On Symmetric Key Broadcast Encryption

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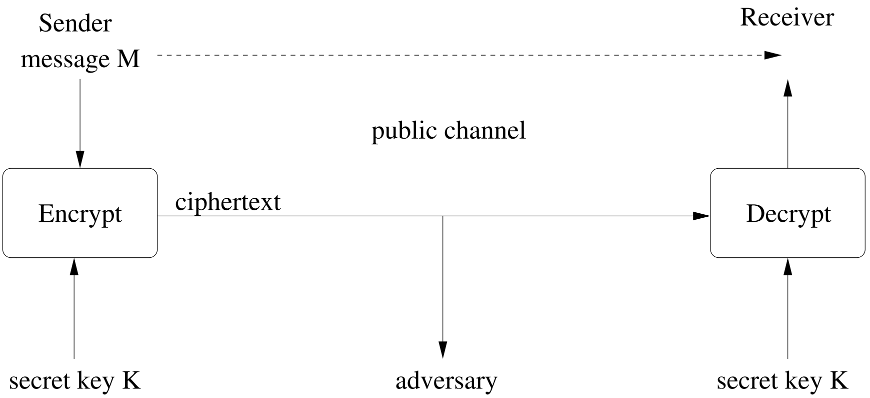
Indian Statistical Institute, Kolkata

Elliptic Curve Cryptography
(This is not)
2014

Conventional Symmetric Key Encryption



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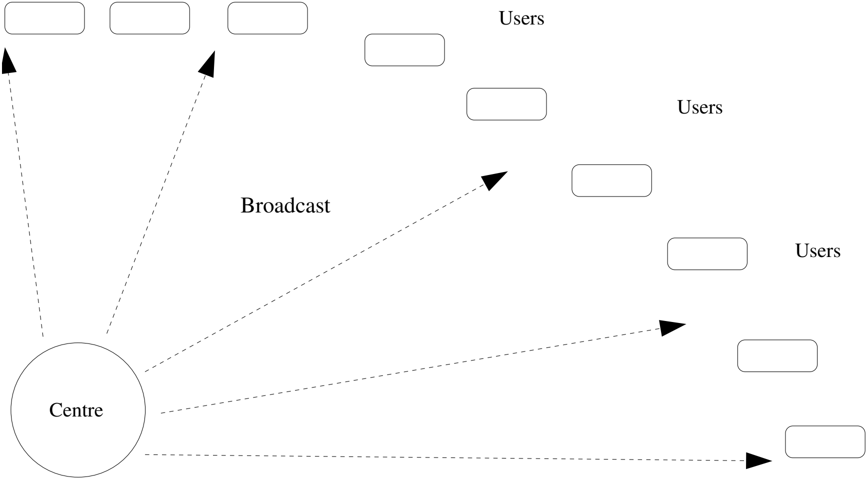


Symmetric Key BE Functionality

- The centre pre-distributes secret information to the users.
- A broadcast takes place in a session.
- For each session:
  - Some users are privileged and the rest are revoked.
  - The actual message is encrypted once using a session key.
  - The session key undergoes a number of separate encryptions. This determines the header.
  - Only the privileged users are able to decrypt. A coalition of all the revoked users get no information about the message.

Parameters of Interest

- Size of the header.
- Size of the secret information required to be stored by the users.
- Time required by the centre to encrypt.
- Time required by a user to decrypt.

Hdr sz and enc time are proportional to # enc of the session key.

Requirement: Reduce header size, user storage and decryption time.

Applications of BE

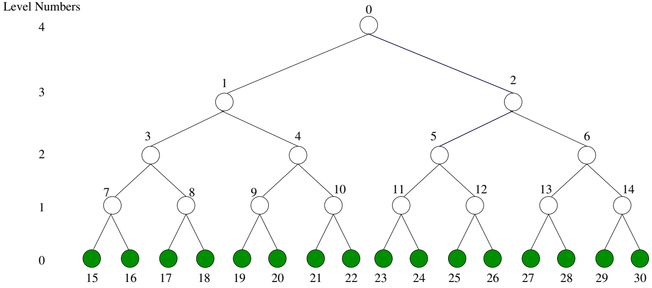
- **AACS** standard: content protection in optical discs: *Disney, Intel, Microsoft, Panasonic, Warner Bros., IBM, Toshiba and Sony.*
- **Pay-TV**: BSkyB in UK and Ireland has a subscriber base of over 10 million;
- Cable Television Networks (Regulation) Amendment Act, 2011 (India).
- File Sharing in Encrypted File Systems.
- Encrypted Email to Mailing Lists.
- Military Broadcasts: Global Broadcast Service (US), Joint Broadcast System (Europe).
- ...

Subset Cover Schemes

- Identify a collection $\mathcal{S}$ consisting of subsets of users.
- Assign keys to each subset in $\mathcal{S}$.
- To each user, assign secret information such that it is able to generate secret keys for each subset in $\mathcal{S}$ to which it belongs; and no more.
- During a broadcast, form a partition $\{S_1, \ldots, S_h\}$ of the set of privileged users with $S_i \in \mathcal{S}$.
- The session key is encrypted using the keys for $S_1, \ldots, S_h$.
- Each privileged user can decrypt; no coalition of revoked users gains any information about the session key (or the message).

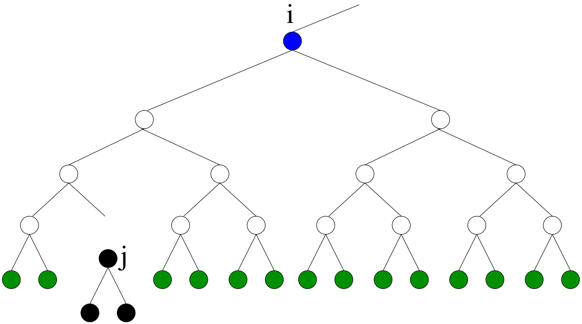
Subset Difference Scheme

Naor-Naor-Lotspiech (2001): patented, AACS standard.
Assumes an underlying full binary tree



Subsets in the collection $\mathcal{S}$

$S_{i,j} = \mathcal{T}_i \setminus \mathcal{T}_j$: has all users that are in $\mathcal{T}_i$ but not in $\mathcal{T}_j$



Collection $\mathcal{S}$: has all subsets $S_{i,j}$ such that $j(\neq i)$ is in the subtree $\mathcal{T}_i$.

Key Assignment

Pseudo-random generator (PRG): $G : \{0,1\}^k \rightarrow \{0,1\}^{3k}$

$G(seed) = G_L(seed) || G_M(seed) || G_R(seed)$

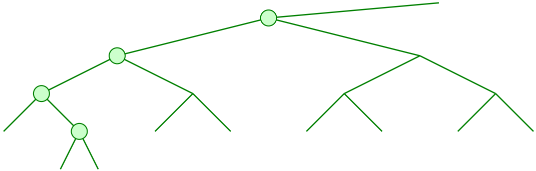


Figure : Key of $S_{i,j}$: $L_{i,j} = G_M(G_R(G_L(G_L(seed_i))))$

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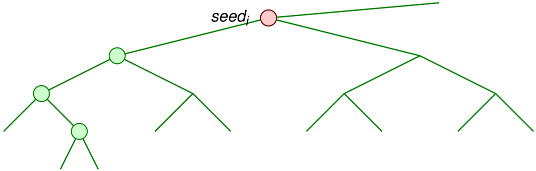

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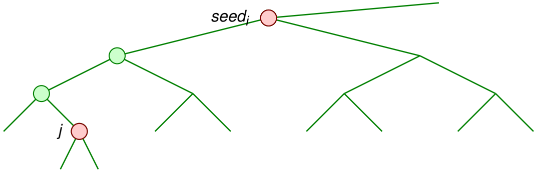

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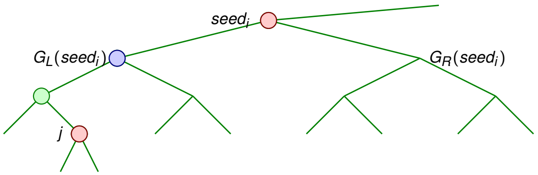

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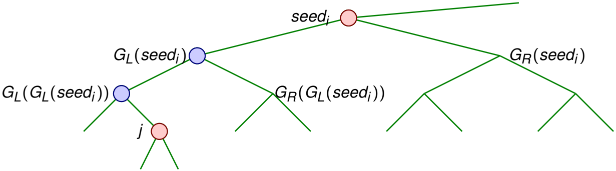

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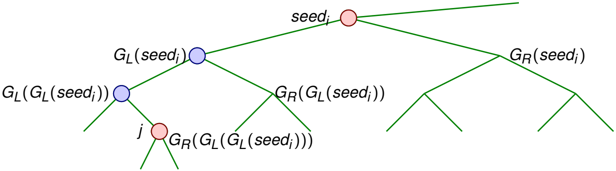

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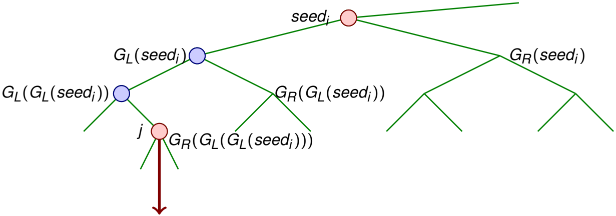

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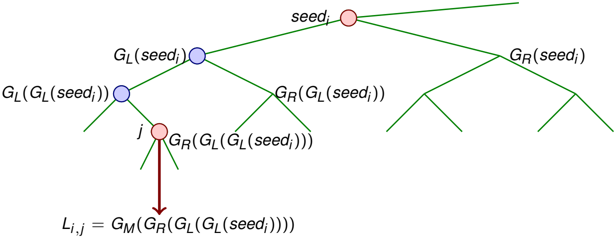

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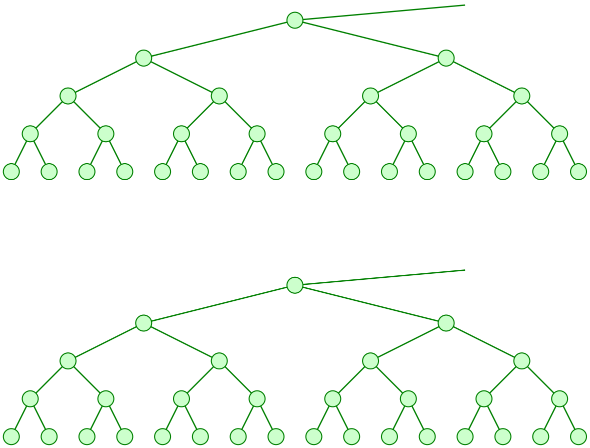

Figure : From one derived seed, keys of many subsets can be generated

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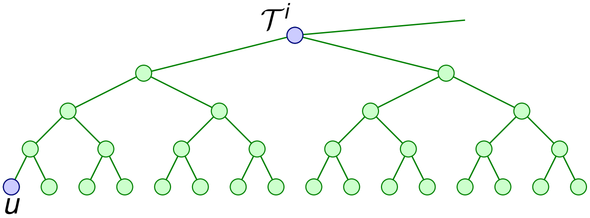

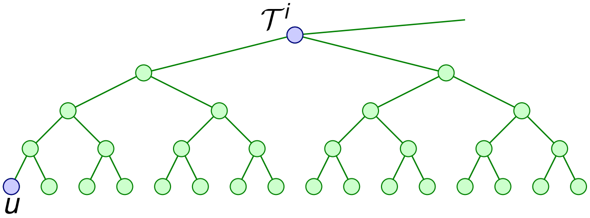

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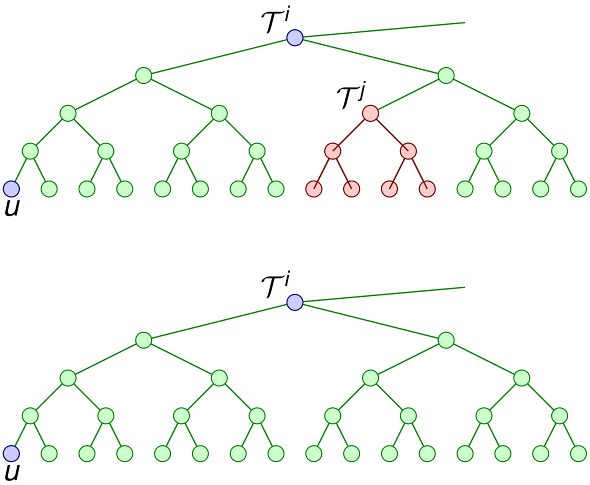

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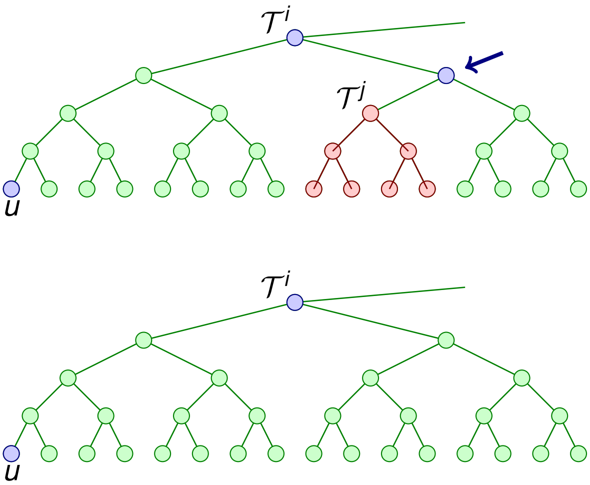

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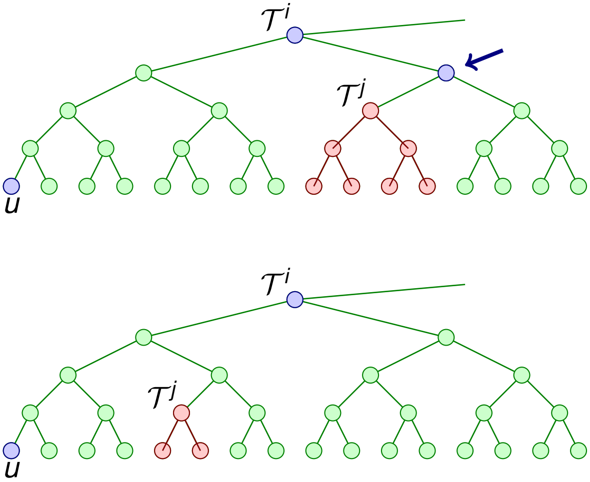

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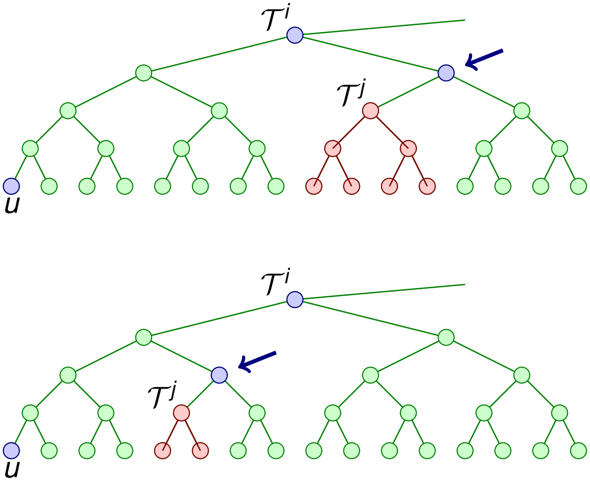

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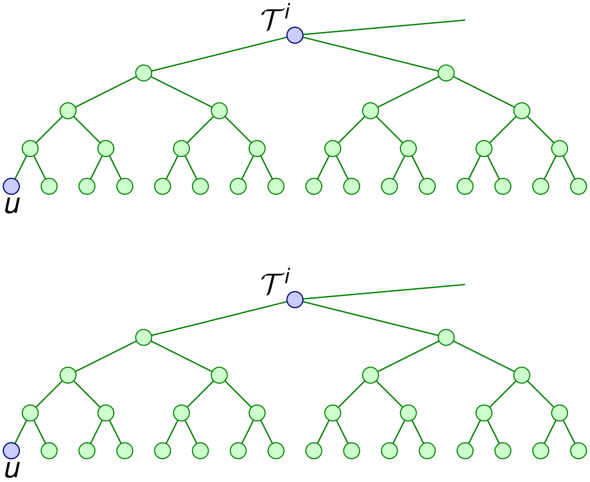

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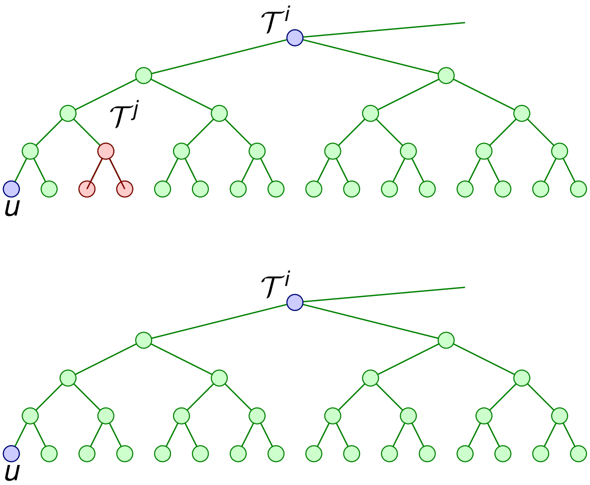

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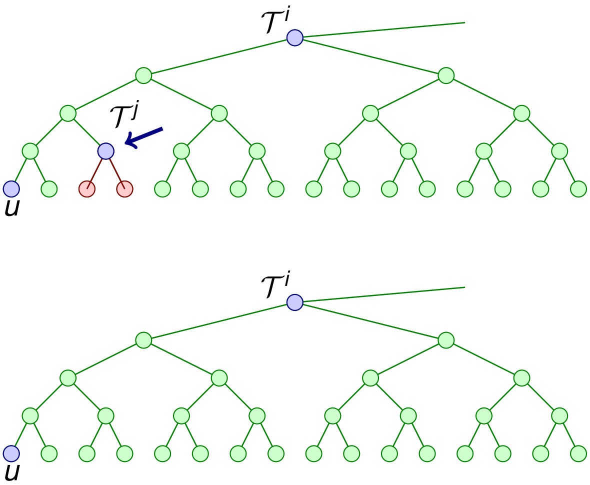

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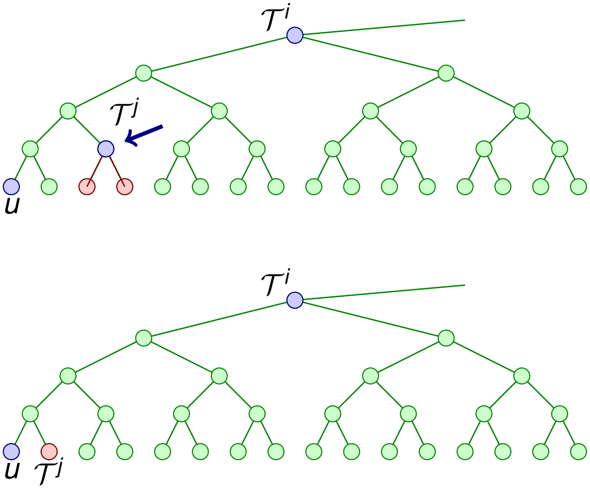

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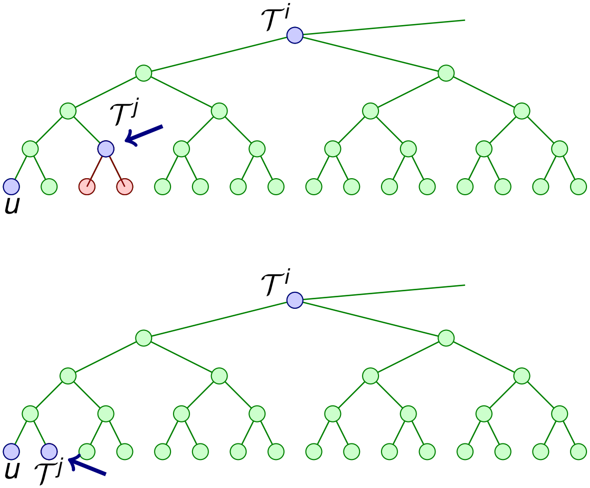

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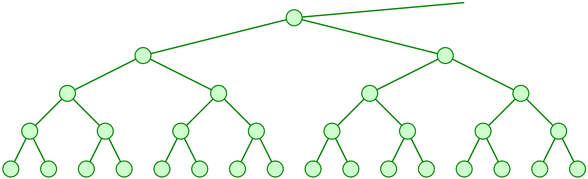

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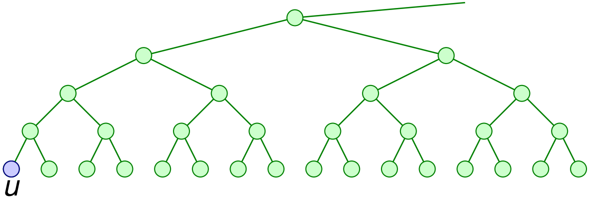

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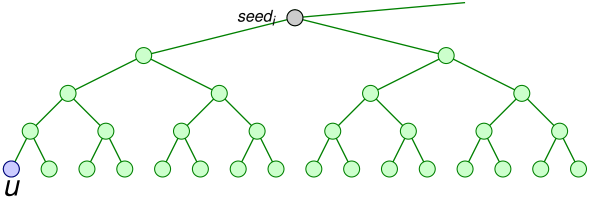

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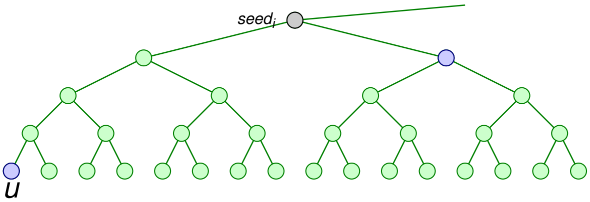

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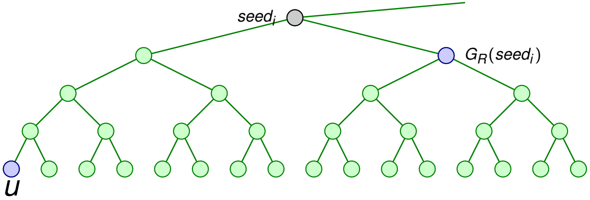

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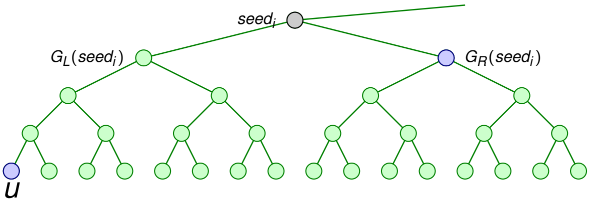

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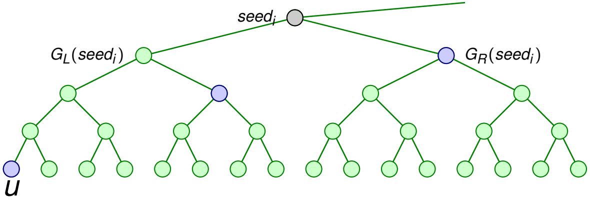

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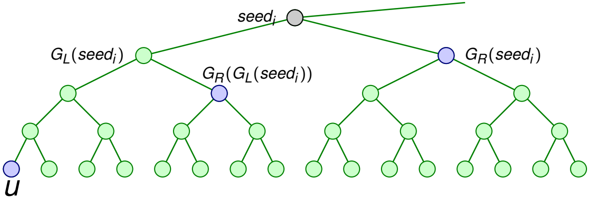

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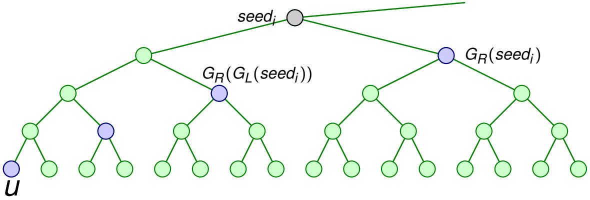

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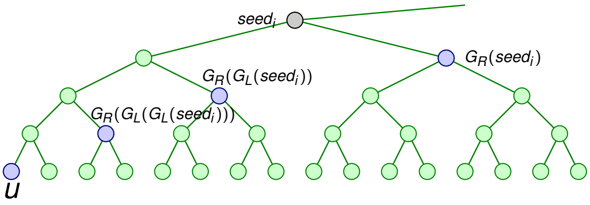

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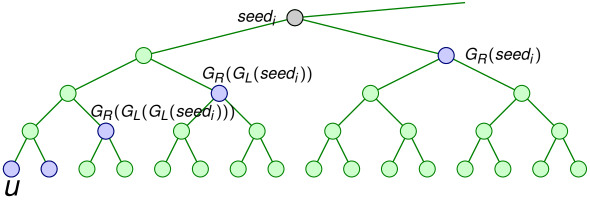

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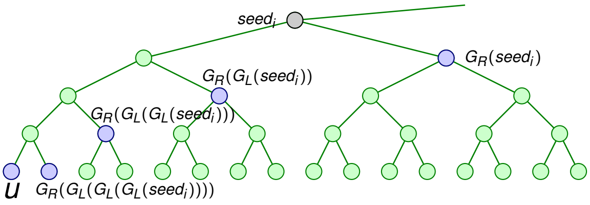

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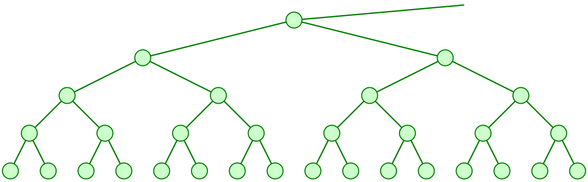

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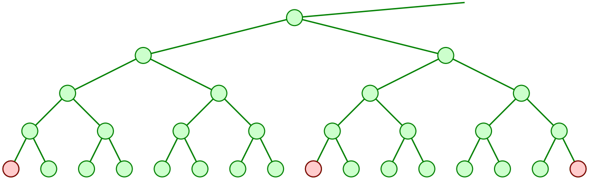

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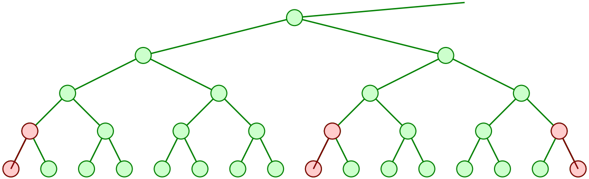

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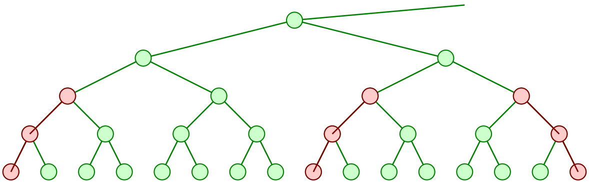

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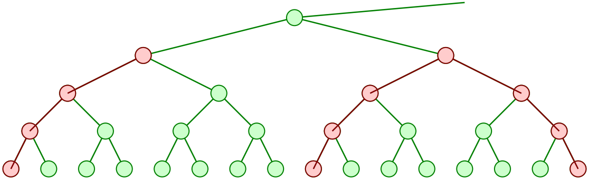

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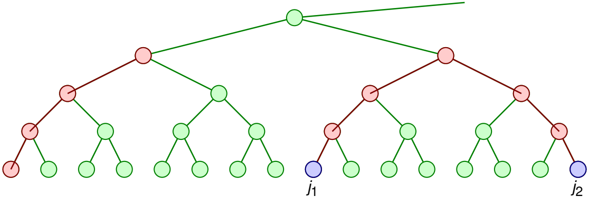

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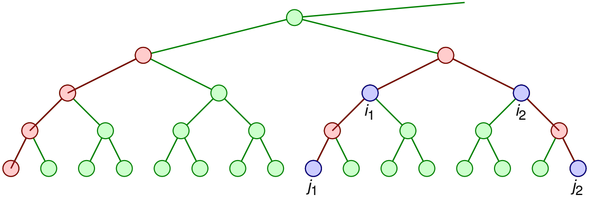

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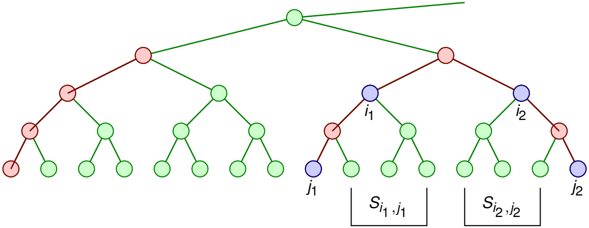

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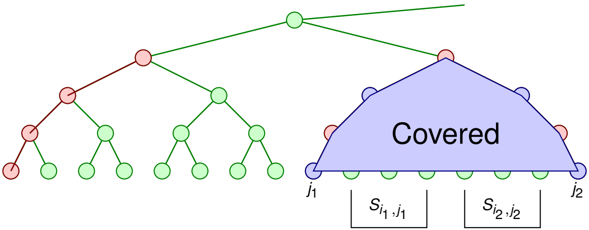

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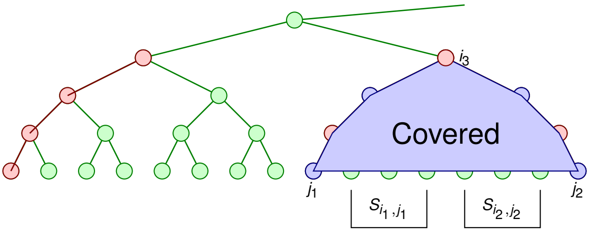

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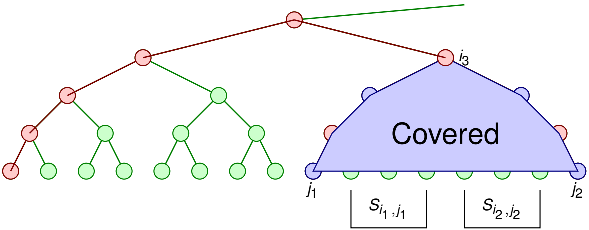

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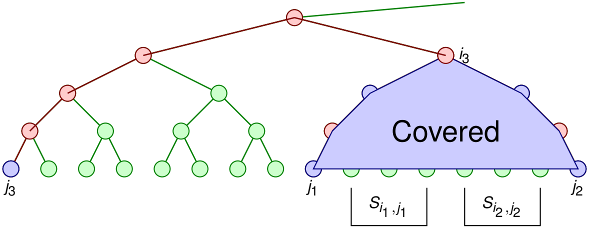

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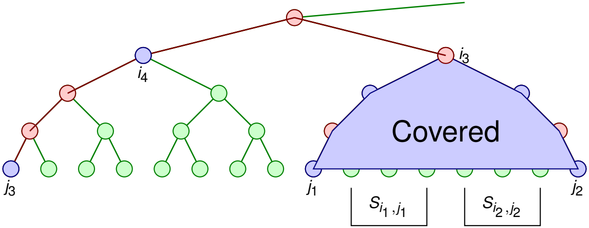

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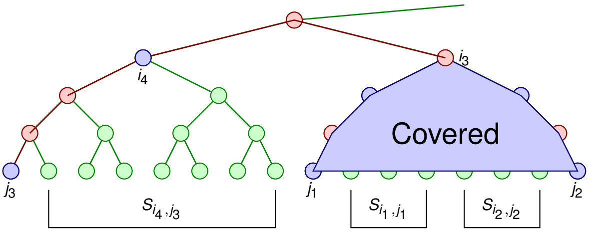

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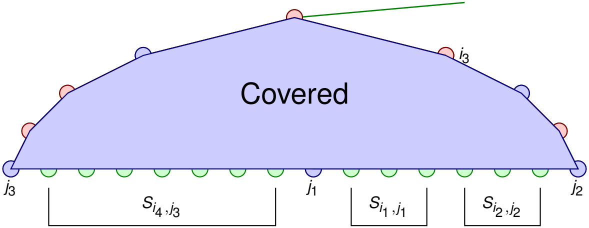
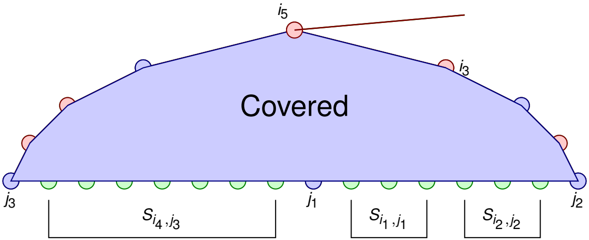


$$S_{i,j} = \mathcal{T}_i \setminus \mathcal{T}_j$$

Subset Cover Finding Algorithm



$$S_{i,j} = \mathcal{T}_i \setminus \mathcal{T}_j$$

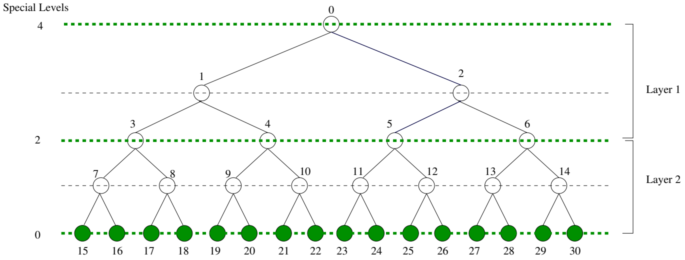
NNL-SD Parameters

For n users out of which r are revoked:

- User storage needed: $O(\log^2(n))$.
- Header length in the worst case: $2r - 1$.
- Decryption time in the worst case: $O(\log n)$.

Layered Subset Difference Scheme

Halevy-Shamir (CRYPTO, 2002) Some levels are marked as *"special"*.



Layered SD Scheme

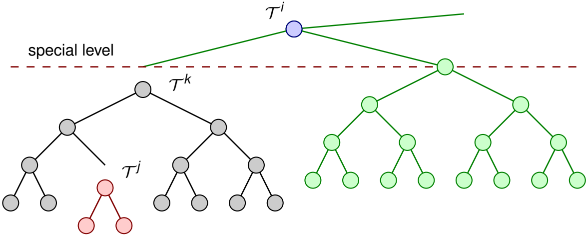


Figure : The subset $S_{i,j}$ split into $S_{i,k}$ (green leaves) and $S_{k,j}$ (grey leaves).

Layered SD Scheme

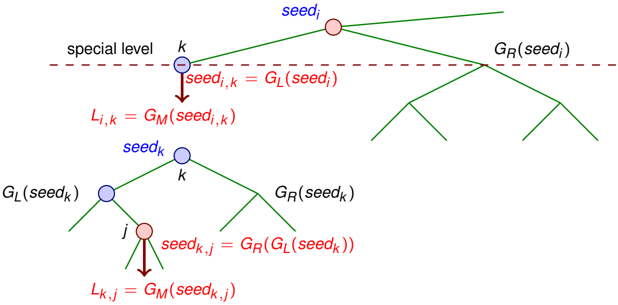


Figure : Key for $S_{i,k}$ is $L_{i,k} = G_M(G_L(seed_i))$ and for $S_{k,j}$ is $L_{k,j} = G_M(G_R(G_L(seed_k)))$.

Important Parameters

NNL-SD scheme:

- User storage needed: $O(\log^2(n))$.
- Maximum Header Length: $2r - 1$.

HS-LSD scheme:

- User Storage needed: $O(\log^{3/2} n)$.
- Maximum header length: $4r - 2$.

Some Questions

- What is the expected header length of the NNL scheme?
- The NNL and the HS schemes are based on *full* binary trees; What happens if the number of users is not a power of two?
- Is the user storage achieved in the HS scheme the minimum possible?
- Is the (expected) header length achieved in the NNL scheme the minimum possible?
- What happens if we use trees of arity higher than 2?

Tackling Arbitrary Number of Users

Complete Tree SD Scheme

Question: What happens when the number of users is not a power of two?

Answer: Add dummy users to get to the next power of two.

- If the dummy users are considered revoked, then the effect on the header length is disastrous.
- If the dummy users are privileged, the situation is better but, there is still a measureable effect on the header length.

Solution: Use a complete binary tree.

- "Completes" (and also subsumes) the NNL-SD scheme to work for any number of users.
- Conceptually simple; working out the details is a bit involved.

CTSD Scheme: Header Length Analysis

$N(n, r, h)$: number of revocation patterns with n users, out of which r users are revoked and the header length is h.

Recurrence relation for $N(n, r, h)$.

- $N(\lambda_i, r_1, h_1) = T(\lambda_i, r_1, h_1) + \sum_{j \in \mathrm{IN}(i)} T(\lambda_j, r_1, h_1 - 1)$ where $\mathrm{IN}(i)$ is the set of all internal nodes in the subtree $\mathcal{T}^i$ excluding the node i.
- $T(\lambda_i, r_1, h_1) = \sum_{r'=1}^{r_1-1} \sum_{h'=0}^{h_1} N(\lambda_{2i+1}, r', h') \times N(\lambda_{2i+2}, r_1 - r', h_1 - h')$ where λ_{2i+1} (respectively λ_{2i+2}) is the number of leaves in the left (respectively right) subtree of $\mathcal{T}^i$.

Boundary Conditions

$T(\lambda_i, r_1, h_1)$	$r_1 < 0$	$r_1 = 0$	$r_1 = 1$	$2 \leq r_1 < n$	$r_1 = n$	$r_1 > n$
$h_1 = 0$	0	0	0	0	1	0
$h_1 \geq 1$	0	0	0	from rec.	0	0
$N(\lambda_i, r_1, h_1)$	$r_1 < 0$	$r_1 = 0$	$r_1 = 1$	$2 \leq r_1 < n$	$r_1 = n$	$r_1 > n$
$h_1 = 0$	0	0	0	0	1	0
$h_1 = 1$	0	1	n	from rec.	0	0
$h_1 > 1$	0	0	0	from rec.	0	0

Table : Boundary conditions on $T(n, r, h)$ and $N(n, r, h)$.

Computing $N(n, r, h)$

Dynamic Programming:

- $N(n, r, h)$ can be computed in $O(r^2h^2 \log n + rh \log^2 n)$ time and $O(rh \log n)$ space.
- $N(n, r, h)$ for all possible h can be computed in $O(r^4 \log n + r^2 \log n)$ time and $O(r^2 \log^2 n)$ space.
- $N(n, r, h)$ for all possible r and h can be computed in $O(n^4 \log n + n^2 \log^2 n)$ time and $O(n^2 \log n)$ space.
- $N(i, r, h)$ for $2 \leq i \leq n$ and all possible r and h can be computed in $O(n^5 + n^3 \log n)$ time and $O(n^3)$ space.

Previous to our work, the only known method was to enumerate all possible $\binom{n}{r}$ revocation patterns, run the header generation algorithm and count the number of patterns leading to a header of size h.

CTSD: Maximum Header Length

Theorem: The maximum header length in the CTSD method for n users is $\min(2r - 1, \lfloor \frac{n}{2} \rfloor, n - r)$.

- For the NNL-SD scheme, the bound of $2r - 1$ was known.
- Complete picture: if $r \leq n/4$, the bound $2r - 1$ is appropriate; if $n/4 < r \leq n/2$, the bound $n/2$ is appropriate; and for $r > n/2$, the bound $n - r$ is appropriate.
- Using the CTSD method is never worse than individual transmission to privileged users.
- The proof requires extensive use of the recurrence for $N(n, r, h)$.

n_r: The value of n for which the header length of $2r - 1$ is achieved with r revoked users.

- A complete characterisation of n_r is obtained.

CTSD: Expected Header Length

Random experiment: Select a random subset of r users out of n users and revoke them.
Random variable $X^i_{n,r}$: takes the value 1 if $S_{i,j}$ is in the header for some j and 0 otherwise.

- $E[X^i_{n,r}] = \Pr[X^i_{n,r} = 1]$.

$H_{n,r}$: expected header length for n users with r revoked users.

- $H_{n,r} = \sum E[X^i_{n,r}] = \sum \Pr[X^i_{n,r} = 1]$ where the sum is over all the $n-1$ internal nodes i in the tree.

CTSD: Expected Header Length

- For all nodes i at the same level, $\Pr[X^i_{n,r} = 1]$ takes at most 3 possible values.
- As a consequence, the sum can be re-written to vary over the levels of the tree.
- $H_{n,r}$ can be computed in $O(r \log n)$ time and $O(1)$ space.
- Provides granular information: expected number of subsets in the header from all the nodes at a certain level.
- Since CTSD subsumes NNL-SD, all the results also hold for NNL-SD.

NNL-SD: Expected Header Length

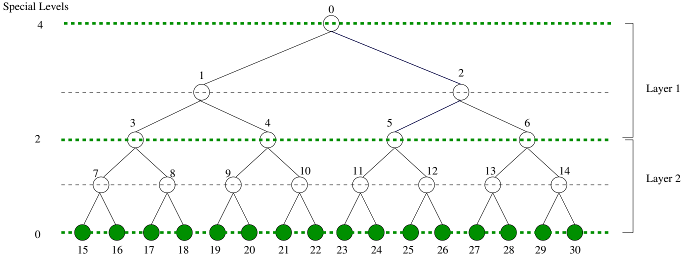
Theorem: For all $n \geq 1$, $r \geq 1$, the expected header length $H_{n,r} \uparrow H_r$, as n increases through powers of two, where

$$H_r = 3r - 2 - 3 \times \sum_{i=1}^{r-1} \left(\left(-\frac{1}{2} \right)^i + \sum_{k=1}^{i} (-1)^k \binom{i}{k} \frac{(2^k - 3^k)}{(2^k - 1)} \right).$$

r	2	3	4	5	6
H_r/r	1.25	1.25	1.2455	1.2446	1.2448

Reducing User Storage Below Halevy-Shamir Scheme

Halevy-Shamir LSD Scheme



"The root is considered to be at a special level, and in addition we consider every level of depth $k \cdot \sqrt{\log(n)}$ for $k = 1 \ldots \log(n)$ as special (wlog, we assume that these numbers are integers)."

Works for 2^{ℓ_0} users with $\ell_0 = 4, 9, 16, 25$ (in the practical range).

Halevy-Shamir LSD Scheme

- For the case of $n = 2^{28}$, HS suggests special levels to be 28, 22, 16, 10, 5, 0.
- Nothing is mentioned about how to choose the layer lengths when ℓ_0 is not a perfect square.

Extending the HS Scheme

Residual bottom layer: Write $\ell_0 = d(e-1) + p$ where $1 \leq p \leq d$. Then the special levels are

$$\ell_0, \ell_0 - d, \ell_0 - 2d, \ldots, \ell - d(e-1), 0.$$

Balanced layering: Write $\ell_0 = d(e-1) + p = (e-d+p)d + (d-p)(d-1)$. Define the layer lengths from the top to be

$$(\underbrace{d, \ldots, d}_{e-d+p}, \underbrace{d-1, \ldots, d-1}_{d-p}).$$

Extending the HS Scheme

- Both strategies (residual bottom; balanced) can be shown to provide the same user storage.
- Having smaller layers nearer the top increases the user storage.
- The balanced layering strategy provides slightly smaller expected header length. We call this the extended-HS (eHS) layering strategy.

Layering Strategy

A choice of special levels is called a layering strategy.

- A layering strategy $\boldsymbol{\ell}$ is denoted by the numbers of the special levels $\ell_0 > \ell_1 > ... > \ell_{e-1} > \ell_e = 0$.
- The layering strategy has $(e + 1)$ special levels.
- Let $\boldsymbol{\ell} = (\ell_0, \ldots, \ell_e)$.
- In general, the layer lengths need not be (almost) equal.

Layering Strategy and User Storage

$$\mathrm{storage}_0(\boldsymbol{\ell}) = \sum_{i=0}^{e-1} \ell_i + \frac{1}{2}\sum_{i=0}^{e-1}(\ell_i - \ell_{i+1})(\ell_i - \ell_{i+1} - 1).$$

Recursive description:

$$\begin{aligned}&\mathrm{storage}_0(\ell_0, \ell_1, \ldots, \ell_e)\\ &= \ell_0 + \frac{(\ell_0 - \ell_1)(\ell_0 - \ell_1 - 1)}{2} + \mathrm{storage}_0(\ell_1, \ldots, \ell_e).\end{aligned}$$

Root as a Non-Special Layer

Observations:

- It can be shown that the probability of the root generating a subset in the header is small.
- Having the root as a special layer increases the user storage.

Layering strategy with root as a non-special layer:

$$\text{storage}_1(\boldsymbol{\ell}) = \text{storage}_0(\boldsymbol{\ell}) - \ell_1.$$

Reduces user storage by ℓ_1 at a negligible increase in the expected header size.

Storage Minimal Layering

- Given ℓ_0, let $\mathsf{SML}_0(\ell_0)$ be a layering strategy which minimises the user storage among all layering strategies;
- $\#\mathsf{SML}_0(\ell_0)$: user storage required by $\mathsf{SML}_0(\ell_0)$;
- $\mathsf{SML}_1(\ell_0)$ and $\#\mathsf{SML}_1(\ell_0)$ corresponds to the case where the root is not special.

Relations/Recurrences for SML

$$\#\mathrm{SML}_0(\ell_0) = \min_{1 \le e \le \ell_0} \#\mathrm{SML}_0(e, \ell_0);$$

where $\#\mathrm{SML}_0(e, \ell_0)$ is the minimum storage that can be achieved with e special levels.

$$\#\mathrm{SML}_0(e, \ell_0) = \min_{(\ell_0, \ldots, \ell_e)} \mathrm{storage}_0(\ell_0, \ell_1, \ldots, \ell_e)$$

where the minimum is over all possible layering strategies $(\ell_0, \ell_1, \ldots, \ell_e)$.

Relations/Recurrences for SML

$$\begin{aligned}
&\#\mathrm{SML}_0(e, \ell_0) \\
&= \min_{1 \le \ell_1 < \ell_0} \left(\ell_0 + \frac{(\ell_0 - \ell_1)(\ell_0 - \ell_1 - 1)}{2} + \#\mathrm{SML}_0(e-1, \ell_1) \right);
\end{aligned}$$

$$\begin{aligned}
&\#\mathrm{SML}_1(\ell_0) \\
&= \min_{e} \min_{\ell_1} \left(\#\mathrm{SML}_0(e-1, \ell_1) + \frac{(\ell_0 - \ell_1)(\ell_0 - \ell_1 + 1)}{2} \right).
\end{aligned}$$

Computing SML

Dynamic Programming:

- An $O(\ell^3)$ time and $O(\ell^2)$ space algorithm to compute $\#\mathrm{SML}_0(\ell_0)$
- The actual layering strategy $\mathrm{SML}_0(\ell_0)$ can also be recovered from the algorithm.
- Once the table has been computed using dynamic programming, it is possible to obtain $\#\mathrm{SML}_1(\ell_0)$ and $\mathrm{SML}_1(\ell_0)$.

Properties of SML

- SML_0 and SML_1 are not necessarily unique; choose the layering for which expected header length is lower.
- Removing ℓ_0 from SML_0 does not necessarily provide SML_1.
- Compared to NNL-SD, eHS reduces storage by a large amount; SML_0 reduces storage below eHS by a small amount; SML_1 reduces storage below eHS by 18% to 24% in the practical range.

Examples of SML

Suppose there are 2^{28} users, i.e., $\ell_0 = 28$:

- NNL-SD: layering: 28,0; storage: 406.
- eHS: layering: 28,22,16,10,5,0; storage: 146.
- SML_0: layering: 28,21,15,10,6,3,1,0; storage: 140.
- SML_1: layering: 22,16,11,7,4,2,0; storage: 119.

Complete Tree LSD Scheme

Question: What if the number of users n is not a power of 2?

Answer: Use a complete tree as in the case of the NNL-SD scheme.

- The notions of layering strategy and storage minimal layering carry over to this case.
- All users would not be required to store the same amount; the requirement is to minimise the maximum of all the user storages.

Header Length

Maximum Header Length:

- At most $\min\left(4r - 2, \left\lceil \frac{n}{2} \right\rceil, n - r\right)$.
- At most $\min\left(4r - 3, \left\lceil \frac{n}{2} \right\rceil, n - r\right)$ if the root level is special.

Expected Header Length:

- The splitting of subsets complicates the analysis.
- An $O(r \log^2 n)$ time algorithm to compute the expected header length.
- A very useful tool to analyse various schemes.

Constrained Minimisation

Question: Is it possible to obtain expected header length close to that of NNL-SD, but, with lower user storage?

- For each level, we have an expression for the expected number of subsets arising from the nodes at that level.
- Suppose ℓ is a level which maximises the above quantity.

Question: How to choose ℓ?
Answer: How to do this analytically is not clear. Extensive experimentation has shown that $\ell = \ell_0 - \log_2 r$ is a good choice.

Constrained Minimisation Layering

Fix a value of r and set $\ell = \ell_0 - \log_2 r$.

- Level ℓ is made special, so that subsets arising from level ℓ are not split.
- All levels below ℓ are made non-special.
- At most one level above ℓ (mid-way between ℓ and the root) is made special; all other levels are made non-special.

How to Choose r?

- Depending on the application, make an *assumption* on the minimum value of r, say r_{min}.
- If the actual r is greater than r_{min}, then there is no problem.
- If the acutal r is smaller than r_{min}, then the benefits on the header length is not attained.
- Choosing r_{min} to be too small will not lead to substantial savings in user storage; choosing r_{min} to be too large will not provide the desired reduction on header storage.

A CML Example

Number of users is $n = 2^{\ell_0}$ with $\ell_0 = 28$ and suppose $r_{\min} = 2^{10}$.

- NNL-SD: layering: 28,0; storage: 406.
- eHS: layering: 28,22,16,10,5,0; storage: 146;
  header lengths:
  $(1.69, 1.63, 1.64, 1.67, 1.69, 1.72, 1.73, 1.74, 1.75, 1.75)$.
- CML: layering: 23, 18,0; storage: 219;
  header lengths:
  $(1.14, 1.08, 1.04, 1.03, 1.01, 1.01, 1.00, 1.00, 1.00, 1.00)$.

Header lengths for 10 equispaced values of r from 2^{10} to 2^{14} normalised by the header length of the NNL-SD scheme.

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Uploaded on 13th August, 2014.

Thank you for your attention!