \approx 1.38$, $\beta z \approx 0.75$.

Model II: Critical behavior

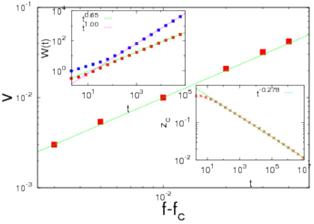
- In this case
$$\frac{\partial h_i}{\partial t} = F\hat{C}_i + f_i^{pinning} + f^{ext},$$
we take $f_i^{pinning}$ to be independent of $h_i(t)$.
- Applying this model for flock of birds flying in a line.
- It is well known that birds often form a 'V' shape patterns in order to increase their efficiency (upto 70%) (Lissaman, Shollenberger, Science **168**, 1003 (1970)).

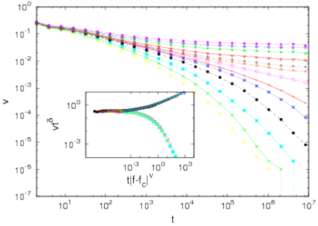


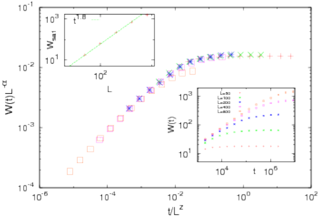


Model II: Results

- Results of numerical simulations:







$\beta = 0.65 \pm 0.05$, $\theta = 1.00 \pm 0.01$, $\nu = 2.95 \pm 0.05$, $\delta = 0.34 \pm 0.01$, $\alpha = 1.8 \pm 0.1$, $z = 2.9 \pm 0.1$.
$\delta + \beta \approx 0.99$, $\theta/\delta \approx 2.94$, $\beta z \approx 1.885$.

Comparisons with other univ. classes

- Comparison with other universality classes:

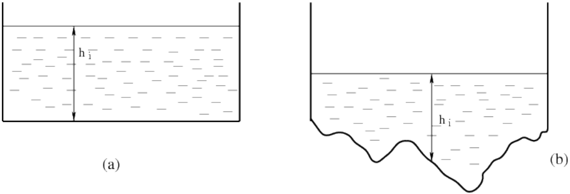
Models	β	θ	ν	δ	α	z
EW	0.85 ± 0.03	0.24 ± 0.03	1.73 ± 0.04	-	0.92 ± 0.04	1.45 ± 0.07
KPZ	0.67 ± 0.05	0.64 ± 0.12	1.35 ± 0.04	-	0.63 ± 0.03	1.01 ± 0.01
$1/r^2$	0.495 ± 0.005	0.625 ± 0.005	1.625 ± 0.005	-	-	0.770(5)
MH	0.841 ± 0.005	0.289 ± 0.008	1.81 ± 0.1	0.160 ± 0.005	1.50 ± 0.06	1.78
Model I	0.50 ± 0.01	0.83 ± 0.01	1.35 ± 0.05	0.60 ± 0.01	0.75 ± 0.05	1.5 ± 0.1
Model II	0.65 ± 0.05	1.00 ± 0.01	2.95 ± 0.05	0.34 ± 0.01	1.8 ± 0.1	2.9 ± 0.1

- References:

1. A. L. Barabási and H. E. Stanley, *Fractal Concepts in Surface Growth* (Cambridge University Press, 1995).
2. D. Bonamy, E. Bouchaud, Phys. Rep. **498**, 1 (2011).
3. L. A. N. Amaral, A. L. Barabási, H. A. Makse, H. E. Stanley, Phys. Rev. E **52**, 4087 (1995).
4. A. Tanguy, M. Gounelle, S. Roux, Phys. Rev. E **58**, 1577 (1998).
5. S. Biswas, BKC, arXiv:1108.1707

Fluctuating external force

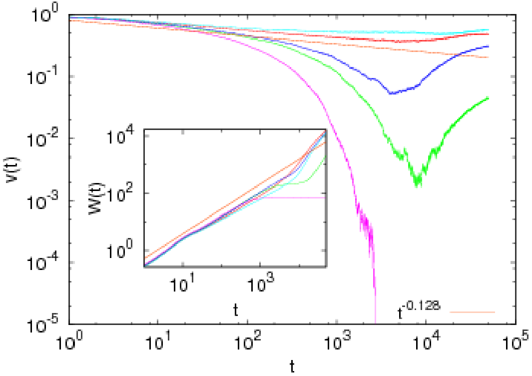
- In earlier studies external force at every site was kept constant.
- Here we keep total external force constant but it can vary from site to site.



- Fluid applying pressure at the base
- External force at every site $f_i^{ext} = \rho h_i$
- The external force becomes correlated in the long range.

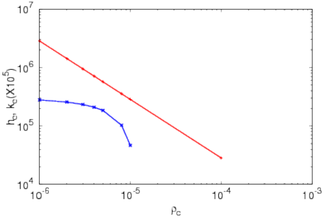
Fluctuating external force (contd.)

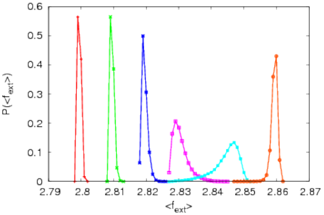
- For the elastic force we keep EW term ($k\nabla^2 h$).
- The velocity of the front is plotted against time



Phase boundary

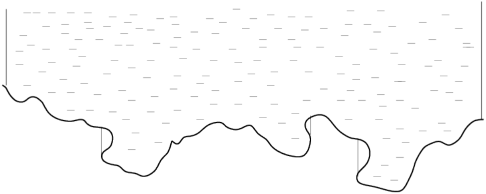
- The depinning takes place much before power-law decay of velocity
- For a given initial volume, there is a phase boundary in $k - \rho$ plane and for given k there is one in the $h - \rho$ plane.
- In the limit $\rho \to 0$, we expect the non-monotonic behavior of velocity to disappear.





Considering overhangs

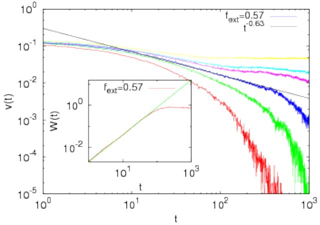
- Considering fracture by fluid pressure, one has to take care of transverse growths, hence overhangs.
- In its simplest form, we consider that any wet site can wet any one of its four neighbors if the surface can overcome (i) quenched pinning force at that point and (ii) linear elastic force directed towards the average height.

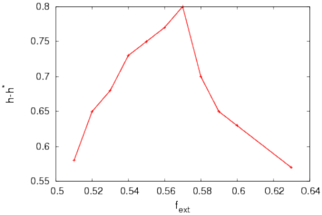


- To keep height as a single valued function, the effect of overhangs can be taken into account in two ways. The difference between these two estimates maximizes at critical point in random field Ising model (Zhou and Zheng (2010)).

Effect of overhangs

- We measure velocity as number of sites moving forward at every step.
- At transition point it decays as $t^{-0.63}$, the 'width' of the surface also scales in a power-law.





- The effect of overhang is maximum at the transition point.

Summary

- We study the effect of fluctuating external force in elastic depinning transition.
- The ‘depinning’ takes place much before the power-law decay of front velocity with time.
- The best power-law fit agrees with corresponding constant force model.
- For fluid pressure one has to consider effect of overhangs.
- In a simple model of elastic string depinning, the effect of overhang is seen to be maximum at depinning transition.

Thank You!