Doubly bottom and bottom- strange tetraquarks from Lattice QCD

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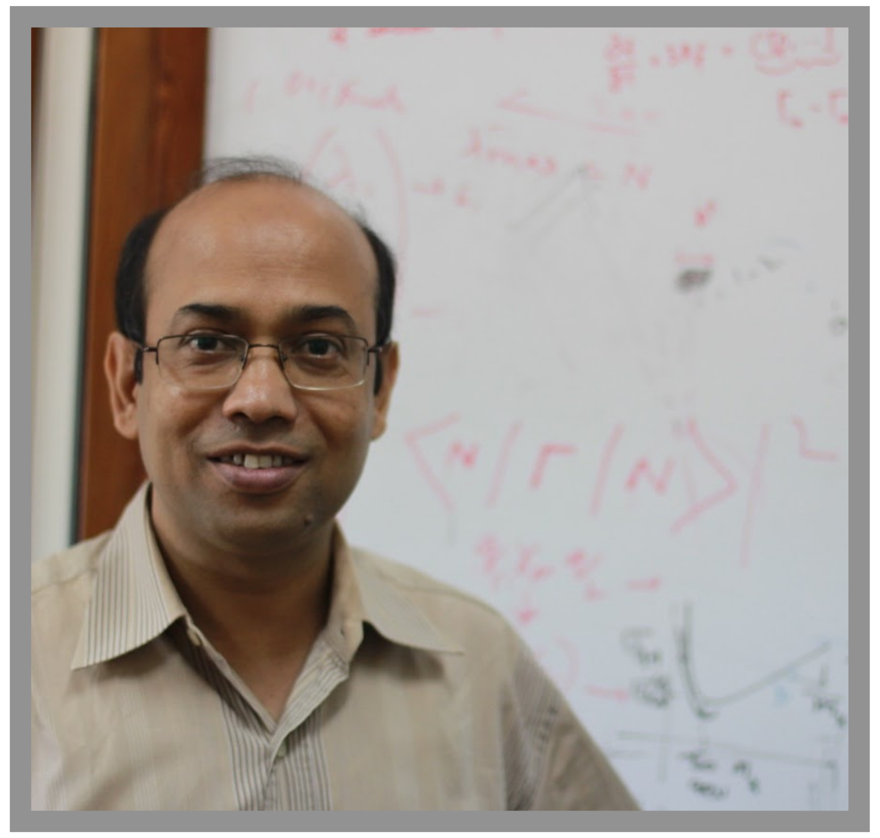
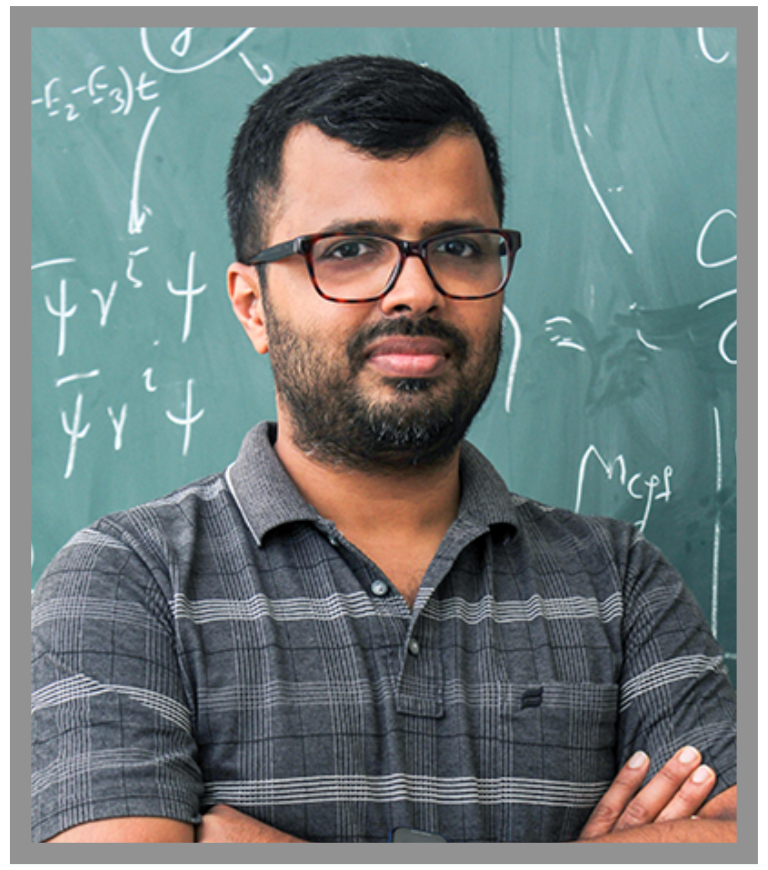
Based on Phys. Rev. D 111, 114504



Nilmani Mathur(TIFR)



M. Padmanath(IMSc)

Introduction and Motivation

Volume 8, number 3 PHYSICS LETTERS 1 February 1964

A SCHEMATIC MODEL OF BARYONS AND MESONS *

M. GELL-MANN
California Institute of Technology, Pasadena, California

Received 4 January 1964

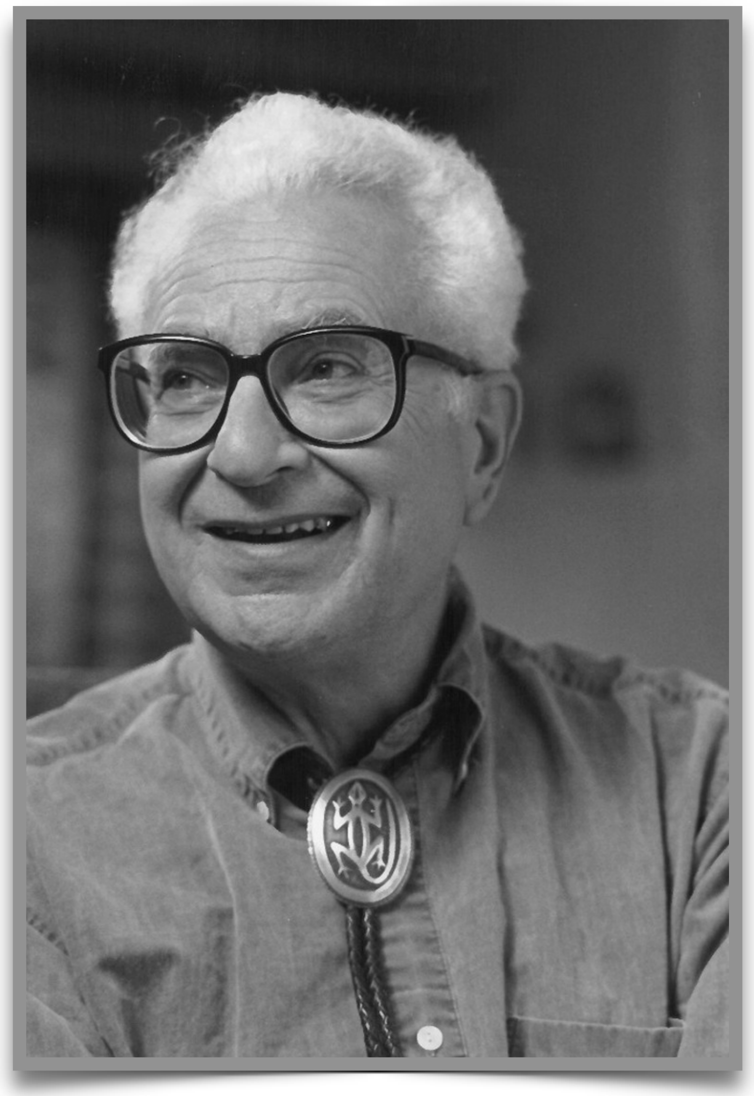
. Baryons can now be constructed from quarks by using the combinations (qqq), $(qqqq\bar{q})$, etc., while mesons are made out of $(q\bar{q})$, $(qq\bar{q}\bar{q})$, etc.

PHYSICAL REVIEW D VOLUME 15, NUMBER 1 1 JANUARY 1977

Multiquark hadrons. I. Phenomenology of $Q^2\bar{Q}^2$ mesons*

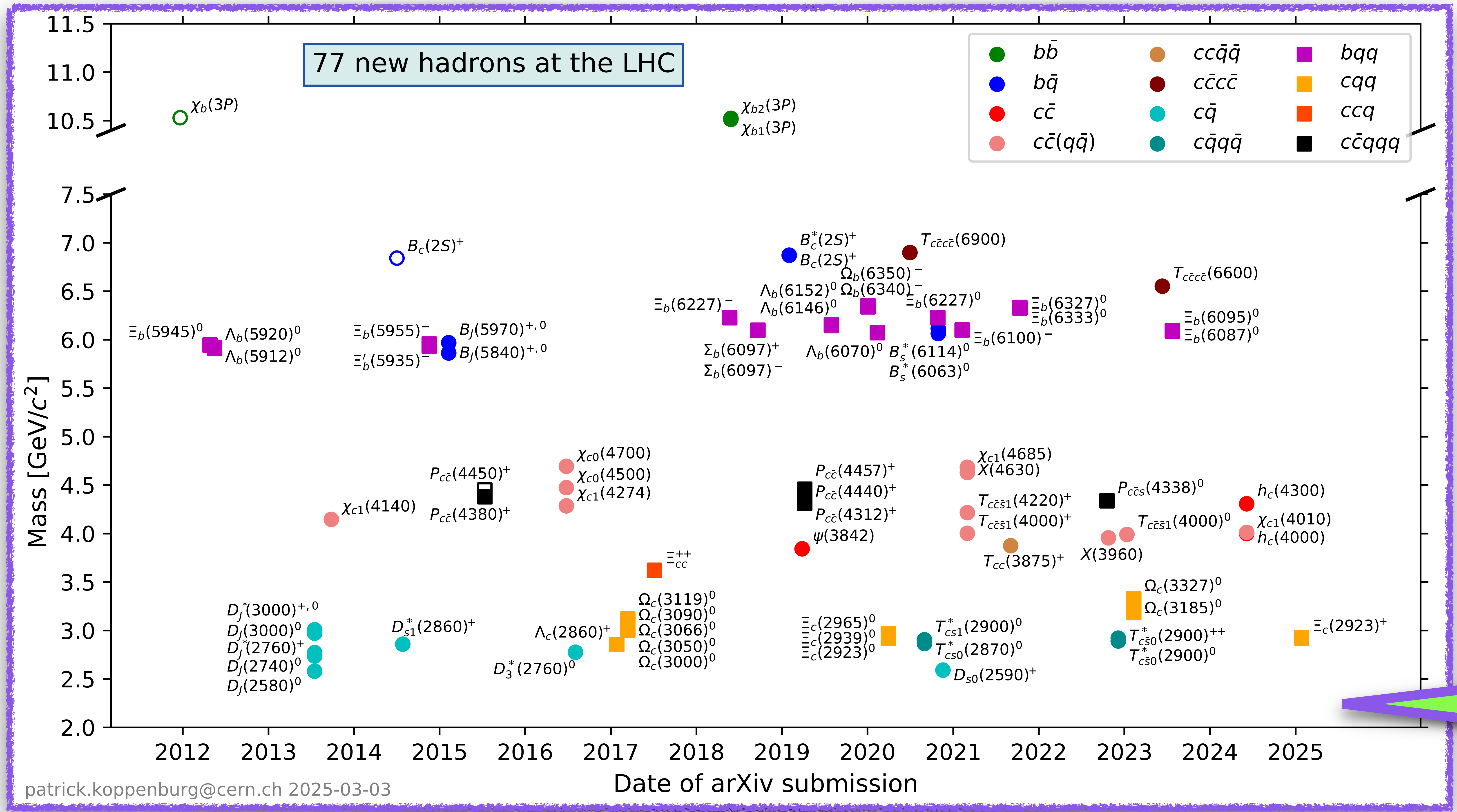
R. J. Jaffe†
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
and Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

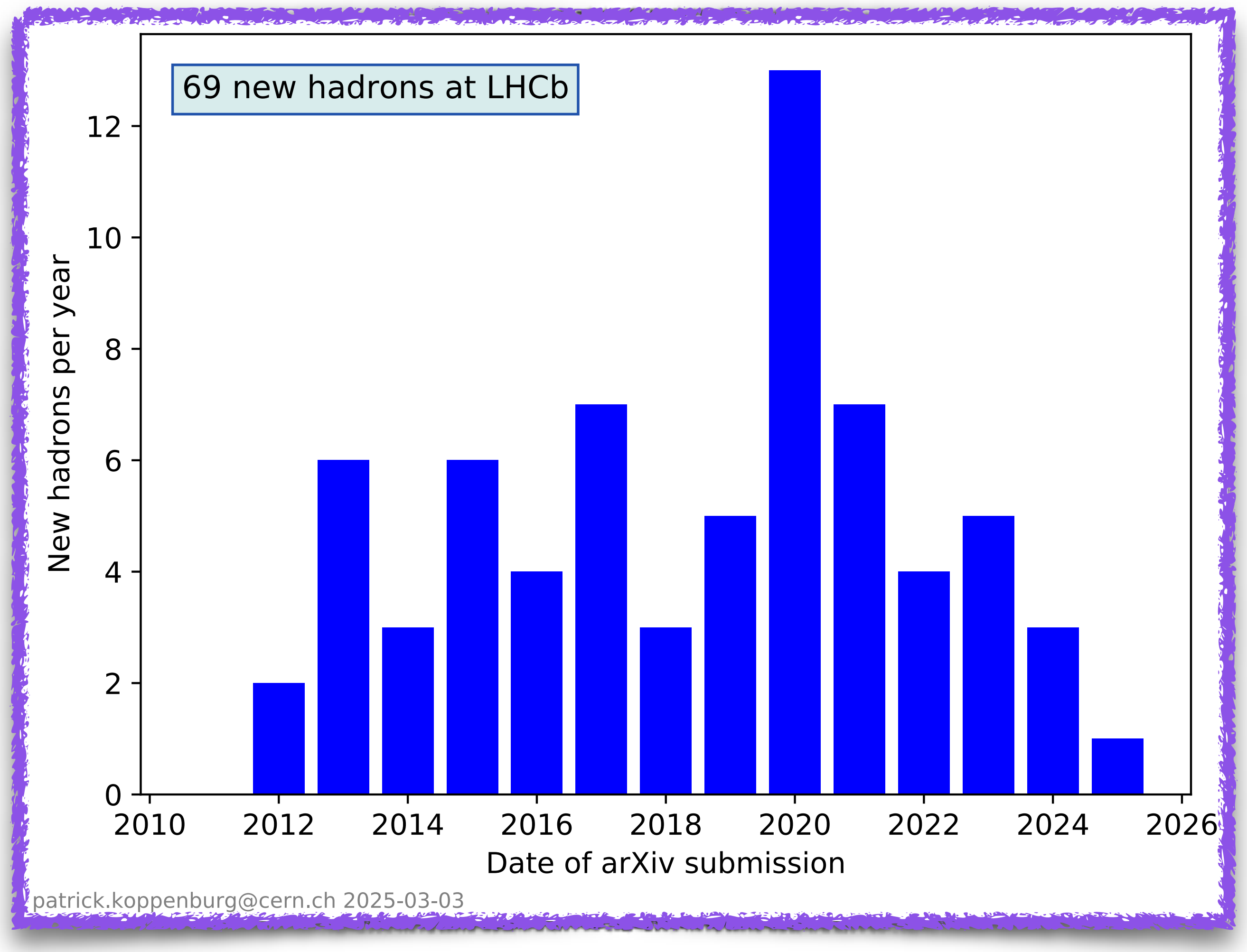
(Received 15 July 1976)



- Murray Gellmann indicated the possibility of multiquark systems.
- Jaffe described it as color neutral states of diquark and antidiquark.
- Currently known as exotic hadrons or XYZ states.

Experimental Results in LHC

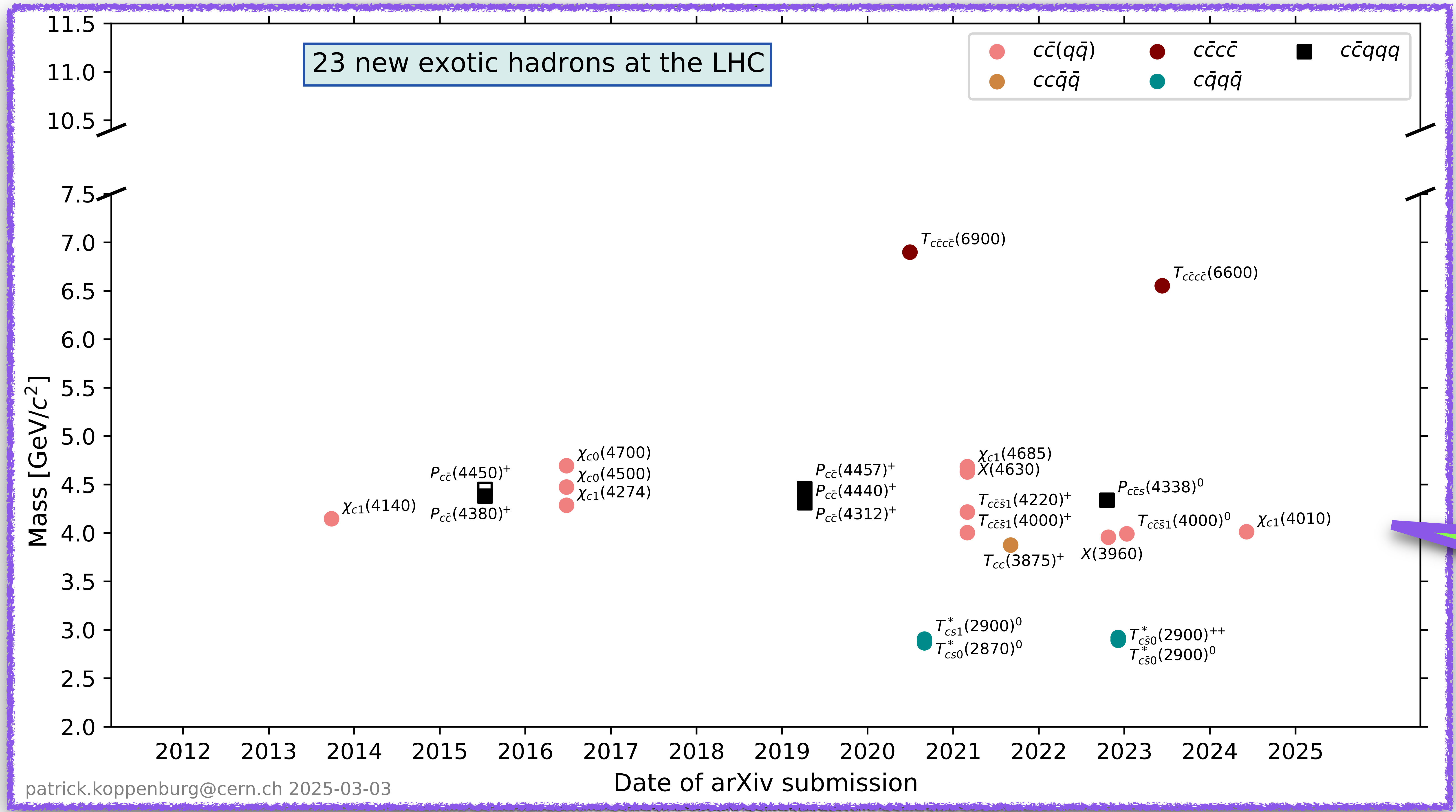


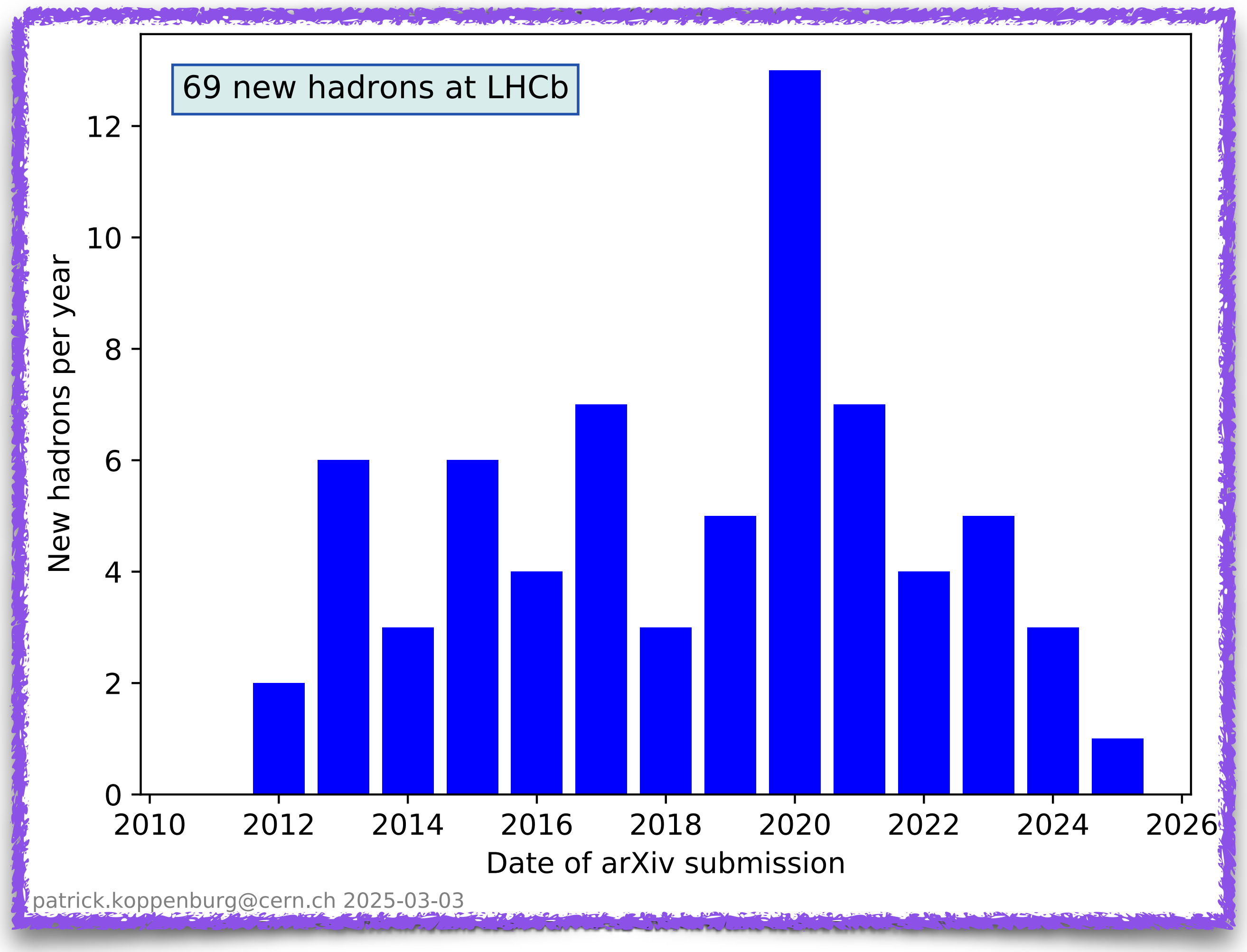


Particles discovered in the last decade including conventional heavy hadrons, open flavor mesons, exotic particles

Other research facilities like Belle, BES-III etc. have active spectroscopy programs.

Experimental Results in LHC





Collection of exotic particles

Other research facilities like Belle, BES-III etc. have active spectroscopy programs.

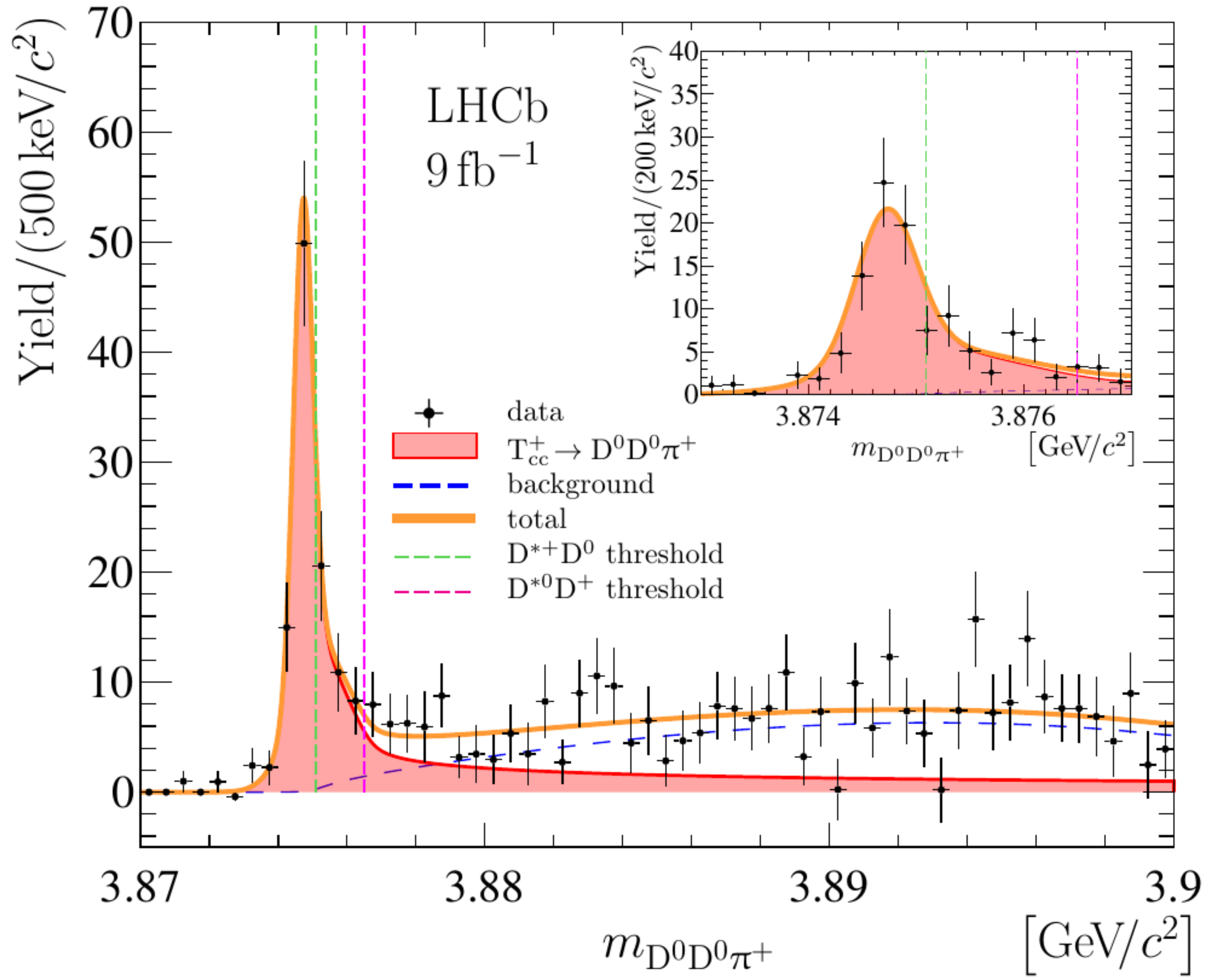
https://qwg.ph.nat.tum.de/exoticshub/

$T_{cc}(cc\bar{u}\bar{d})$ discovery at LHC

- In 2021, LHCb made headlines by discovering the longest-lived exotic state ever, observed close to X(3872).

- It was observed in the channel $I = 0$, $J^P = 1^+$ below D^0D^{*+} threshold(in $D^0D^0\pi^+$).

- Many more exotic tetraquark states discovered recently e.g. T_{cs}, $T_{c\bar{s}}$, Z_c and so on. Scope for T_{bc}, T_{bs} in near future.



Nature phys: https://rdcu.be/dNMRV
Arxiv:2109.01038

$$\delta M = M_{T_{cc^+}} - (M_{D^{*+}} + M_{D^0})$$

$$\delta M_{pole} = -360 \pm 40(^{+4}_{-0}) keV/c^2$$

$$\Gamma_{pole} = 48 \pm 2(^{+00}_{-14}) KeV$$

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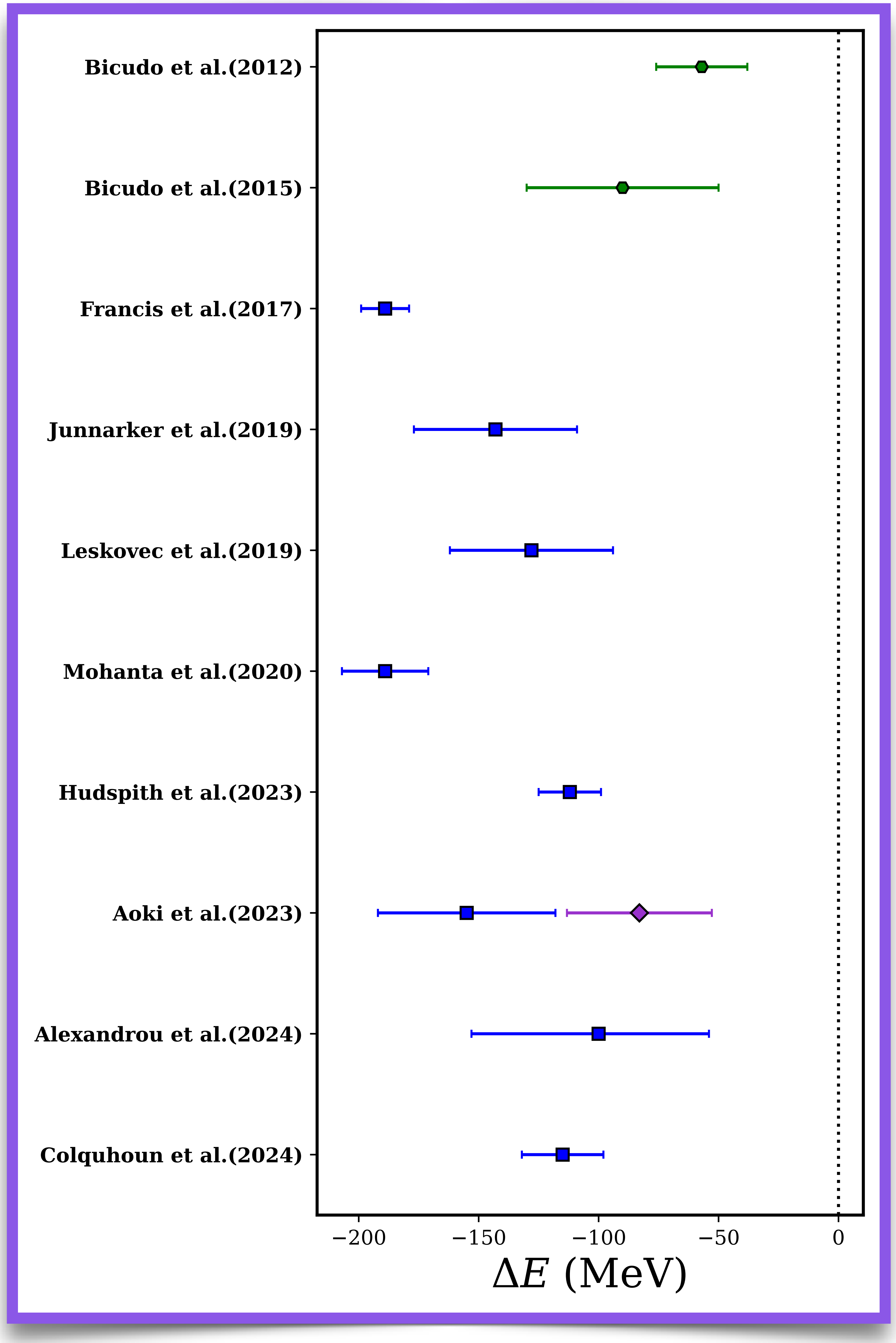
Long History of $T_{bb}(bb\bar{u}\bar{d})$

- Phenomenological calculation of $I(J^P) = 0(1^+)$ T_{bb} can be trace back to the early 80's.
- Prediction of deeply bound state in the heavy quark limit.

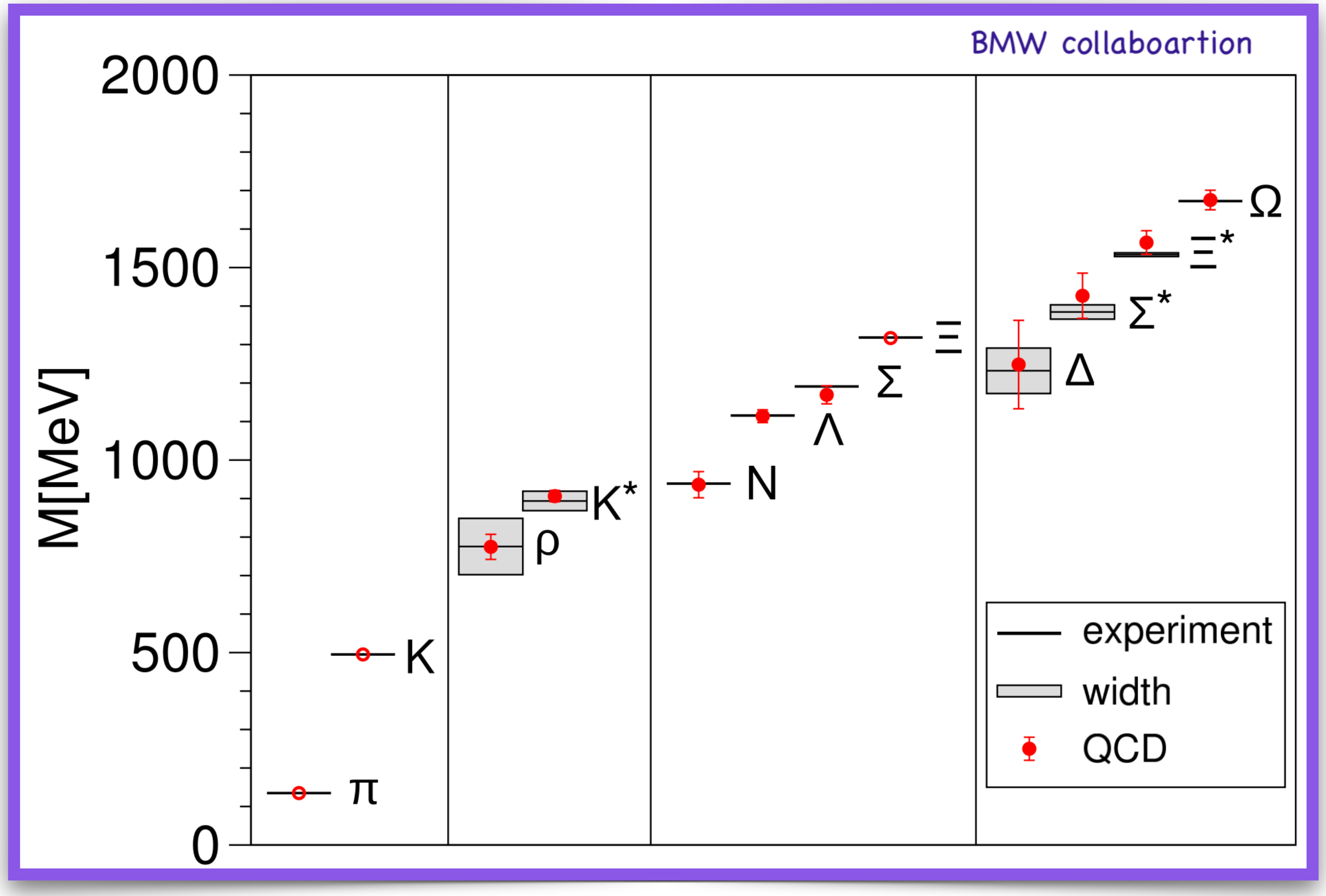
Nucl.Phys.B 399 (1993)

- Results from various phenomenological studies suggest possibility of deeply bound state.
- Previous lattice calculations on $bb\bar{u}\bar{d}$ $I = 0$ shows deep bound state upto systematics.
- Long way to go for experimental verification.

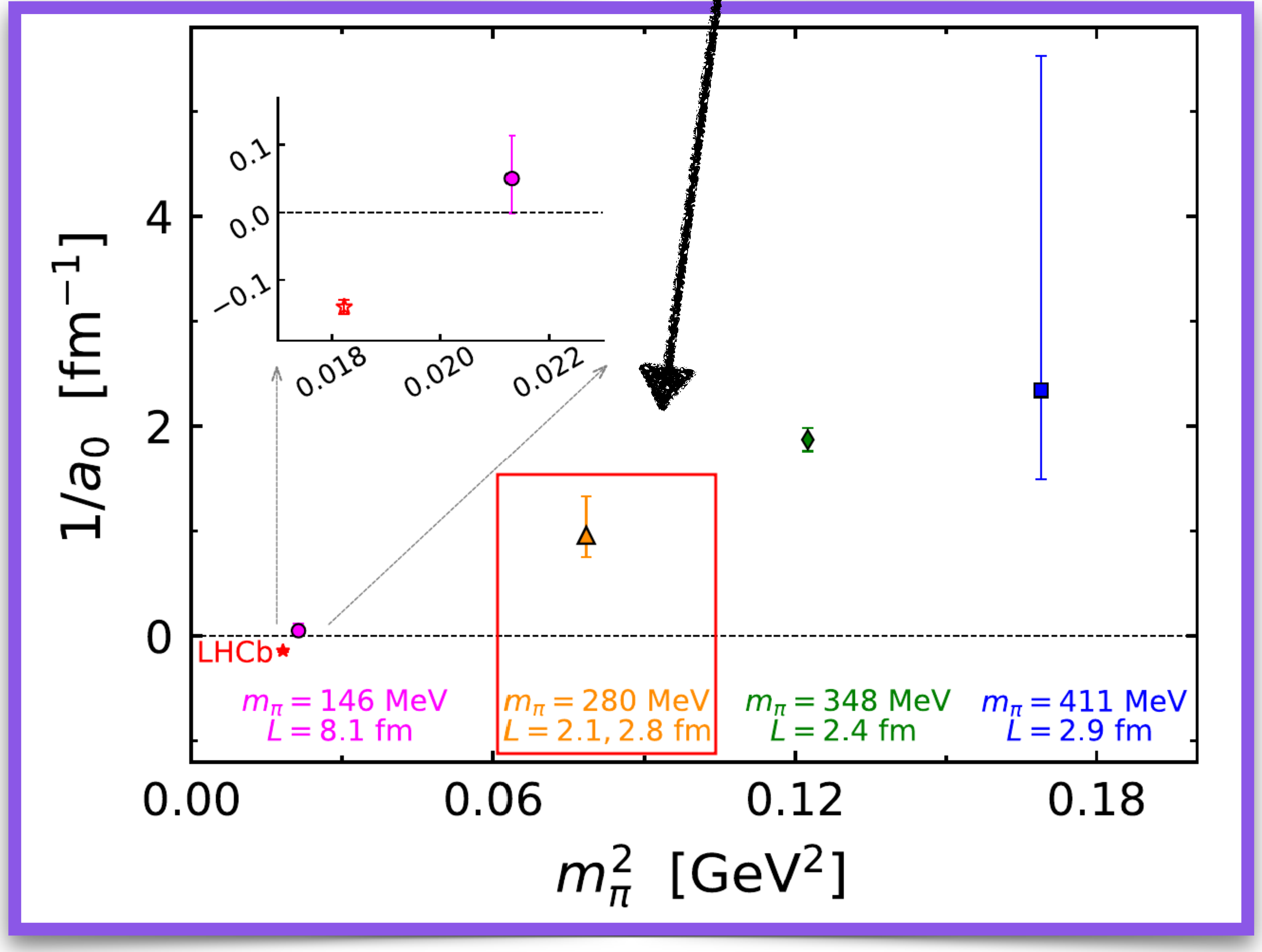


Lattice Validations

Phys. Rev. Lett. 129, 032002 Padmanath, Prelovsek



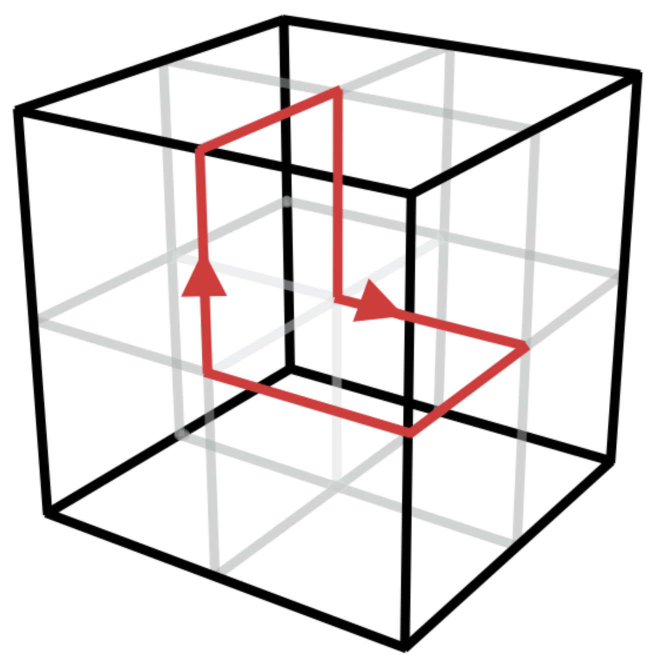
Science 322 (2008) 1224-1227



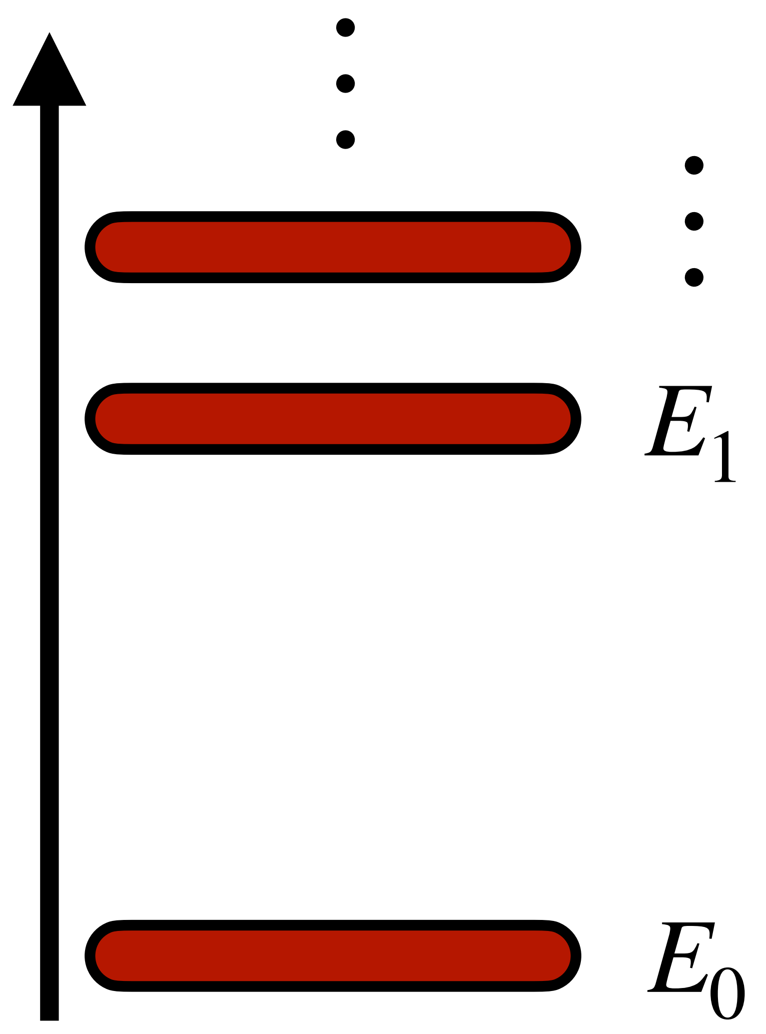
Yan Lyu et al. Phys Rev Lett.131.161901

- **Early Lattice calculations accurately validates masses of known hadrons.**
- T_{cc} **lattice results matches with that of the experimental results as pion mass decreases.**

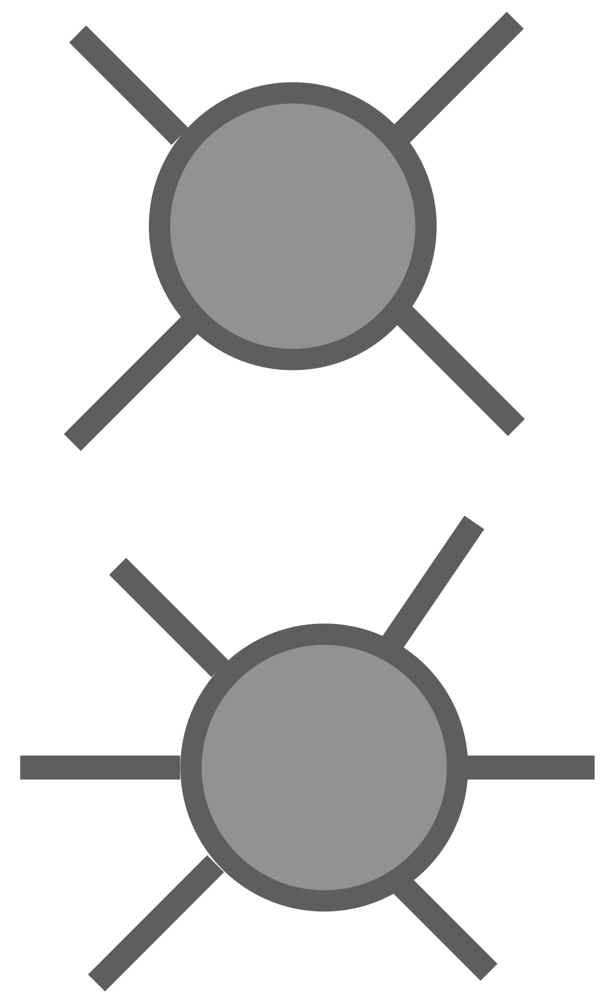
Hadron Spectroscopy in a Nutshell



Ensemble Generation



Finite Volume Spectra



Scattering Amplitude

$a \to 0$

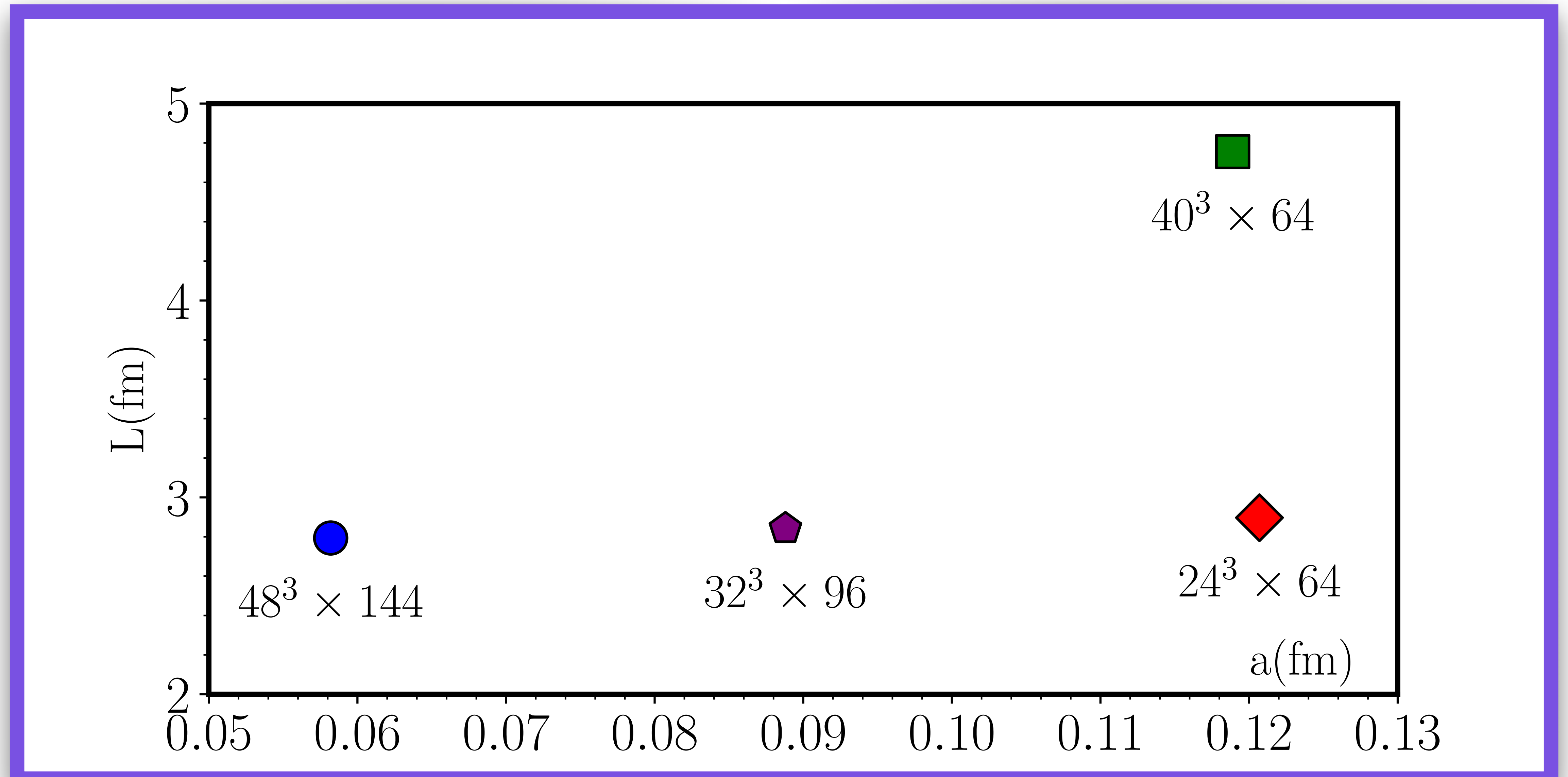
$L \to \infty$

$M_{ps} \to M_{\pi}^{phy}$

Continuum Extrapolation

Lattice Setup

- We are interested in doubly bottom tetra quarks $bb\bar{u}\bar{d}$ with $I(J^P) = 0(1^+)$ and $bs\bar{u}\bar{d}$ with $I(J^P) = 0(1^+)$ and $0(0^+)$.
- Worked with 4 MILC ensembles with $N_f = 2 + 1 + 1$ using HISQ action.
- Ensembles were generated at unphysical light quarks and physical charm and strange quarks.
- Light quark propagators were constructed using Overlap action. For heavy(bottom) quark, we used NRQCD action.
- Wall-source smearing setup.
- Used multiple volumes, box-sink correlators to reduce systematic effects.
- First lattice calculation for $bs\bar{u}\bar{d}$ with finite volume analysis.



NRQCD formalism

- Relativistic propagator not possible as $am_b >> 1$ in our setup. Require bigger lattice size.

- NRQCD becomes most suitable candidate.

- Energy scales here,

$$m_b >> m_b v >> m_b v^2$$

- We exclude rest mass term to make momentum as highest energy scale allow for calculation in larger lattice spacing.

- Offset correction accounted at the time of Analysis.

Building Hadrons

- Goal is to construct interpolating fields coupled with ground state of desired quantum number.

- Properties:-

  1. Flavor Sturture:- Correct combination of quark fields.

  2. Spin and Parity:- Appropriate Dirac bilinear Γ and quadrilinear(Tqs) to match desired spin and parity.

- Lattice breaks O(3) symmetry, operators must transform according to irreps of the cubic group.

- e.g. Pion:- $\Phi_\pi = \bar{d}^\alpha_c (\gamma_5)_{\alpha\beta} u^\beta_c$, c -> color index

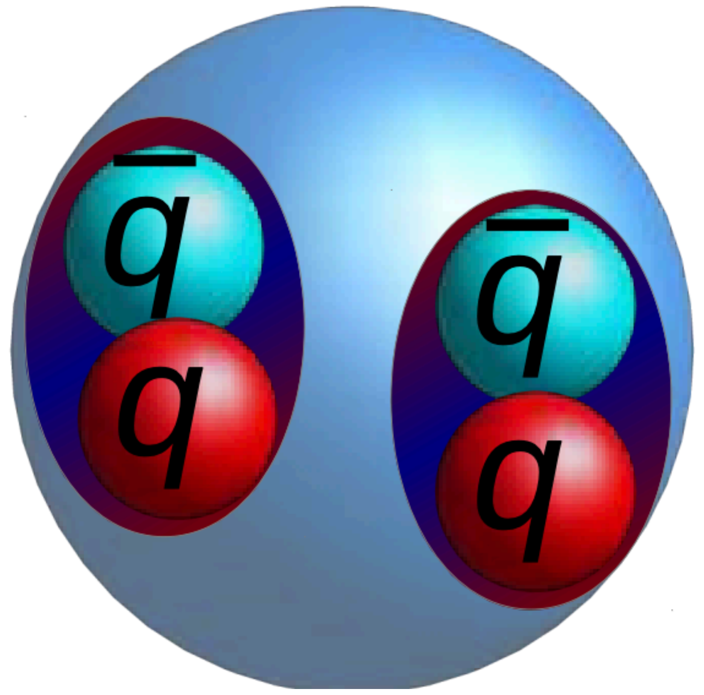
Pseudo scalar

Extracting Finite Volume Spectrum

- Finite volume spectrum can be calculated using Euclidean $\mathscr{C}_{ij}(t)$, between $\Phi's$

$$\mathscr{C}_{ij}(t) = \sum_X \left\langle \Phi_i(\mathbf{x}, t)\tilde{\Phi}_j^\dagger(0) \right\rangle \propto e^{-E_n t}$$

- To extract spectrum we need good interpolating operators.

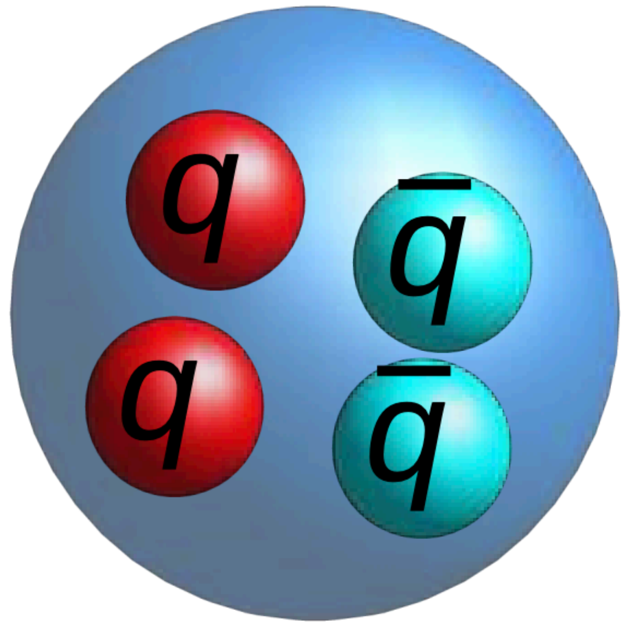


- Here we are using two types of operators for $bb\bar{u}\bar{d}$ TQ.

<u>Meson-Meson</u> :

$$\Phi_{\mathscr{M}_{BB^*}}(x) = [\bar{u}(x)\gamma_i b(x)][\bar{d}(x)\gamma_5 b(x)] - [\bar{u}(x)\gamma_5 b(x)][\bar{d}(x)\gamma_i b(x)]$$

<u>Diquark-antidiquark</u>:



$$\Phi_{\mathscr{D}}(x) = [((\bar{u}(x)^T C\gamma_5 \bar{d}(x)) - (\bar{d}(x)^T C\gamma_5 \bar{u}(x))) \times (b^T(x) C\gamma_i b(x))]$$

Operators for T_{bs}

- For axial-vector $I(J^P) = 0(1^+)$, we use three operators,

$$\Phi_{\mathscr{M}_{KB^*}}(x) = \left[\bar{u}(x)\gamma_i b(x)\right]\left[\bar{d}(x)\gamma_5 s(x)\right] - \left[\bar{u}(x)\gamma_5 s(x)\right]\left[\bar{d}(x)\gamma_i b(x)\right]$$

$$\Phi_{\mathscr{M}_{BK^*}}(x) = \left[\bar{u}(x)\gamma_5 b(x)\right]\left[\bar{d}(x)\gamma_i s(x)\right] - \left[\bar{u}(x)\gamma_i s(x)\right]\left[\bar{d}(x)\gamma_5 b(x)\right]$$

$$\Phi_{\mathscr{D}}(x) = \left[\left(\bar{u}(x)^T C\gamma_5 \bar{d}(x) - \bar{d}(x)^T C\gamma_5 \bar{u}(x)\right) \times \left(b^T(x) C\gamma_i s(x)\right)\right]$$

- For scalar $I(J^P) = 0(0^+)$, we use two operators,

$$\Phi_{\mathscr{M}_{BK}}(x) = \left[\bar{u}(x)\gamma_5 b(x)\right]\left[\bar{d}(x)\gamma_5 s(x)\right] - \left[\bar{u}(x)\gamma_5 b(x)\right]\left[\bar{d}(x)\gamma_5 s(x)\right]$$

$$\Phi_{\mathscr{D}}(x) = \left[\left(\bar{u}(x)^T C\gamma_5 \bar{d}(x) - \bar{d}(x)^T C\gamma_5 \bar{u}(x)\right) \times \left(b^T(x) C\gamma_5 s(x)\right)\right]$$

Wall Sources Point Sink

- Instead of using a single point source, unique source is placed at every spatial point on the source time slice.

$$Q(\bar{x}, t; t') = \sum_{\bar{x}'} Q(\bar{x}, t; \bar{x}', t')$$

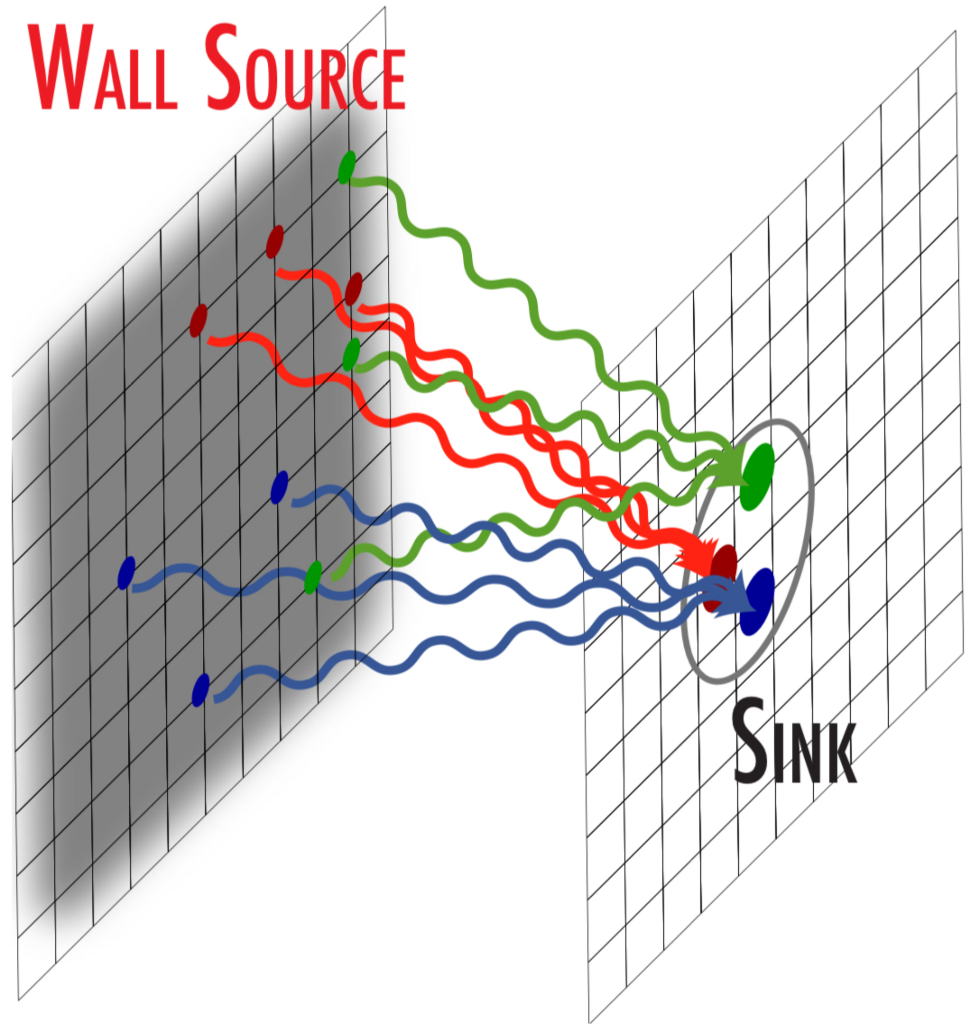


Image Credit:- S. Aoki

- ADVANTAGES:- Better signals in the ground state.
- DISADVANTAGE:-
  1. Asymmetric Correlation Function, Non-Hermitian GEVP Needed.
  2. False Plateau encounter, careful with fitting time window.
- Why not Wall source wall sink correlator? -> Very Noisy signal.

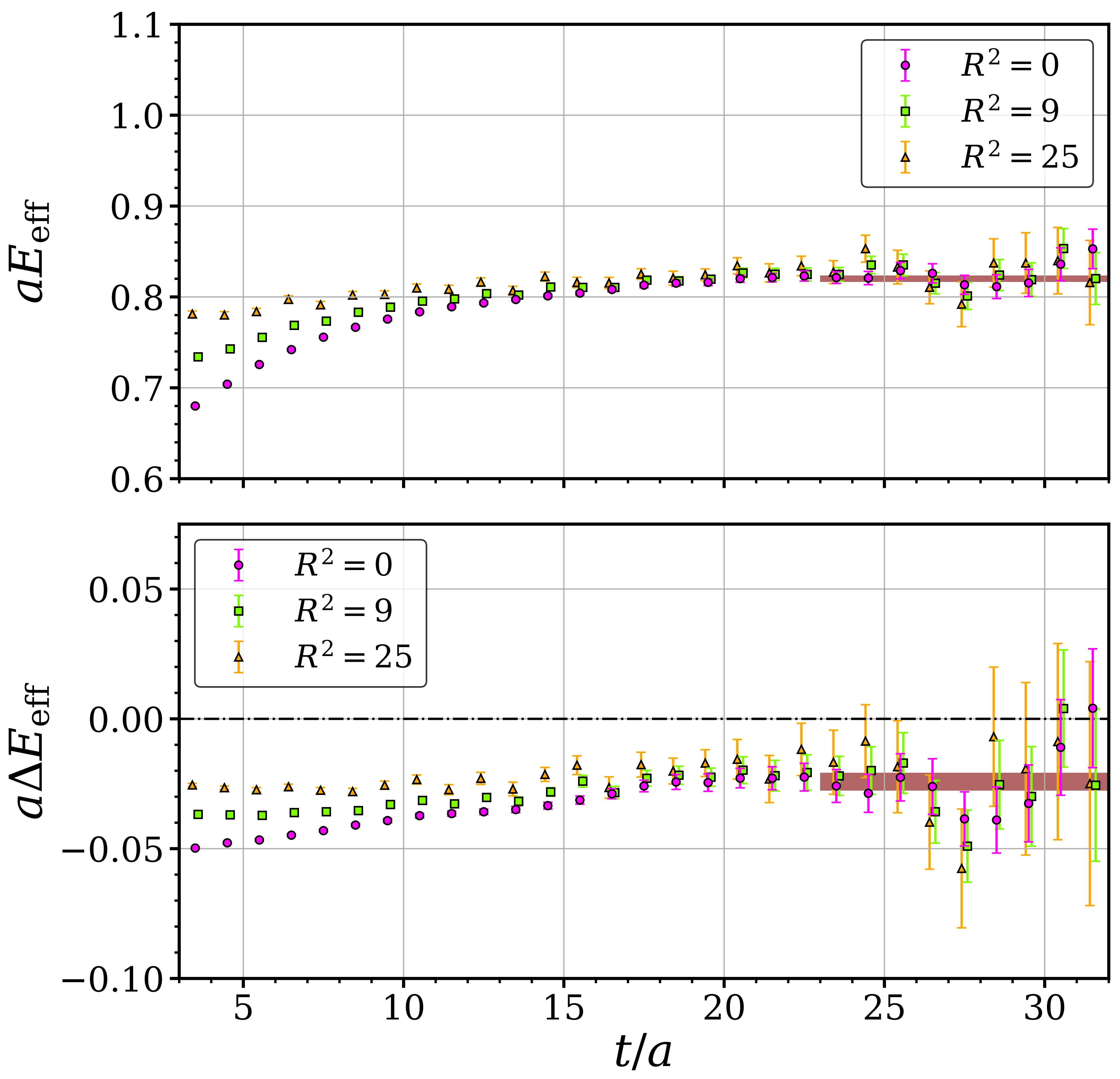
Wall source- Box Sink Correlator

- Instead of wall-sink, we build box-sink correlator.

Phys. Rev. D **102**, 114506

$$Q(\bar{x}, t; t') = \sum_{|\bar{y}-\bar{x}|<R} Q(\bar{y}, t; , t')$$

- As we increase box radius R, it approaches to symmetric correlator.
- Used to make comparative study of the asymptotic signals.
- Validates our energy plateau identification.



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Finite Volume Spectrum cont.

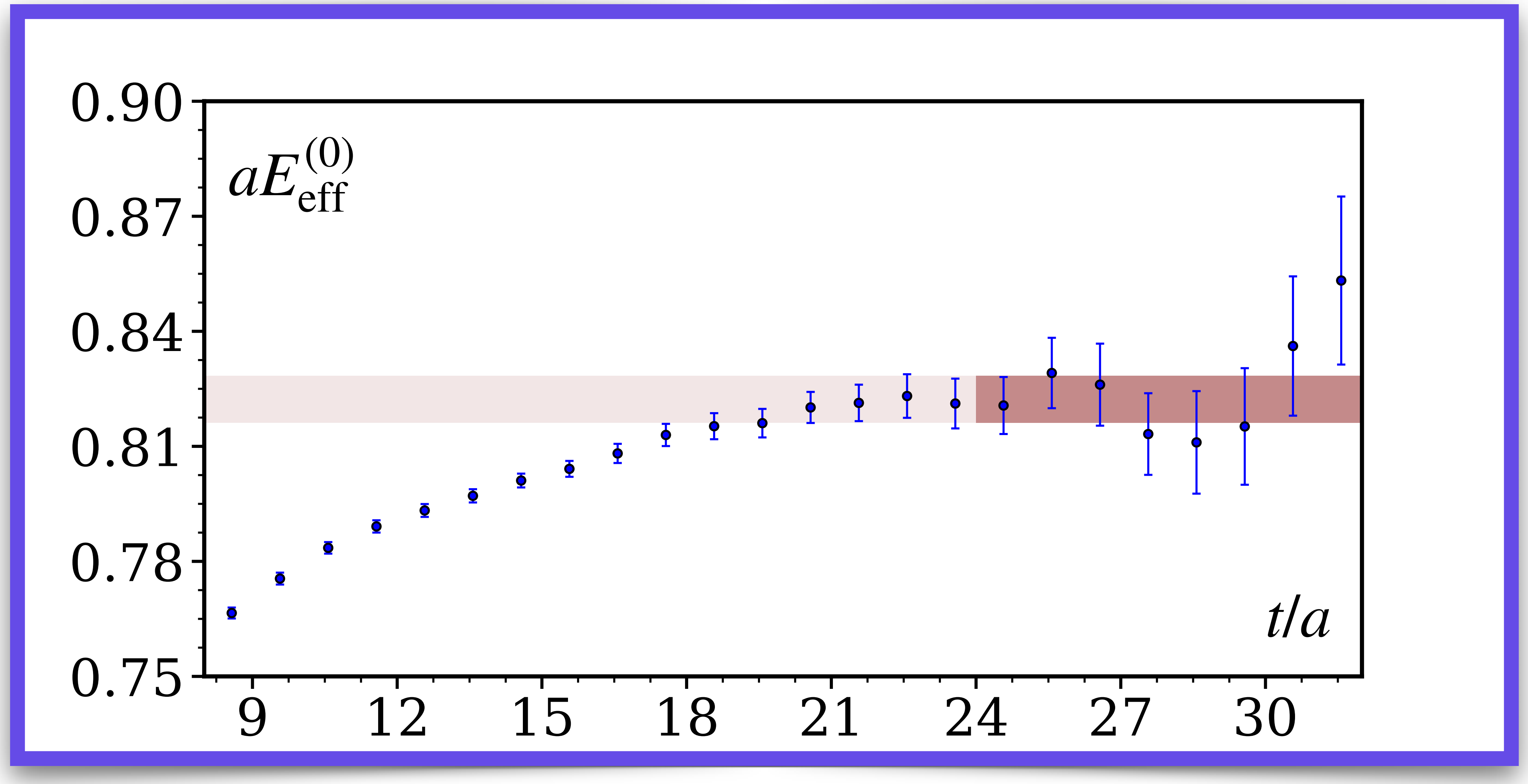
- We use GEVP to extract finite volume spectrum from correlation-matrix variationaly.

$$\mathscr{C}(t)v^{(n)}(t) = \lambda^{(n)}(t, t_0)\mathscr{C}(t_0)v^{(n)}(t)$$

- Fitting the leading exponential of $\lambda^n(t)$, yields the energy Eigen states E_n.

$$\lambda^n(t, t_0) = |A_n|^2 e^{-E_n(t-t_0)}[1 + \mathscr{O}(e^{-\Delta_n(t-t_0)})]$$

- Excited states can be determined with this method.
- Repeated for non-interacting mesons.



Non-Hermitian GEVP

- Modification of GEVP to account non-hermiticity.
- Solving asymmetric correlation matrix with left and right Eigenvector with same Eigenvalue.

$$\mathscr{C}(t_d)v_r^{(n)}(t_d) = \tilde{\lambda}^{(n)}(t_d)\mathscr{C}(t_0)v_r^{(n)}(t_d)$$

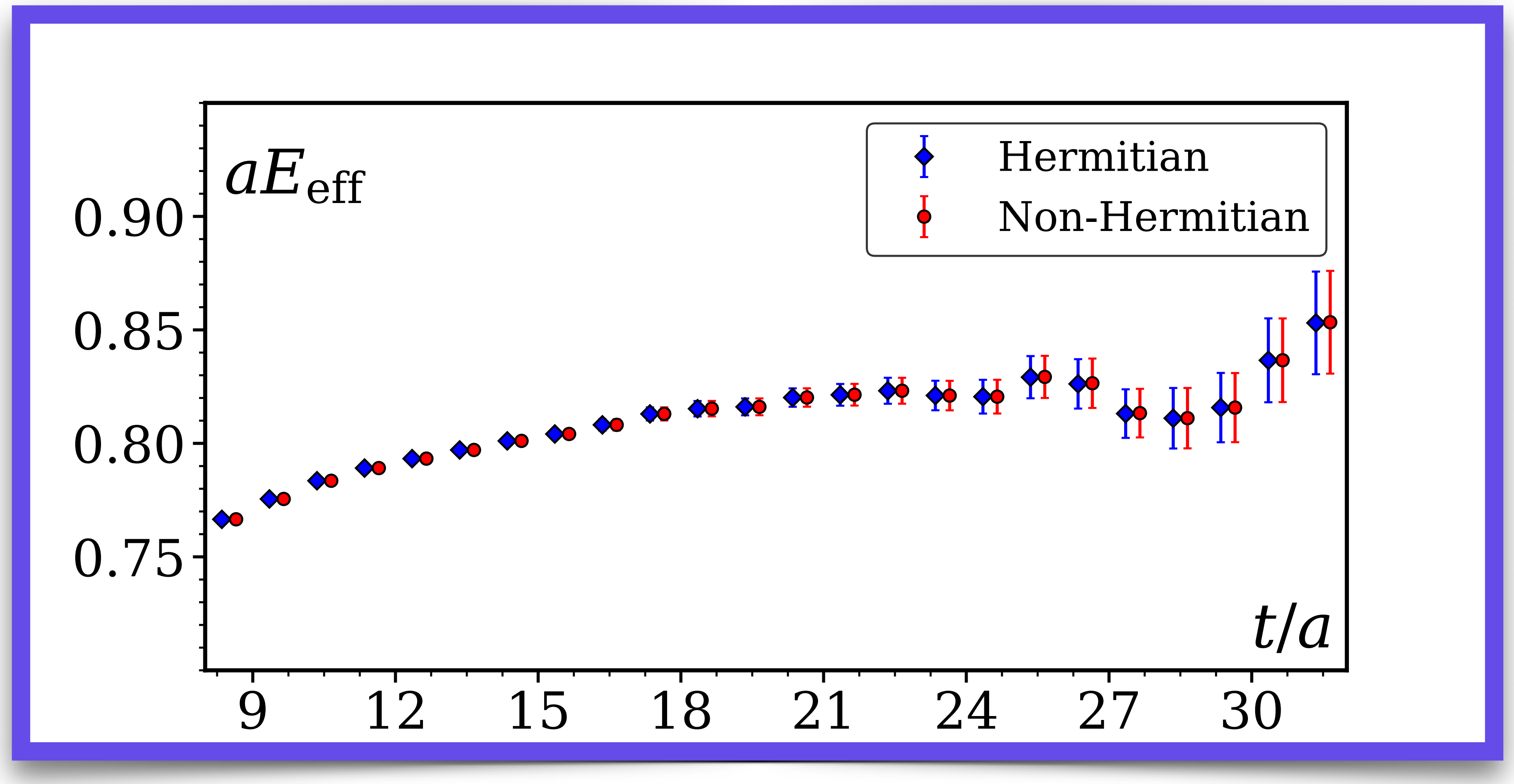
$$v_l^{(n)\dagger}(t_d)\mathscr{C}(t_d) = \tilde{\lambda}^{(n)}(t_d)v_l^{(n)\dagger}(t_d)\mathscr{C}(t_0)$$

t_d -> diagonalization time slice

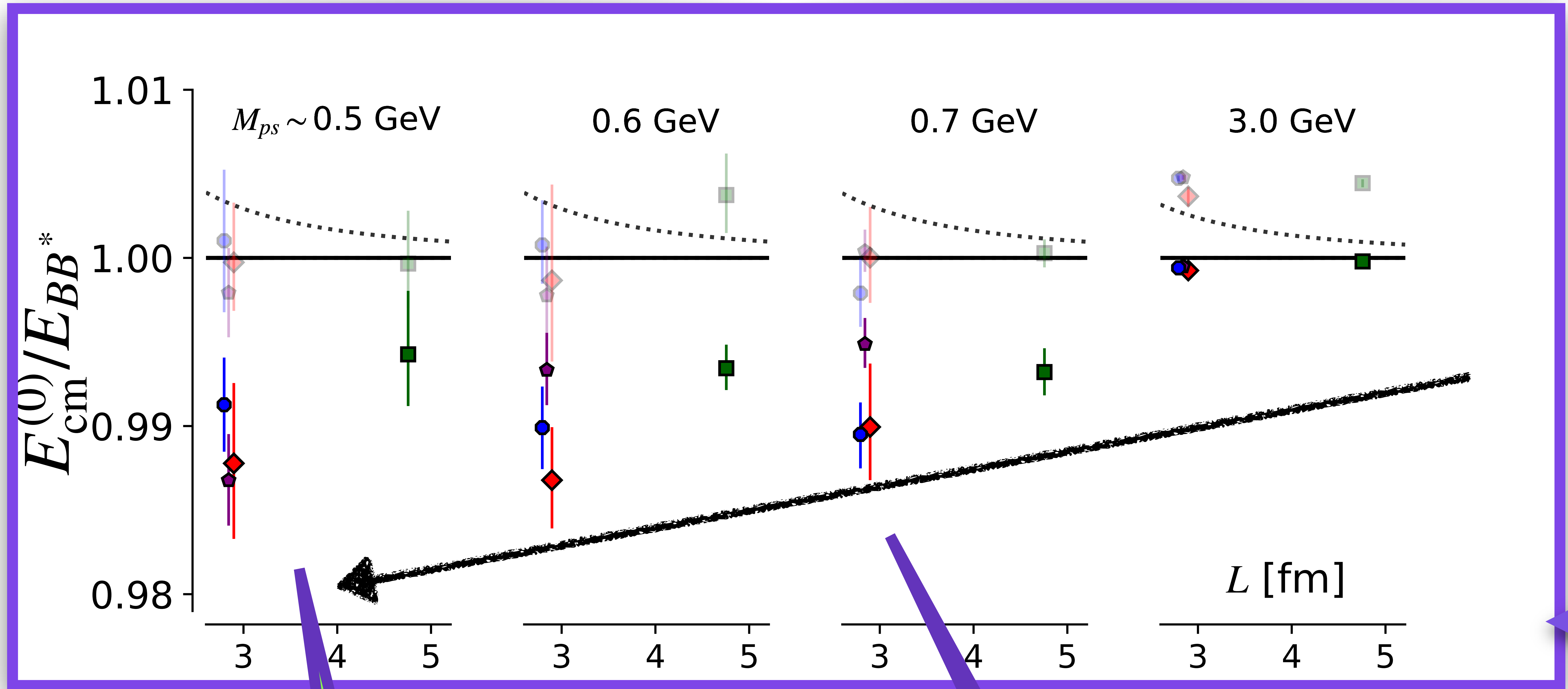
- Solve it for other time slices with $v_l(t_d)$ and $v_r(t_d)$ using

$$\tilde{\lambda}^{(n)}(t) = v_l^{(n)\dagger}(t_d)\mathscr{C}(t)v_r^{(n)}(t_d)$$

- Found $\dfrac{Im(\tilde{\lambda}^{(n)}(t))}{|(\tilde{\lambda}^{(n)}(t))|} < 0.01$ for all correlators used.



Finite Volume Spectrum cont.



- Spectrum extraction repeated for every M_{ps} and every ensemble.

Spectrum were calculated in units of nearest Two body decay threshold $BB*$.

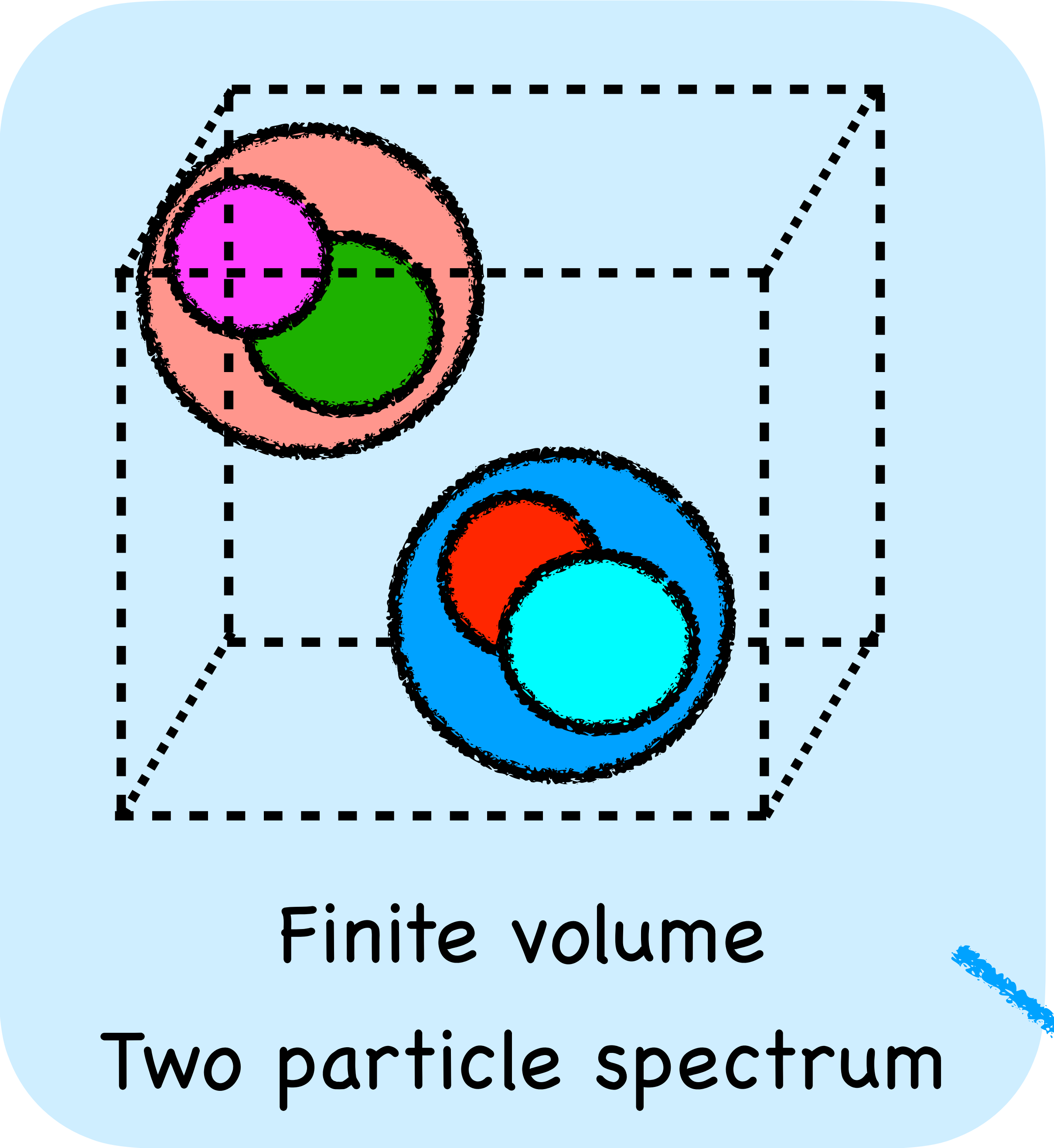
NRQCD Offset corrected

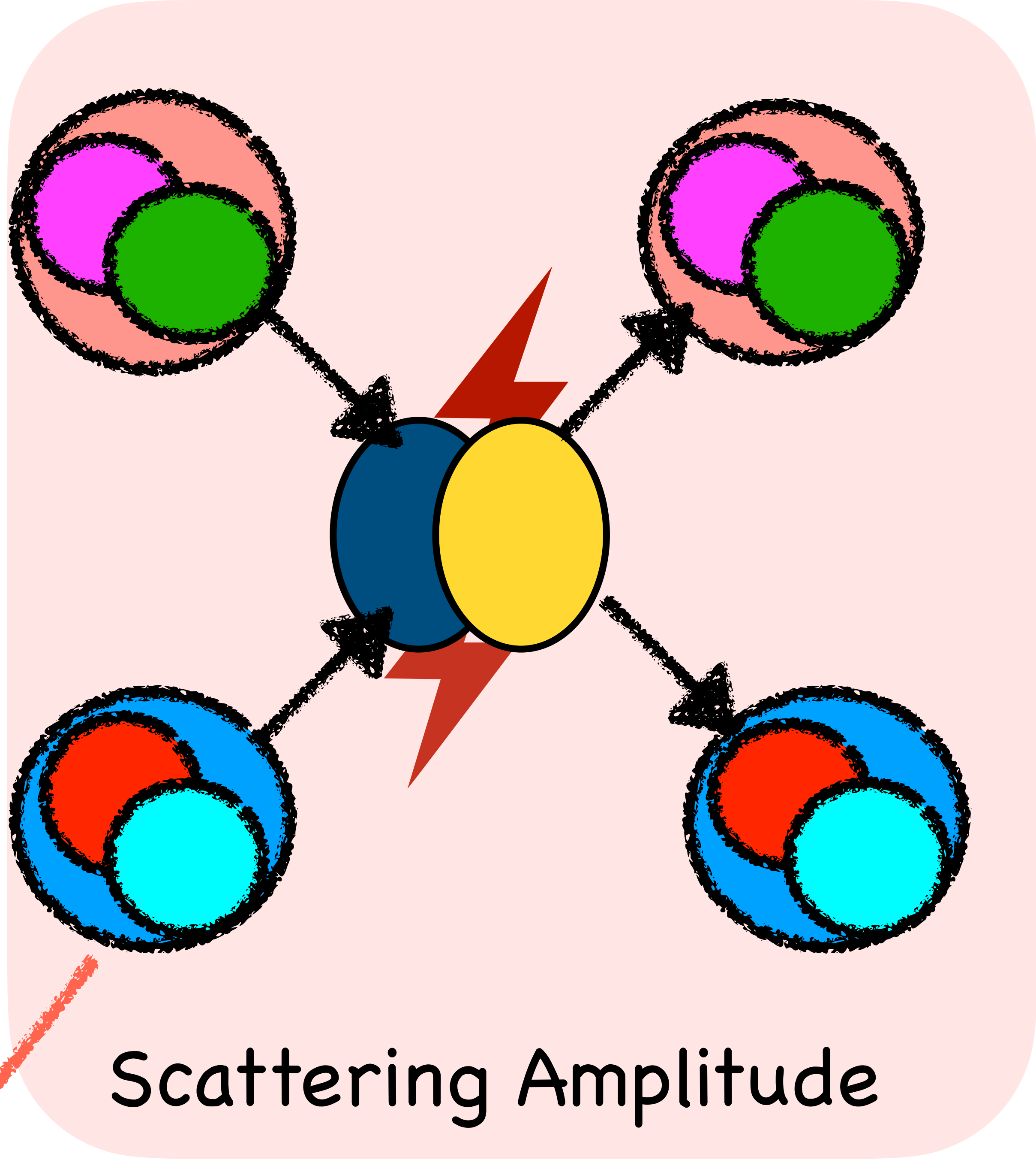
A decreasing trend can be observed

We need continuum extrapolation to have results in physical limit

Amplitude Analysis

Lüscher based relation(1991)





$$\boxed{det\left[1 + i\mathscr{G}(E)\ \mathscr{M}(E)\right] = 0}$$

Amplitude Analysis

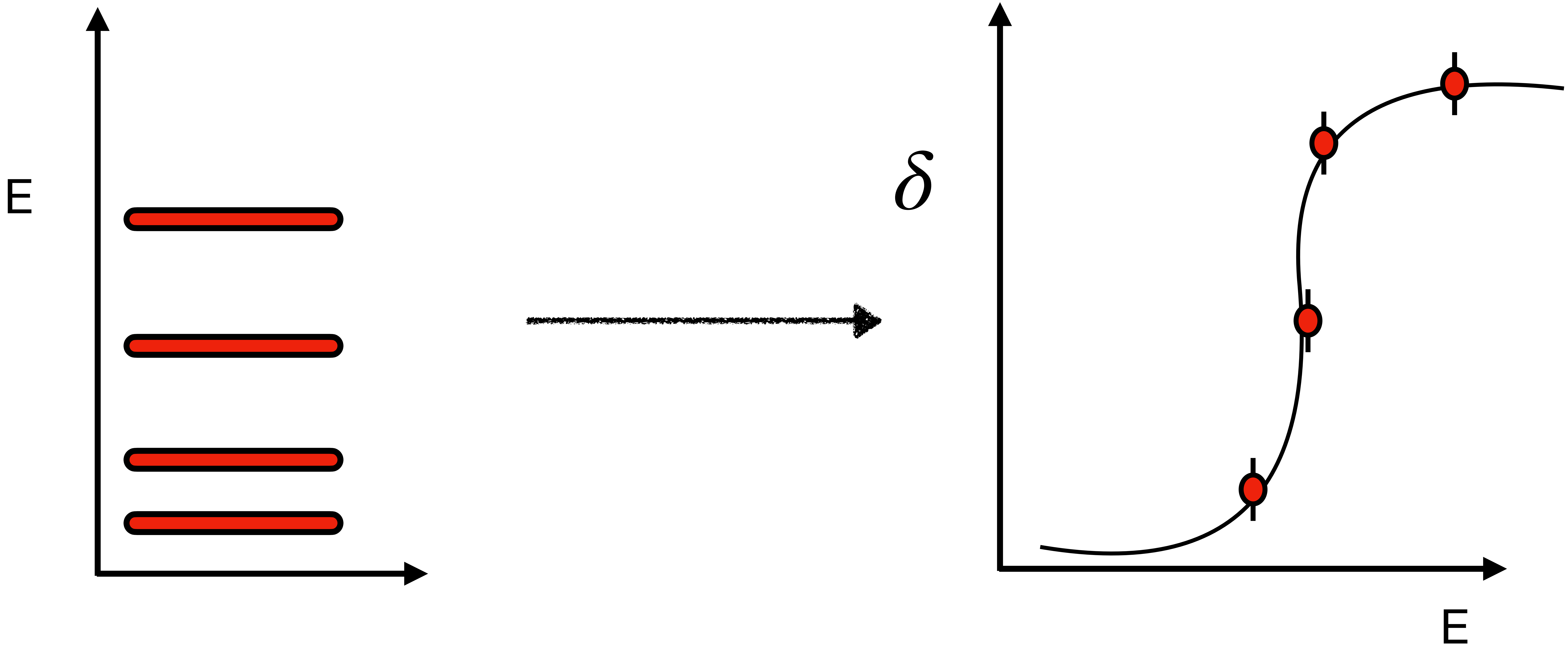
- Considering only s-wave, the simplified quantisation condition becomes,

$$p \cot \delta_0(p) = \frac{2}{\sqrt{\pi} L} \mathcal{Z}_{00}(1; q^2)$$

Here $q = \dfrac{L}{2\pi} p$ and $4sp^2 = (s - (M_1 + M_2)^2)(s - (M_1 - M_2)^2)$, $s = E_{cm}^2$

- $\mathcal{Z}$ is known as Lüscher zeta function -> known mathematical function
- Following the quantisation conditions energy dependence of amplitude is being extracted.
- Search for poles of amplitude near threshold representing bound states.

Amplitude Analysis



- Energy is discrete.
- Parametrisation of phase shift is required.

Amplitude Analysis

- Considering only s-wave, the simplified relation becomes,

$$p \cot \delta_0(p) = \frac{2}{\sqrt{\pi} L} \mathcal{Z}_{00}(1; q^2)$$

- We use zero range approximation for amplitude with lattice spacing a dependence.

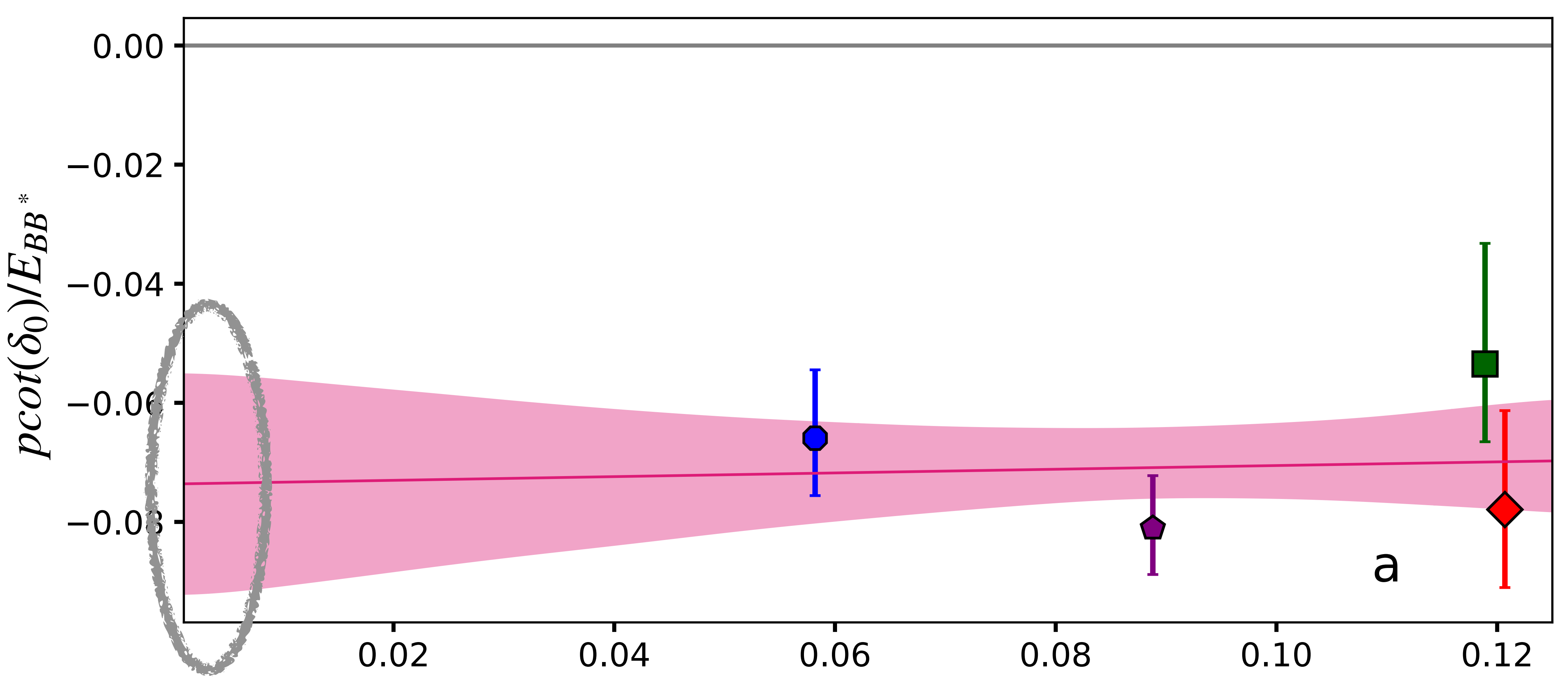
$$pcot\delta_0 = A^{[0]} + a \,.\, A^{[1]}, \qquad A^{[0]} = -\frac{1}{a_0}$$

- The parameters $A^{[0]}$ and $A^{[1]}$ is determined by minimising χ^2

$$\chi^2 = \left[E(L) - E^{sol}(L, A^{[0]}, A^{[1]})\right] C^{-1} \left[E(L) - E^{sol}(L, A^{[0]}, A^{[1]})\right]$$

Continuum Extrapolation

$M_{ps} = 0.5$ GeV



- Scattering Amplitude is given as

$$T \propto (pcot\ \delta - ip)^{-1}$$

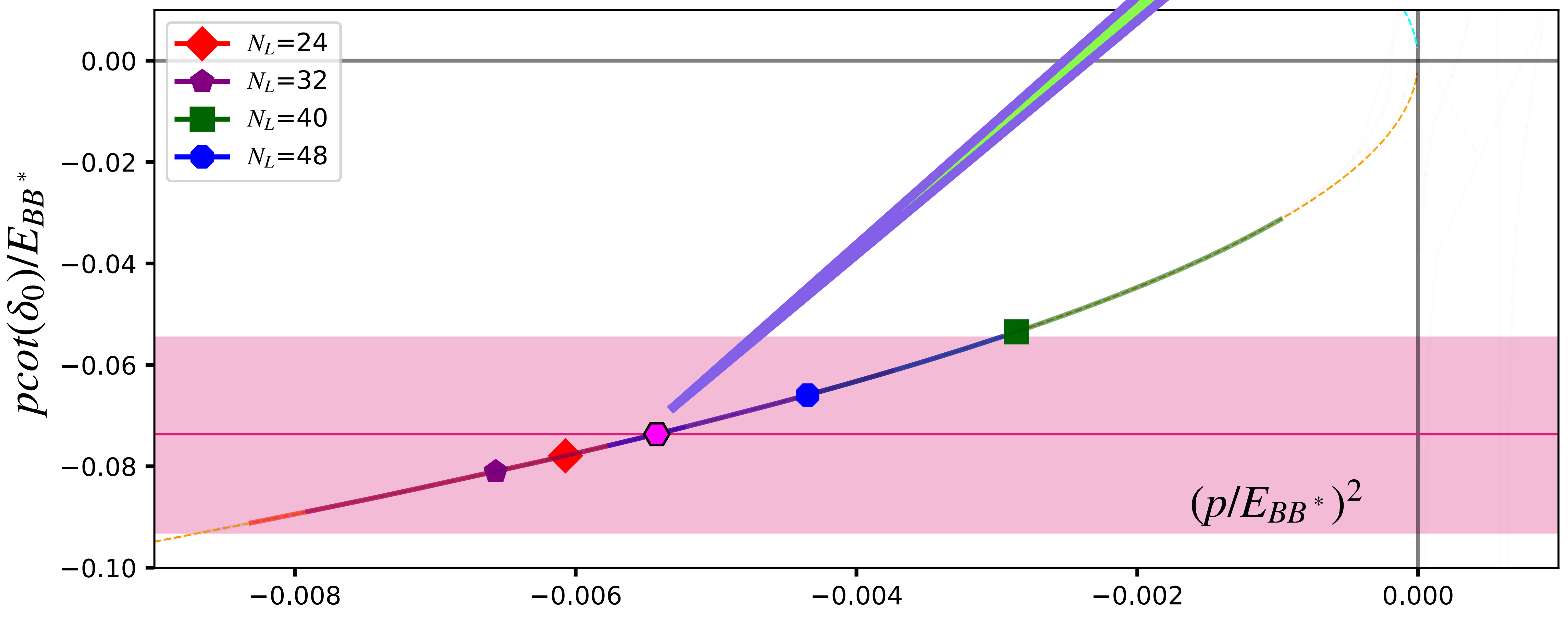
- Phase shift parametrised as

$$pcot\ \delta_0 = -\frac{1}{a_0} + A^{[1]} \cdot a$$

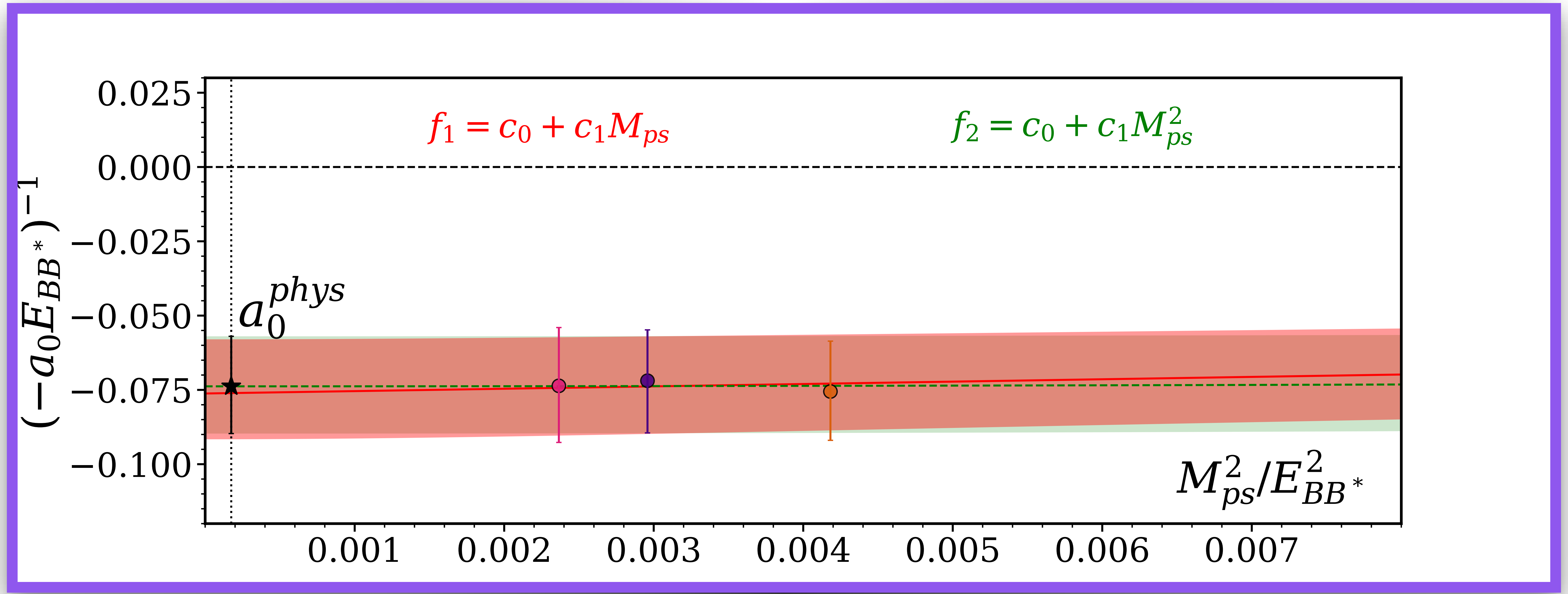
Real Bound State

- Same repeated for other M_{ps}(0.6, 0.7 GeV).

- Consistent Negative values for other M_{ps} as well as real bound state.



Chiral Extrapolation

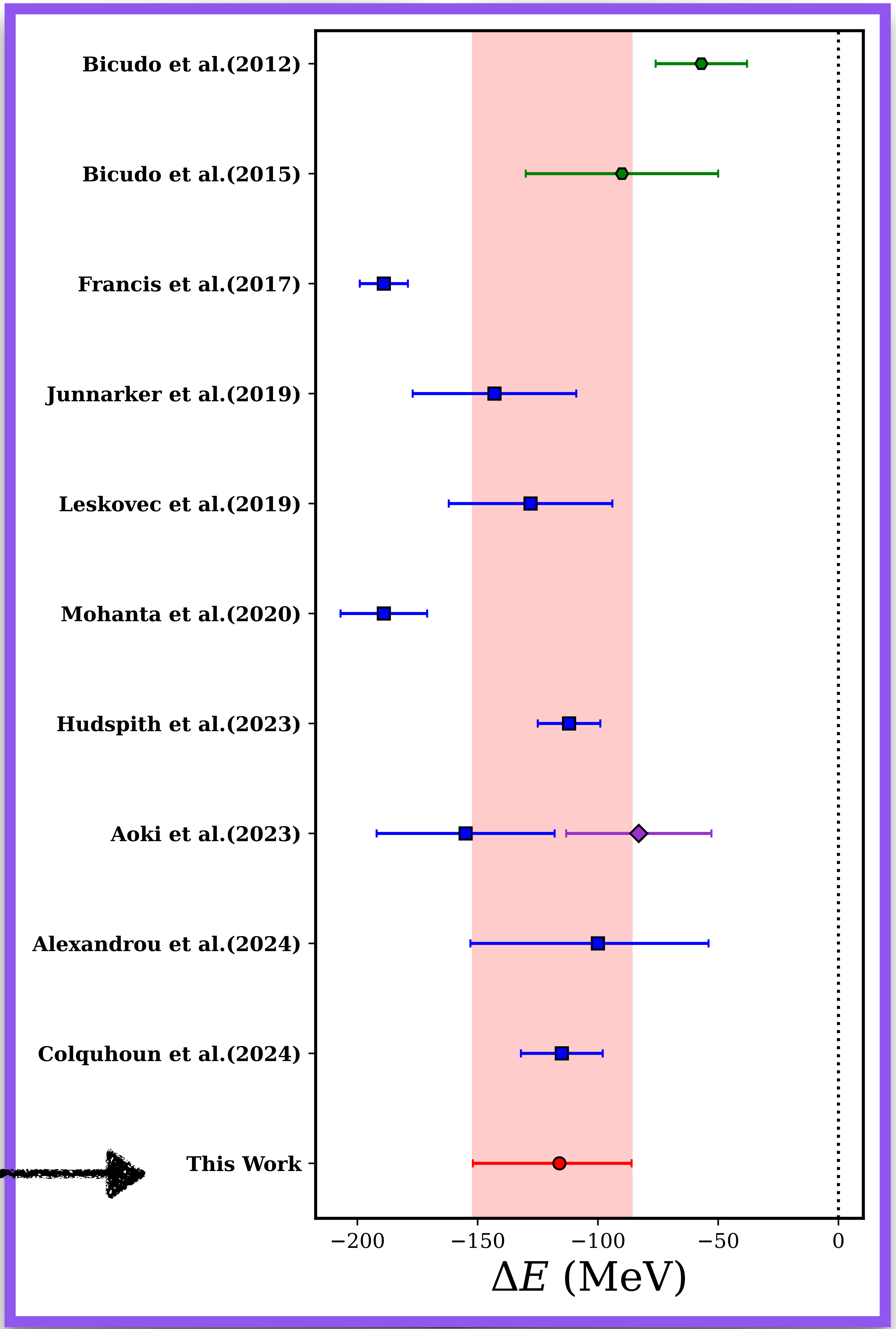


Result:

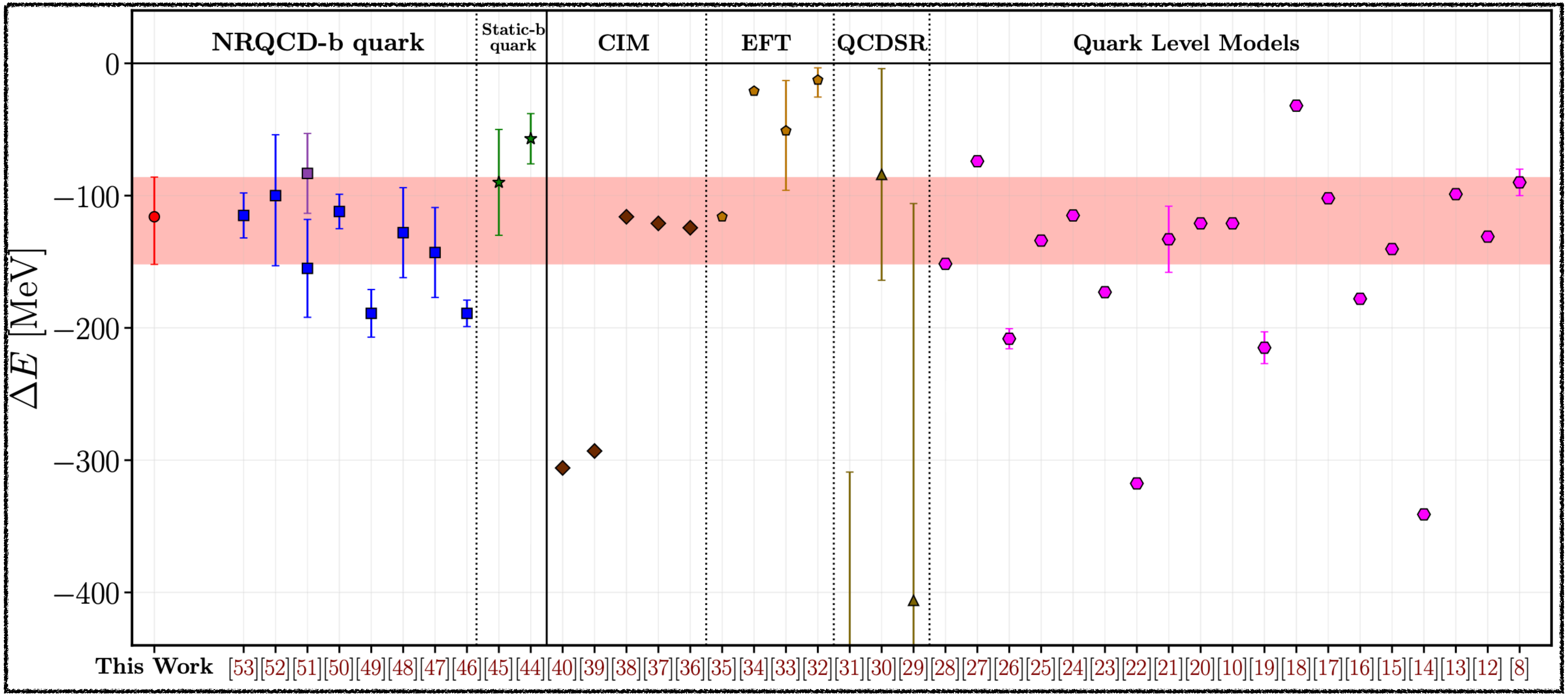
Scattering length at physical limit

$$a_0^{phy} = 0.25(^4_3)\ fm$$

Corresponds to binding energy $\Delta E = -116(^{+30}_{-36})$ **MeV.**



Summary T_{bb}



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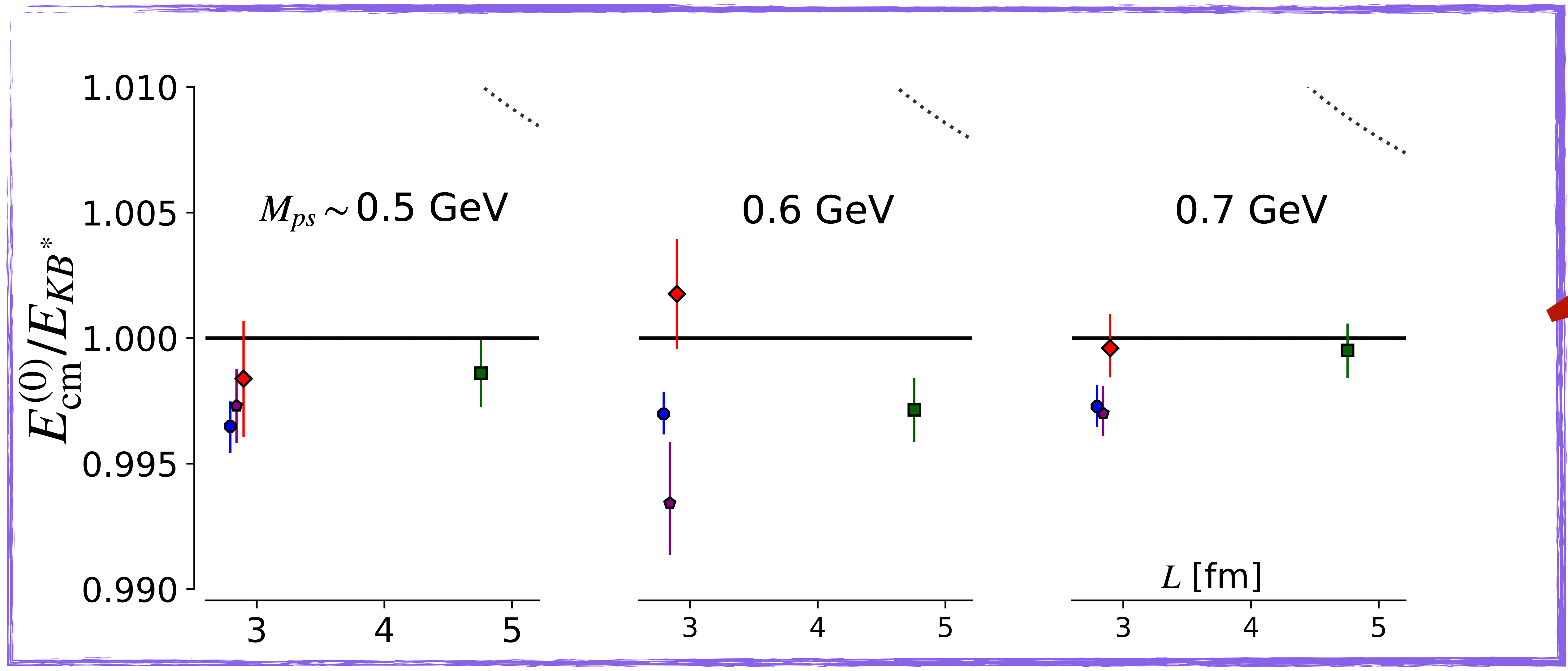
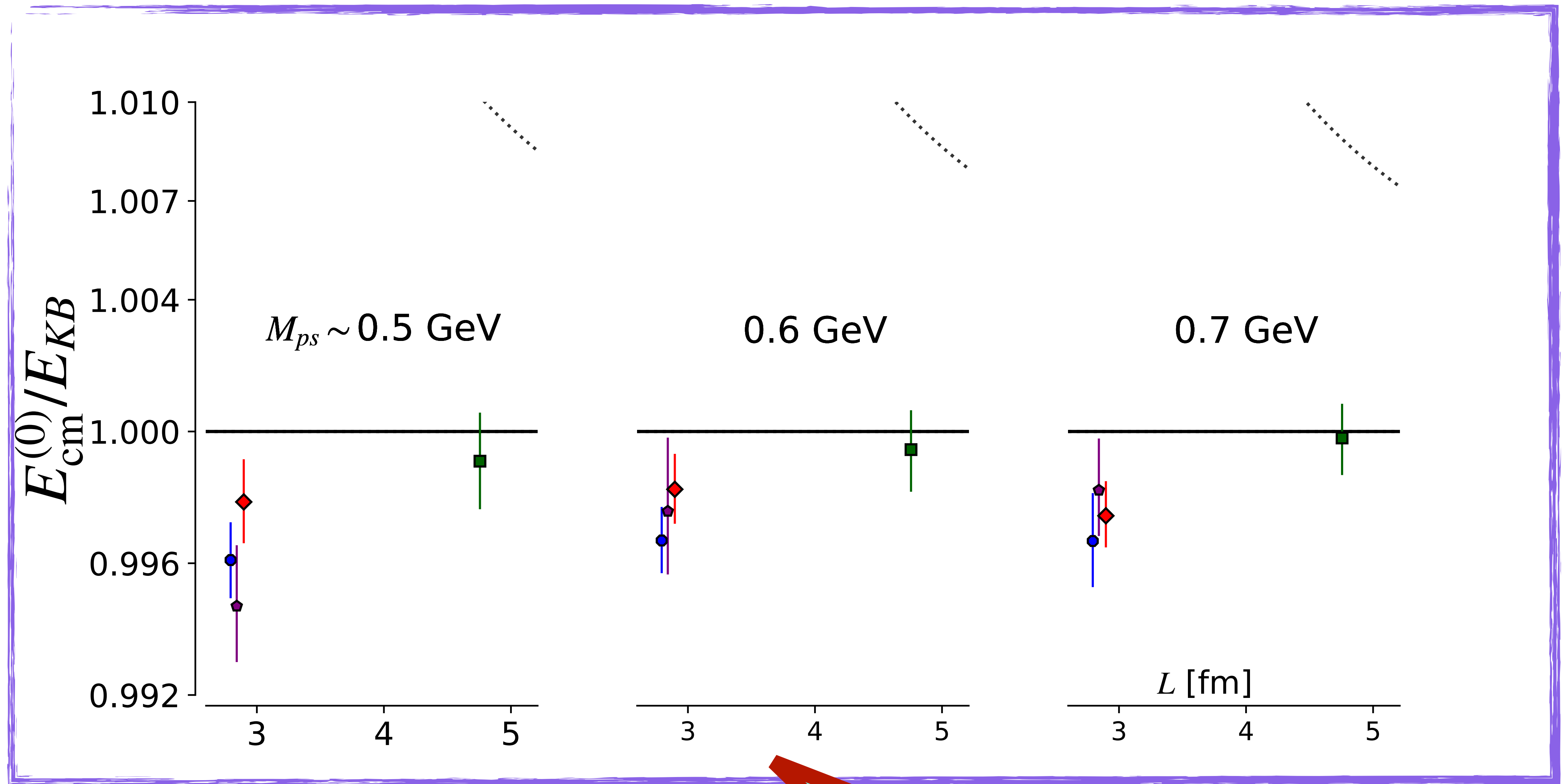
Results of T_{bs}



Axial vector T_{bs}

- Results are consistent with threshold in both the cases for larger volume.

- Need low M_{ps} datas for better results.



Scalar T_{bs}

Results of T_{bs}

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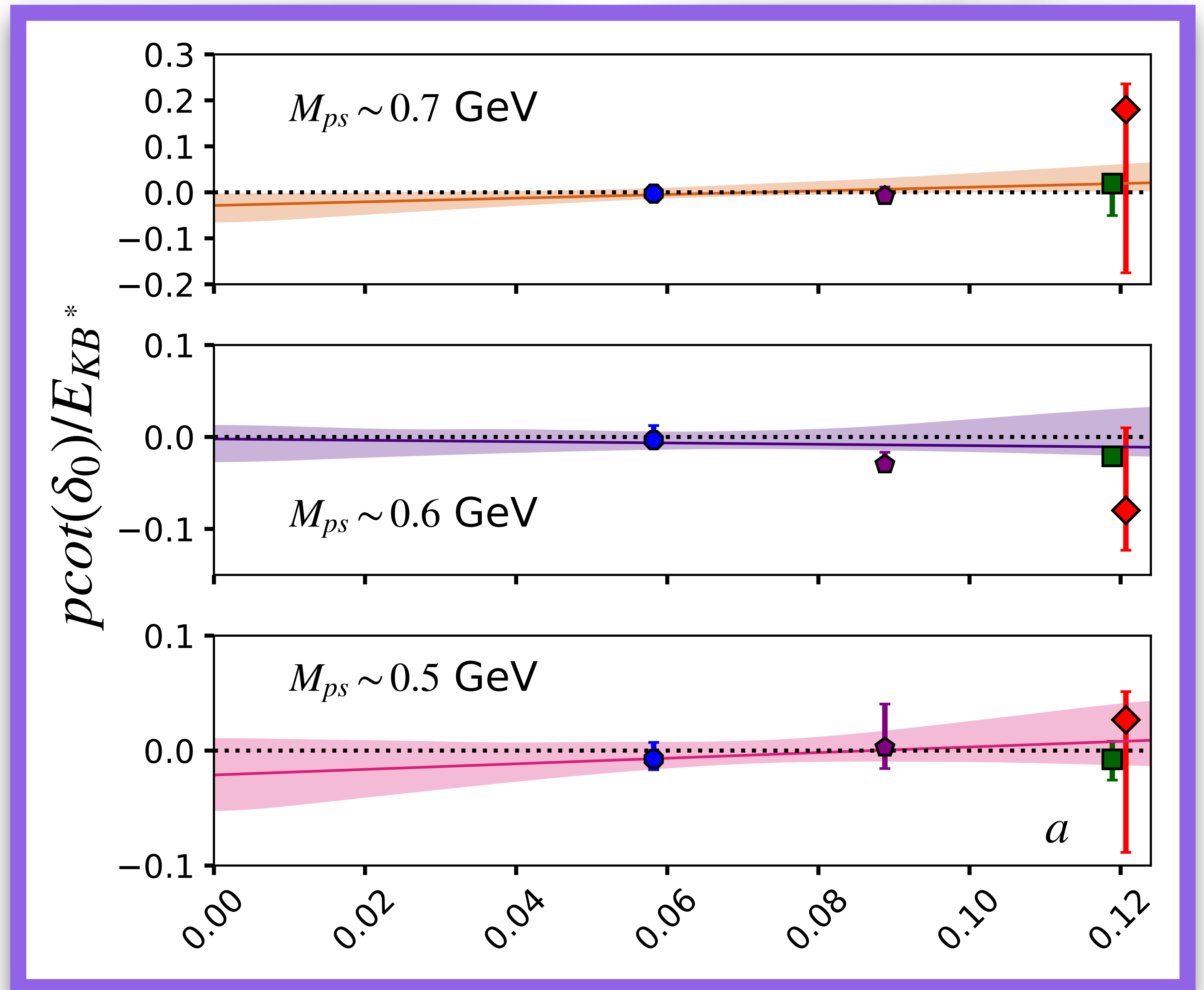
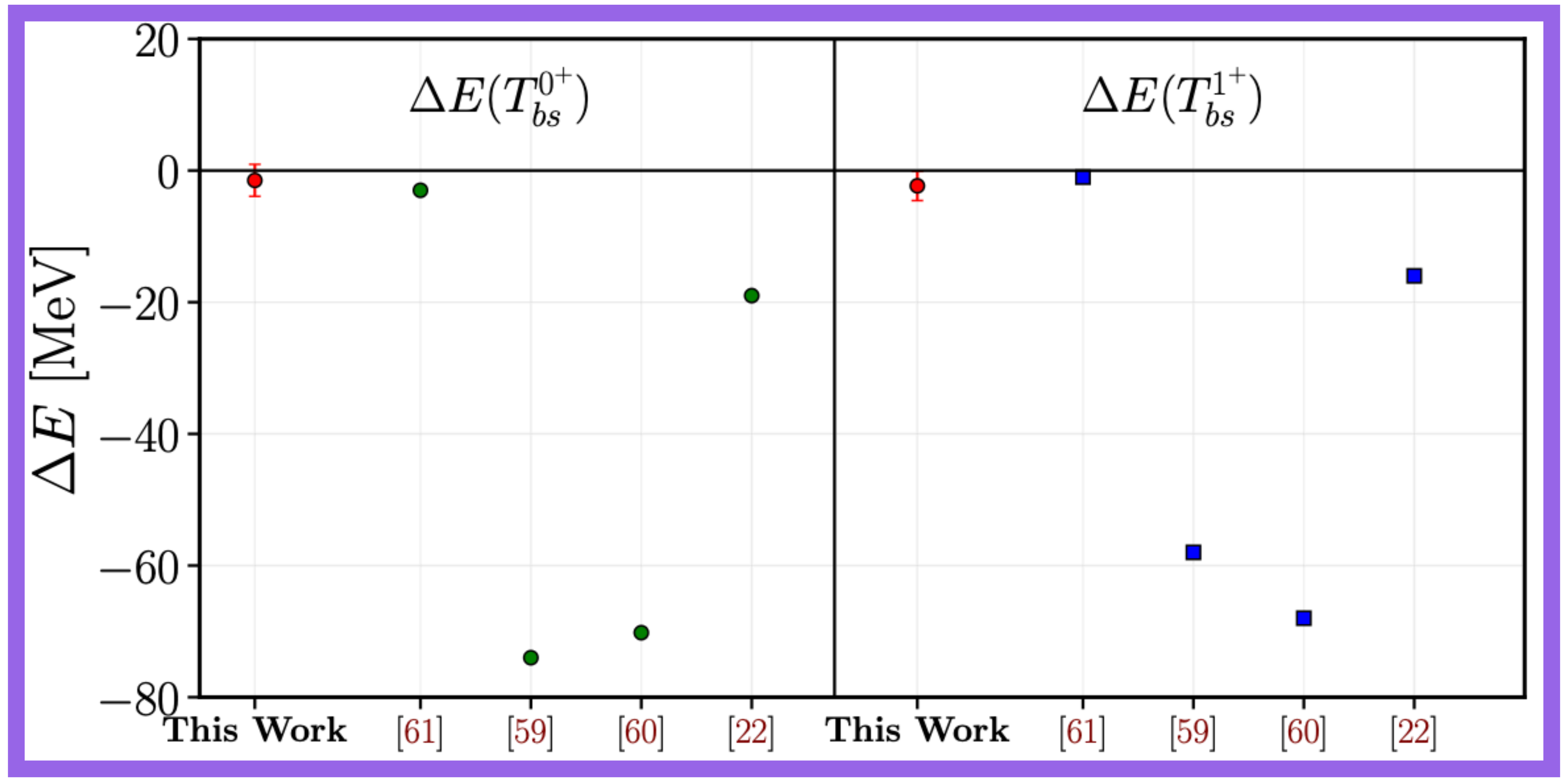
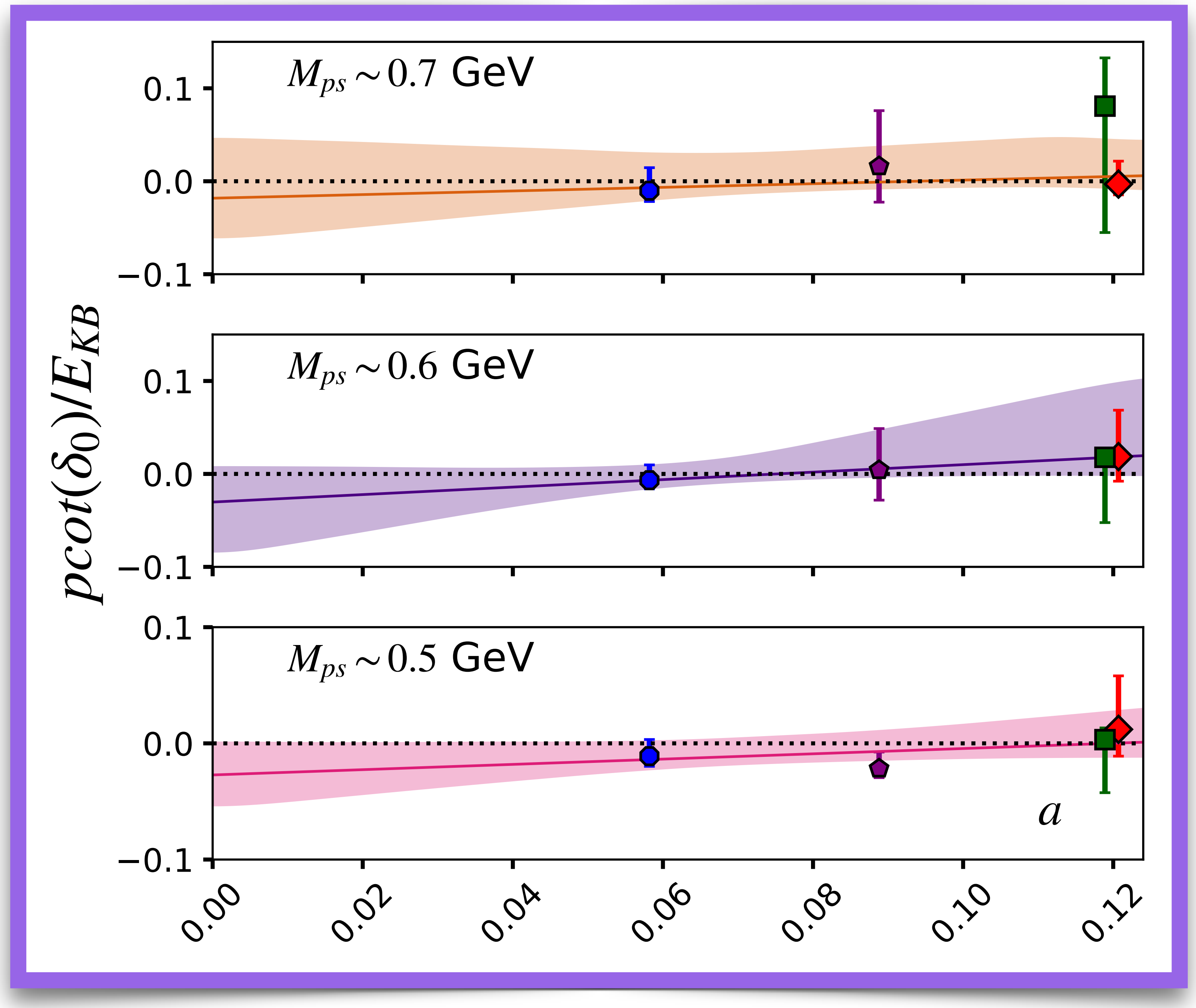


Axial vector T_{bs}

Summary T_{bs}





Scalar T_{bs}

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Summary and Outlook

- We worked with isoscalar axial vector T_{bb} and both scalar and axial-vector T_{bs}.
- Various work widely predicted deep binding in isoscalar axial-vector T_{bb}.
- Rigorous spectrum analysis were done for T_{bb} and T_{bs} tetraquark.
- We worked with multiple lattice spacing, two volumes to control systematics.
- Finite volume spectrum indicates negative energy shift with respect to BB^* threshold.
- Found a possible deeply bound state for T_{bb} not such exciting results in T_{bs}.

Slides will be available at my website https://www.imsc.res.in/~bhabanist/

Vielen Dank für Ihre Aufmerksamkeit