$bb\bar{u}\bar{d}$ and $bs\bar{u}\bar{d}$ tetraquarks from Lattice QCD using two-meson and diquark-antidiquark variational basis

Bhabani Sankar Tripathy

The Institute of Mathematical Sciences, Chennai
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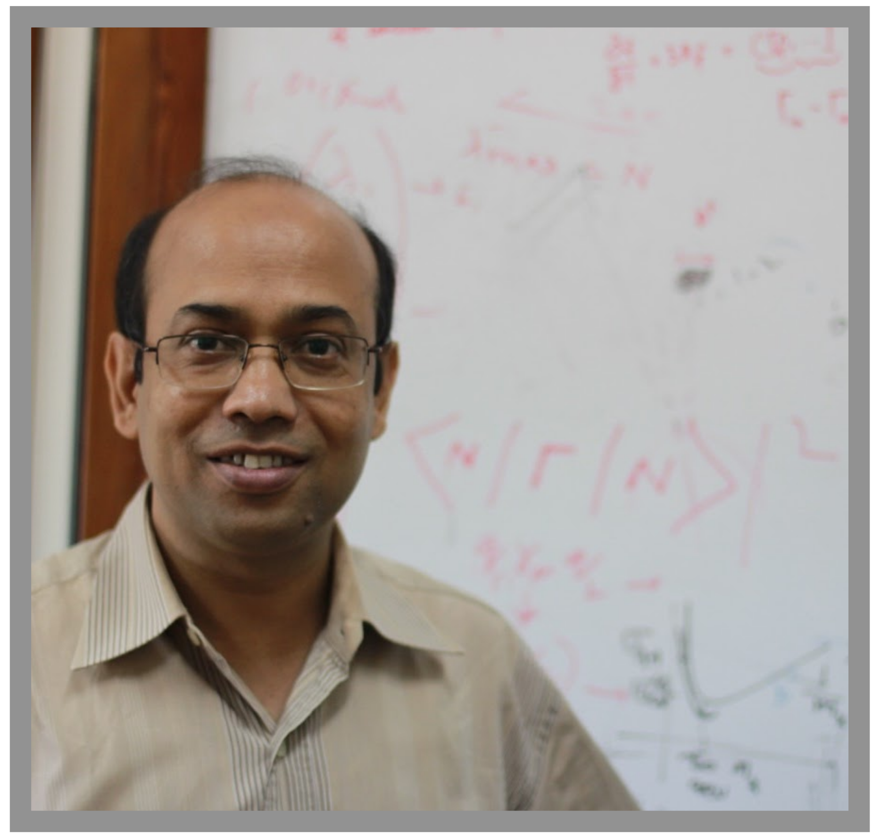
16 May 2025

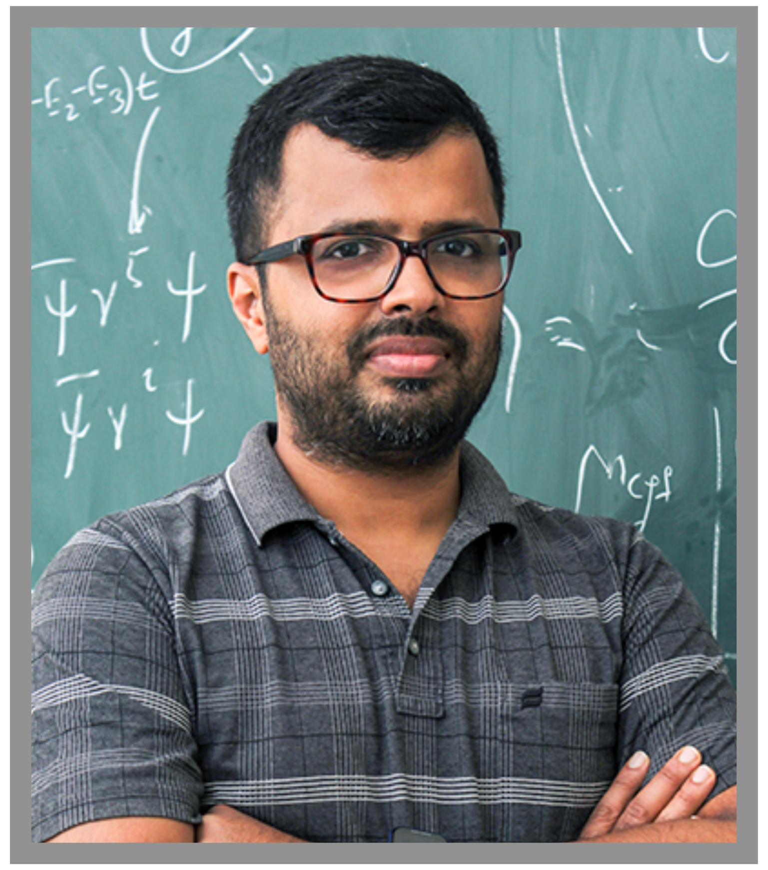


Nilmani Mathur(TIFR)

TIFR HEP JC TALK

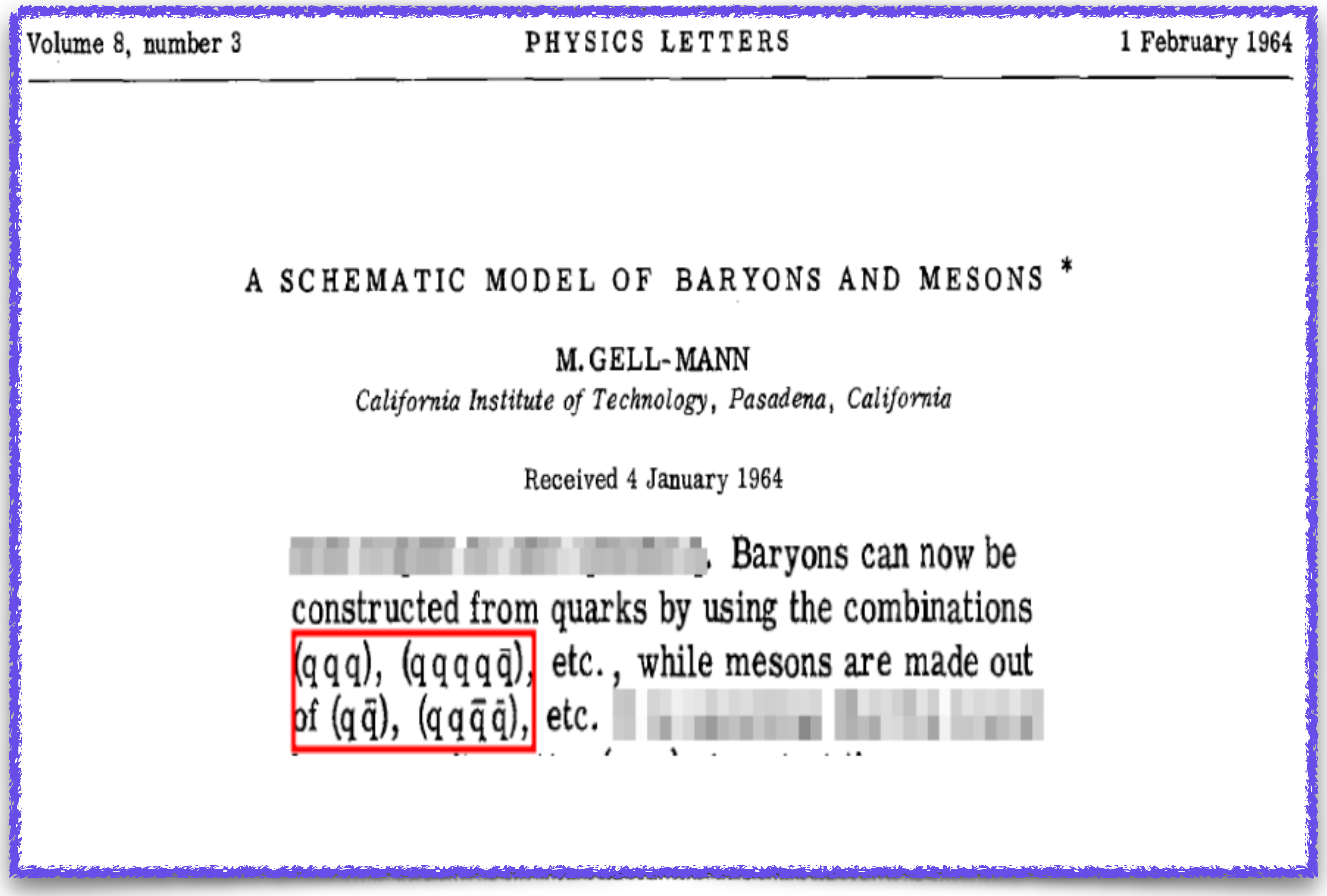
Based on arXiv:2503.09760

Accepted in PRD



M. Padmanath(IMSc)

Introduction and Motivation



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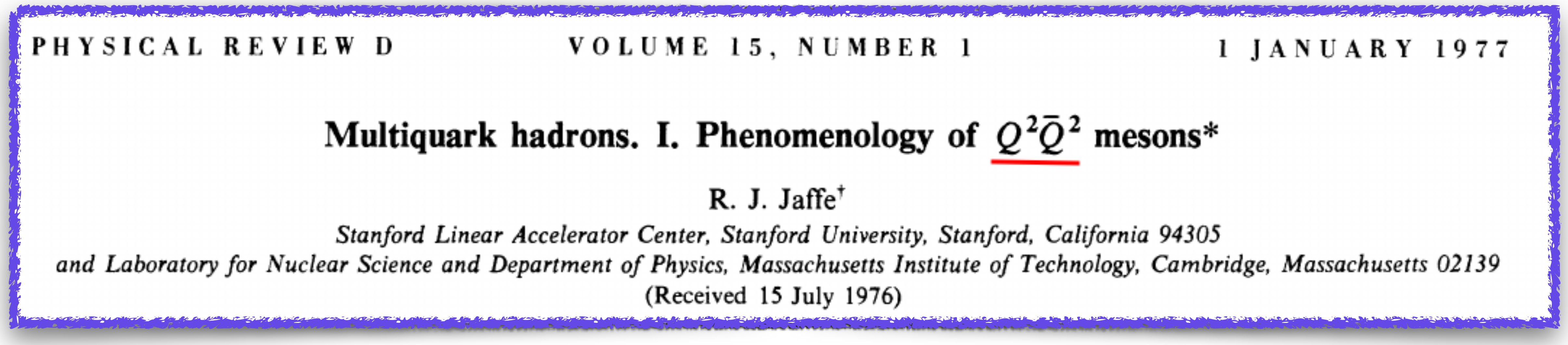

PHYSICAL REVIEW D VOLUME 15, NUMBER 1 1 JANUARY 1977

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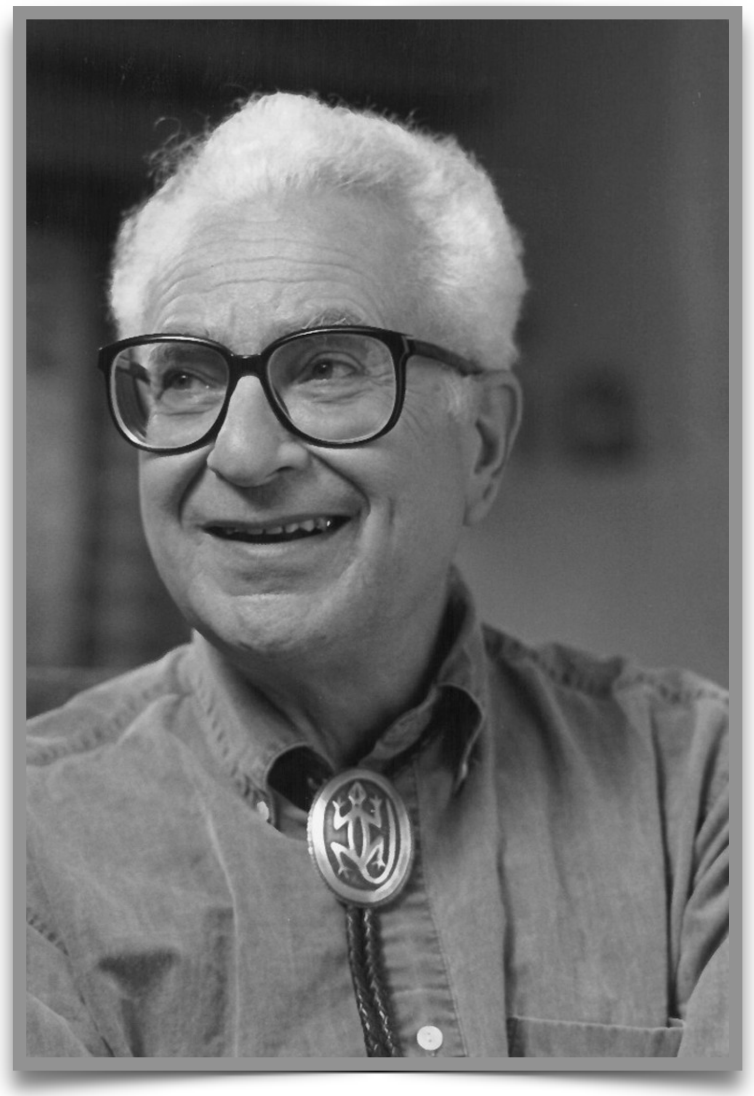

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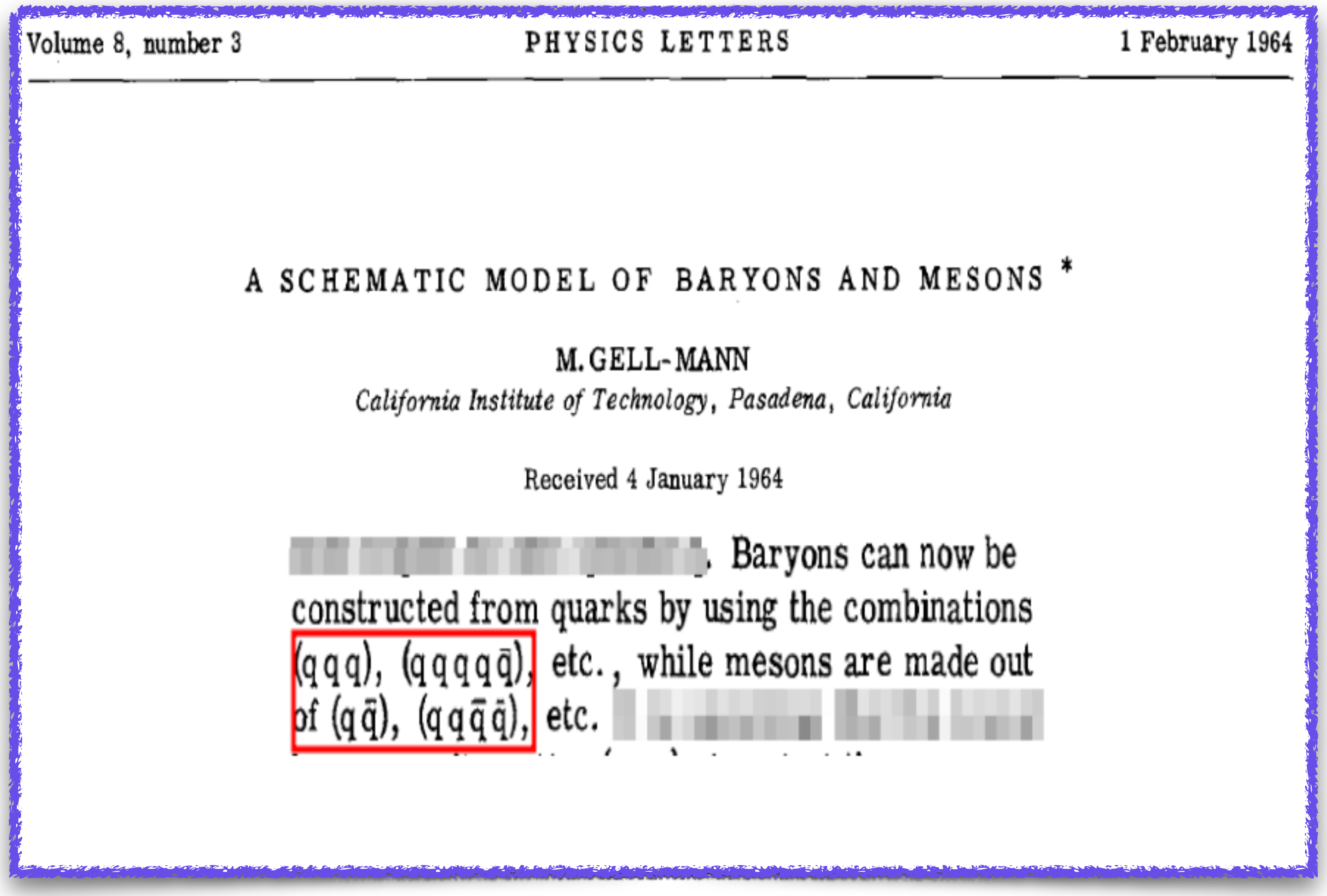
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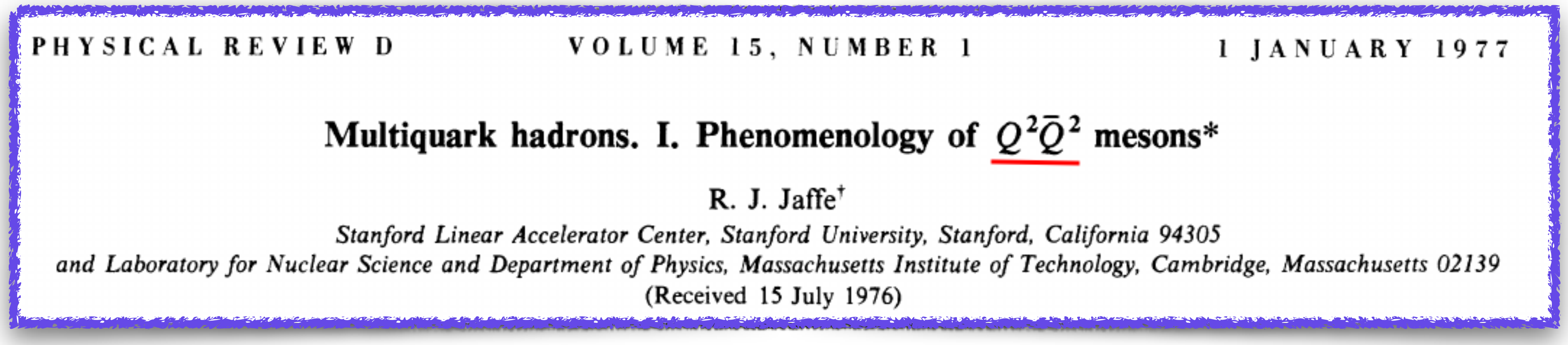
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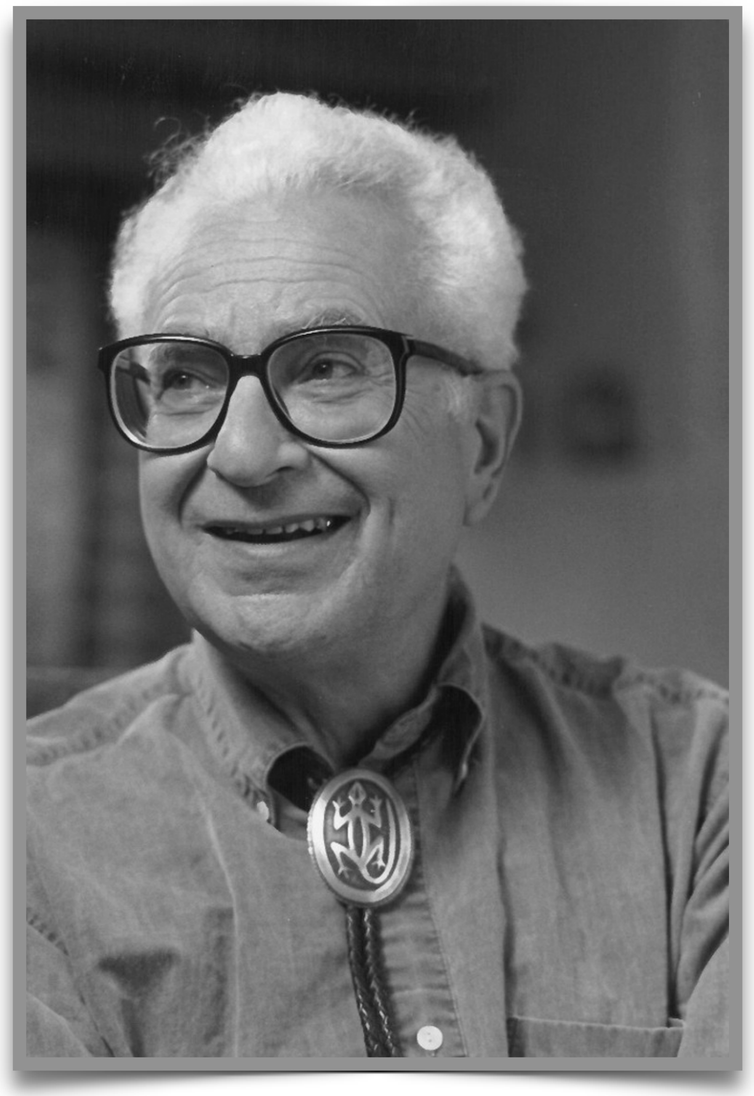

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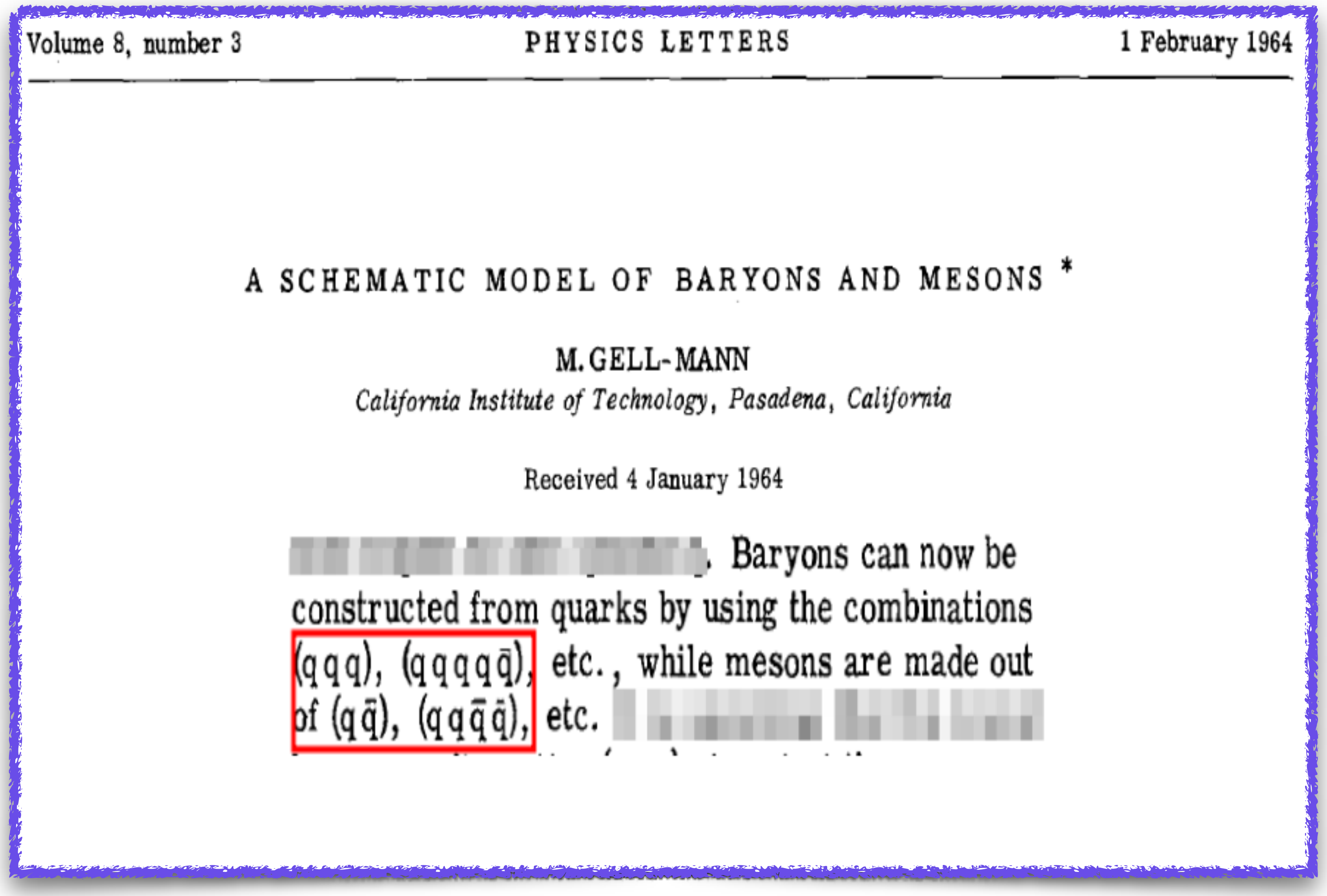
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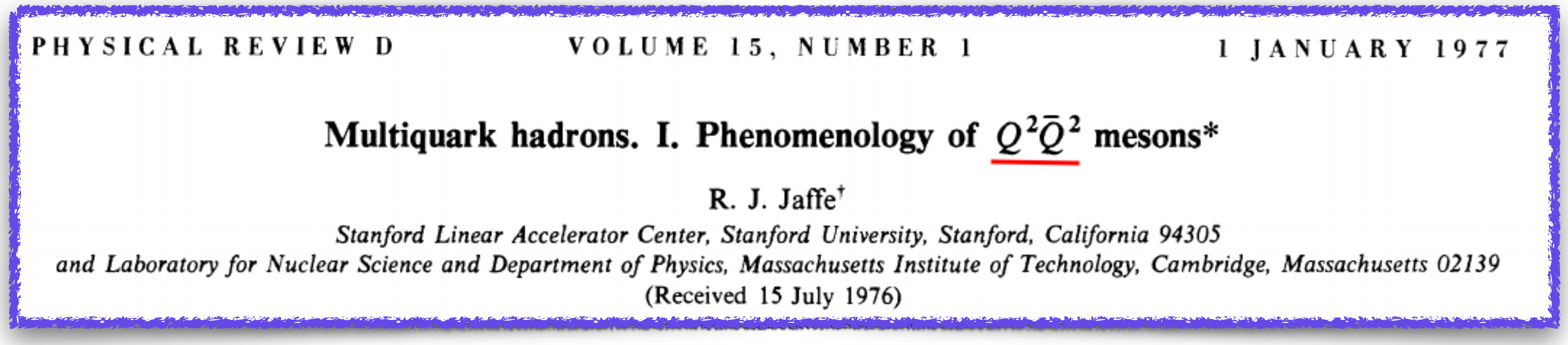
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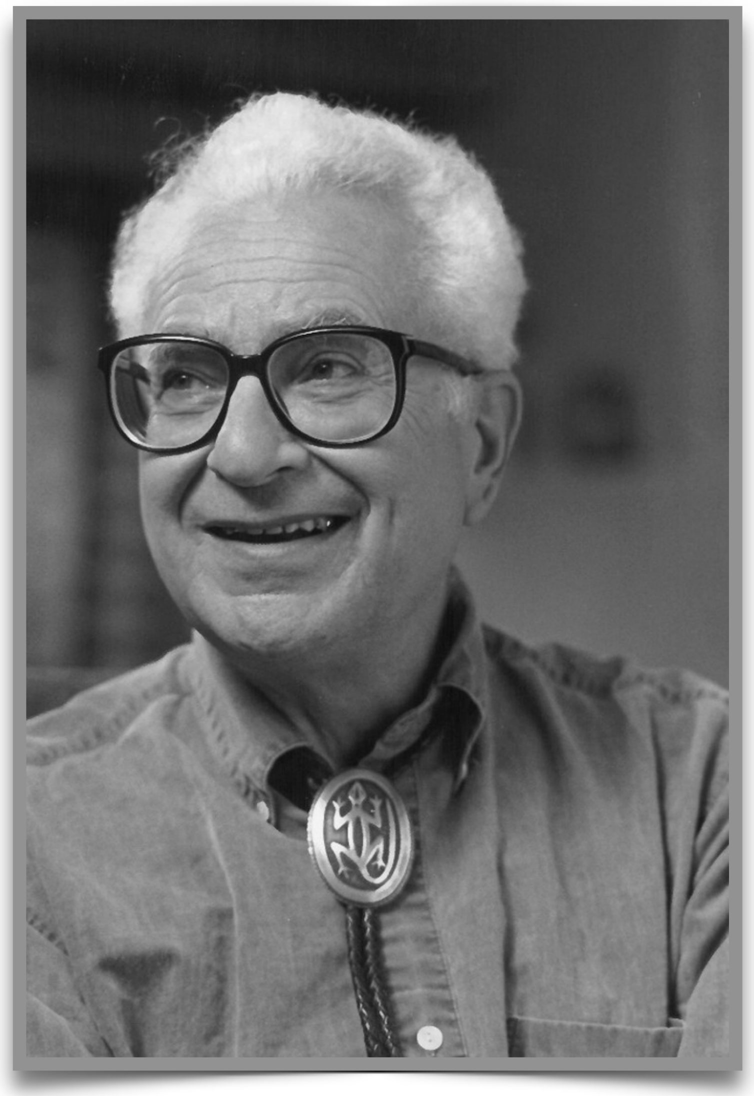

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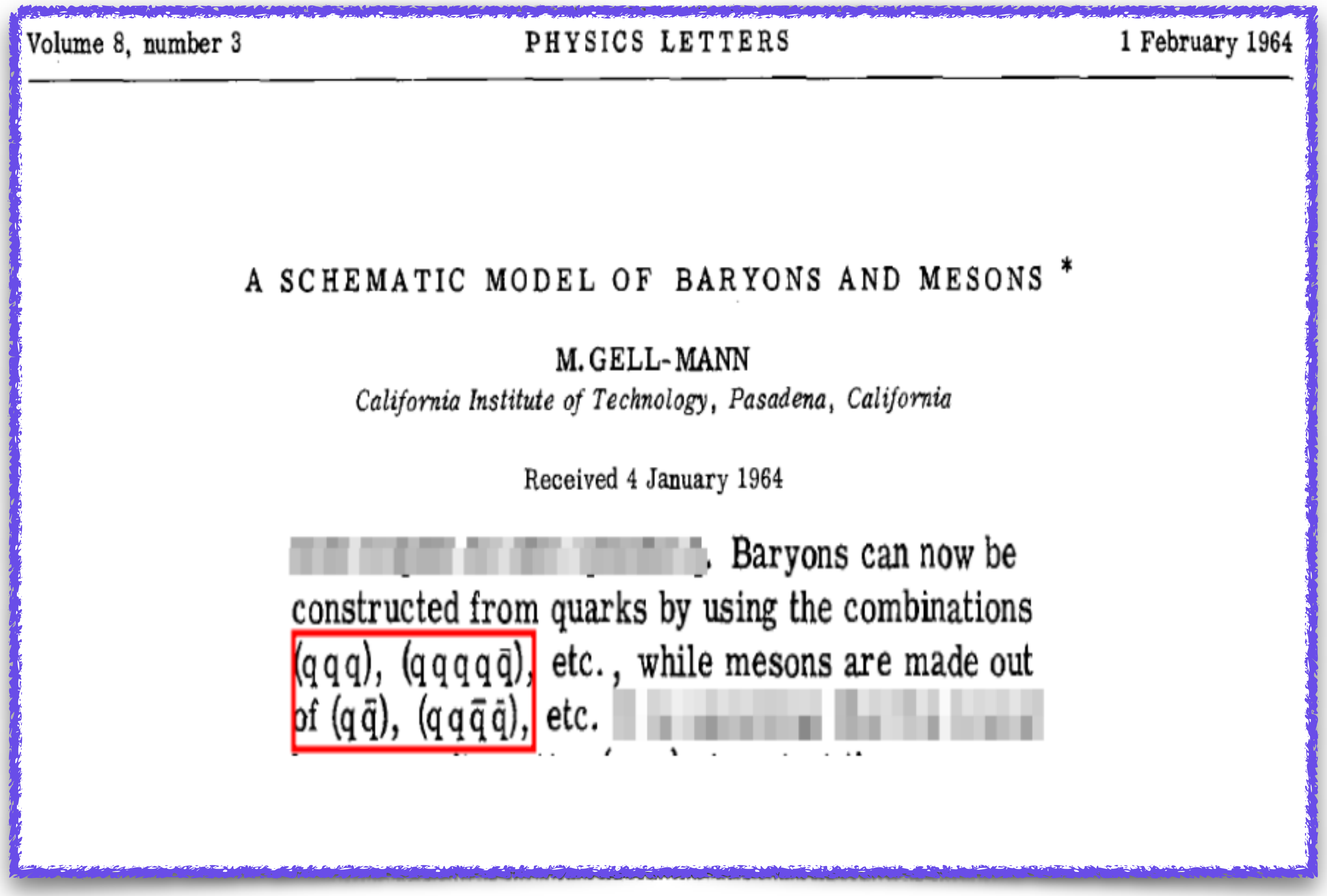

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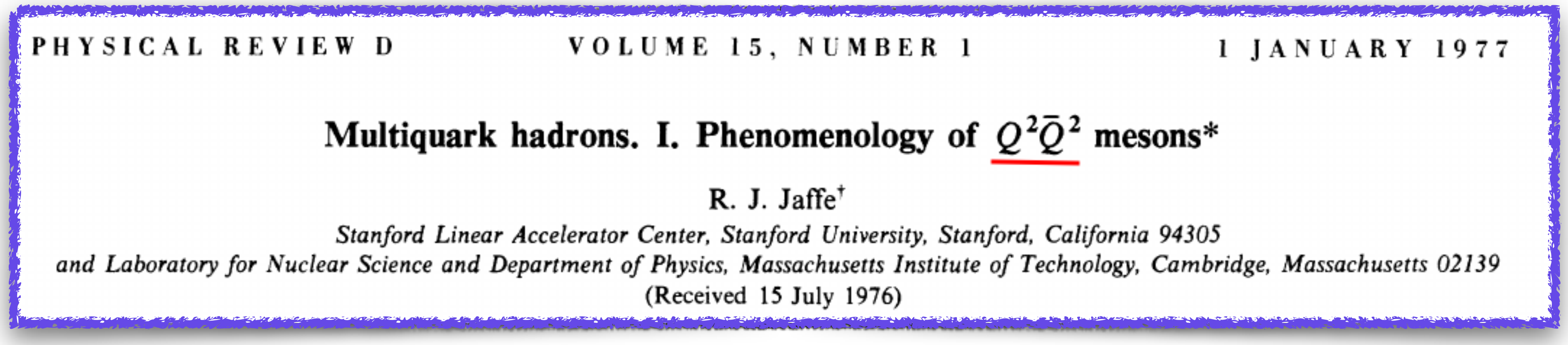

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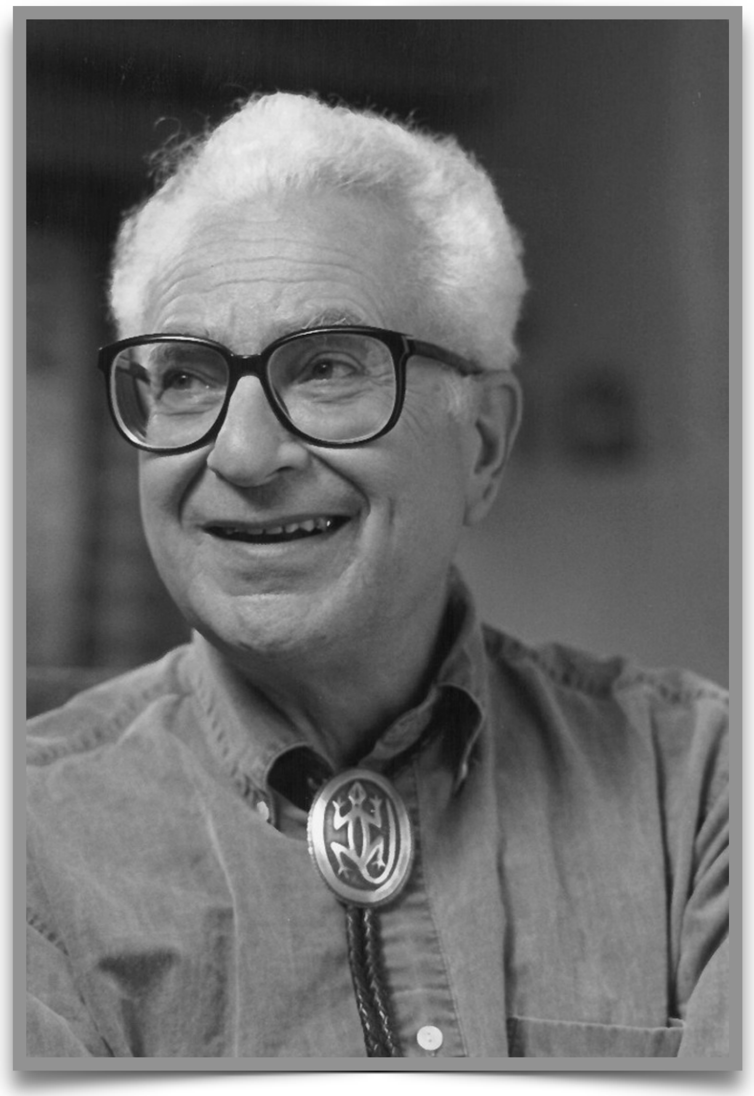

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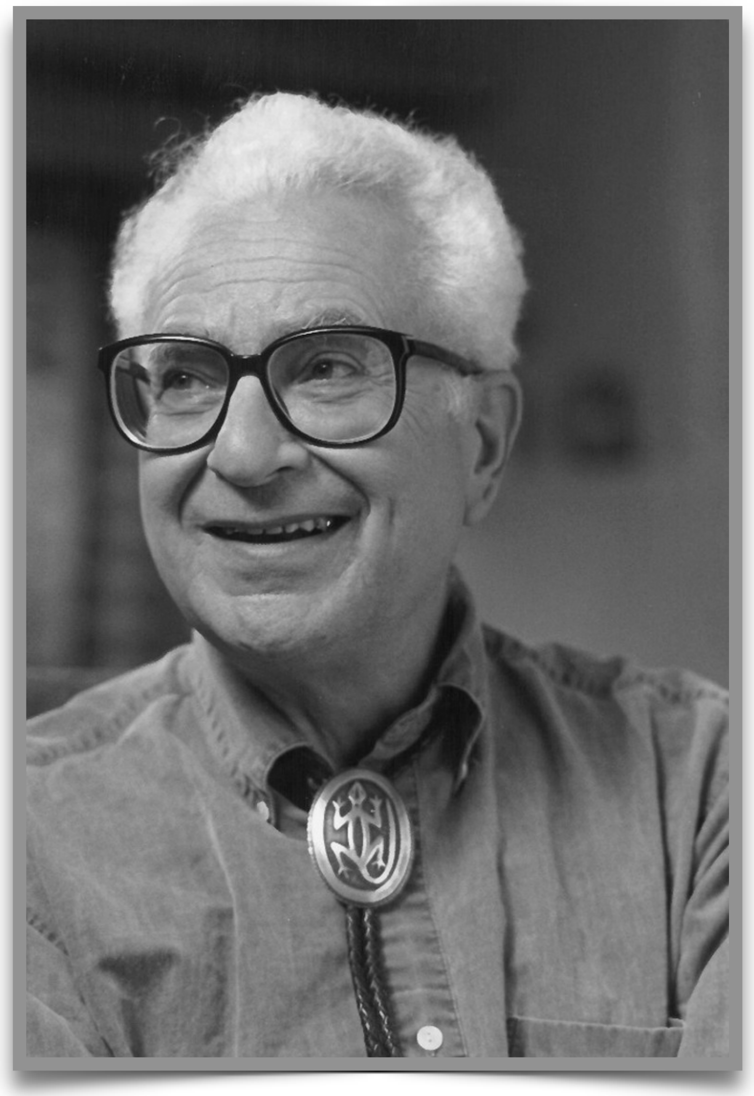

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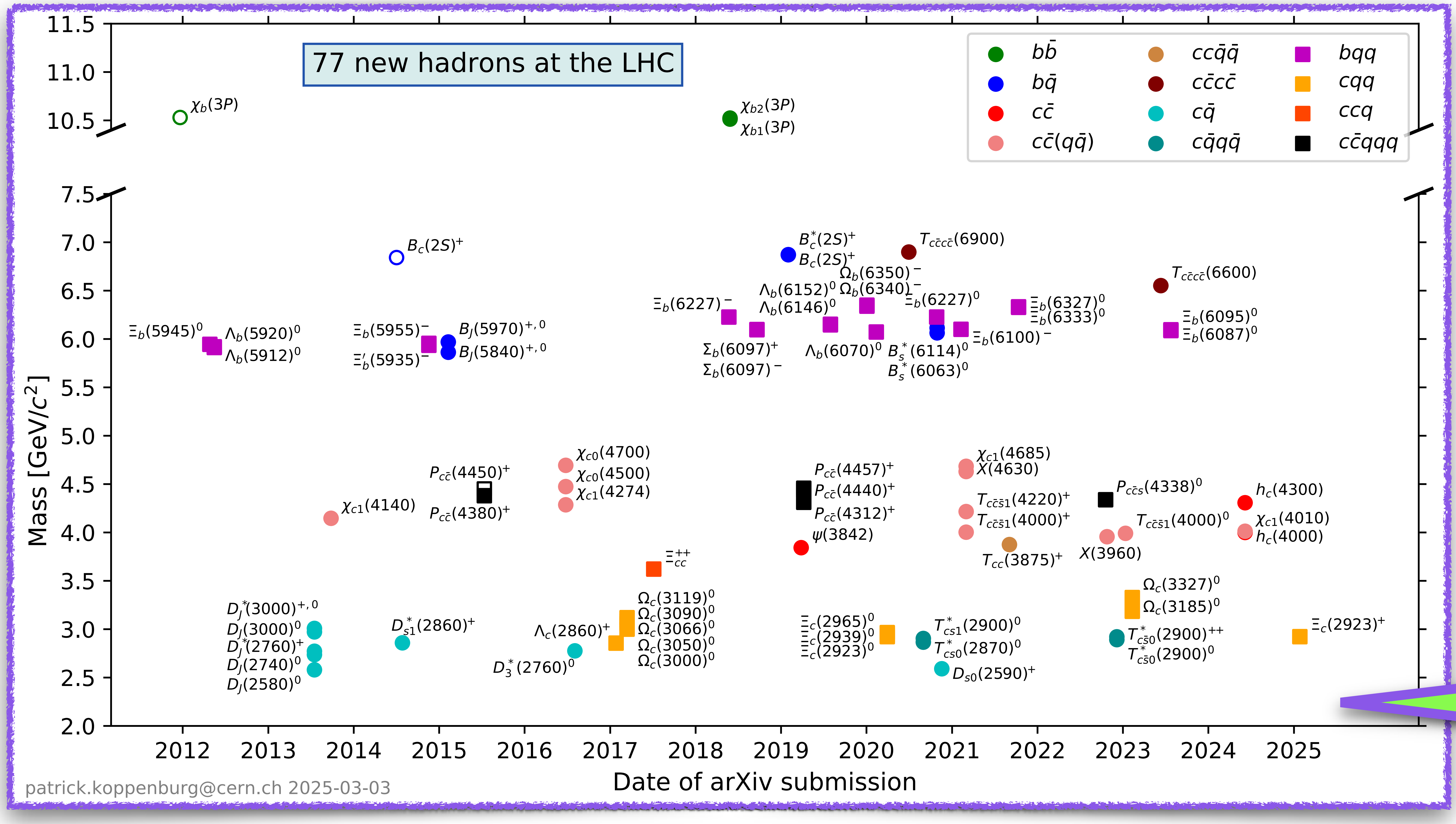

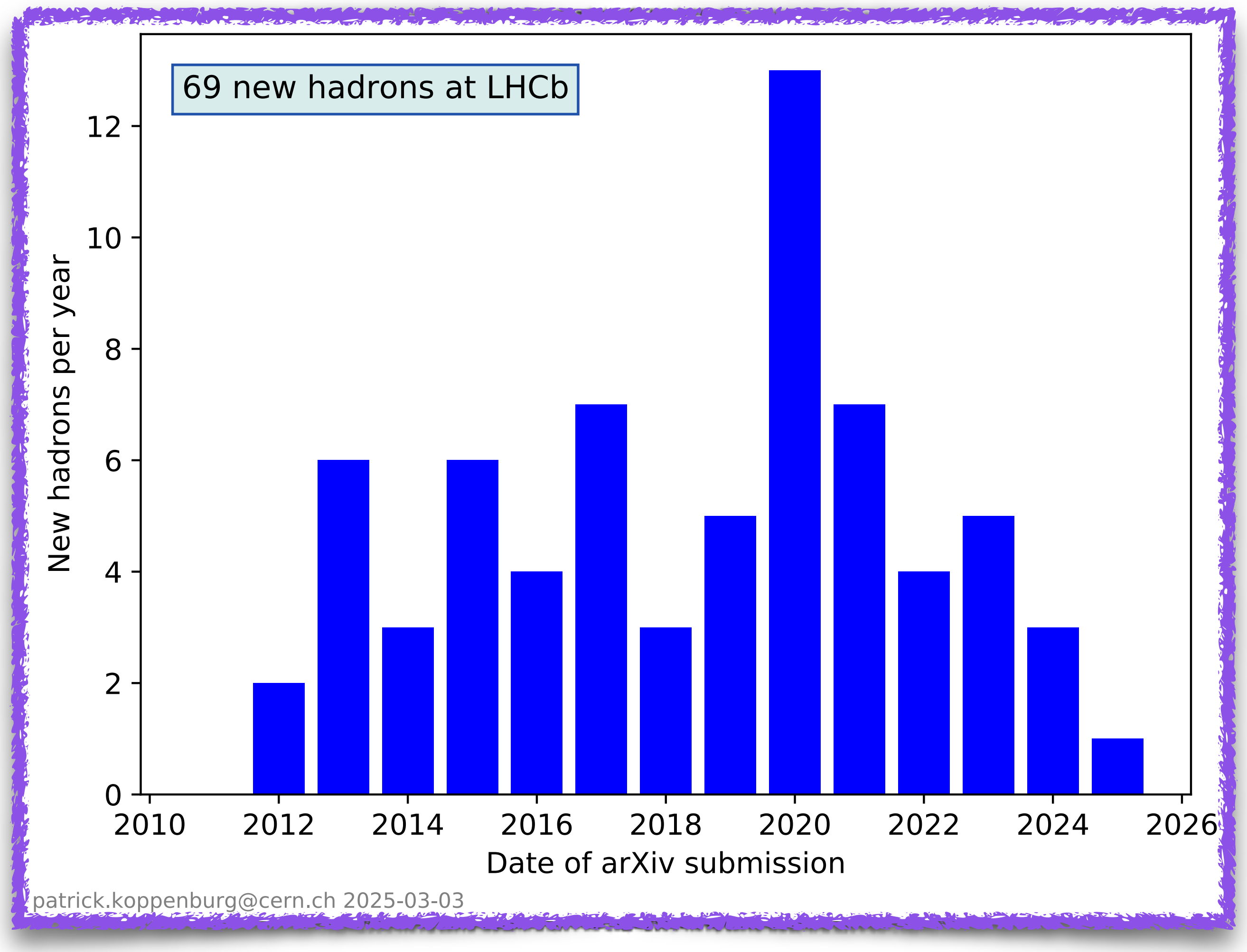

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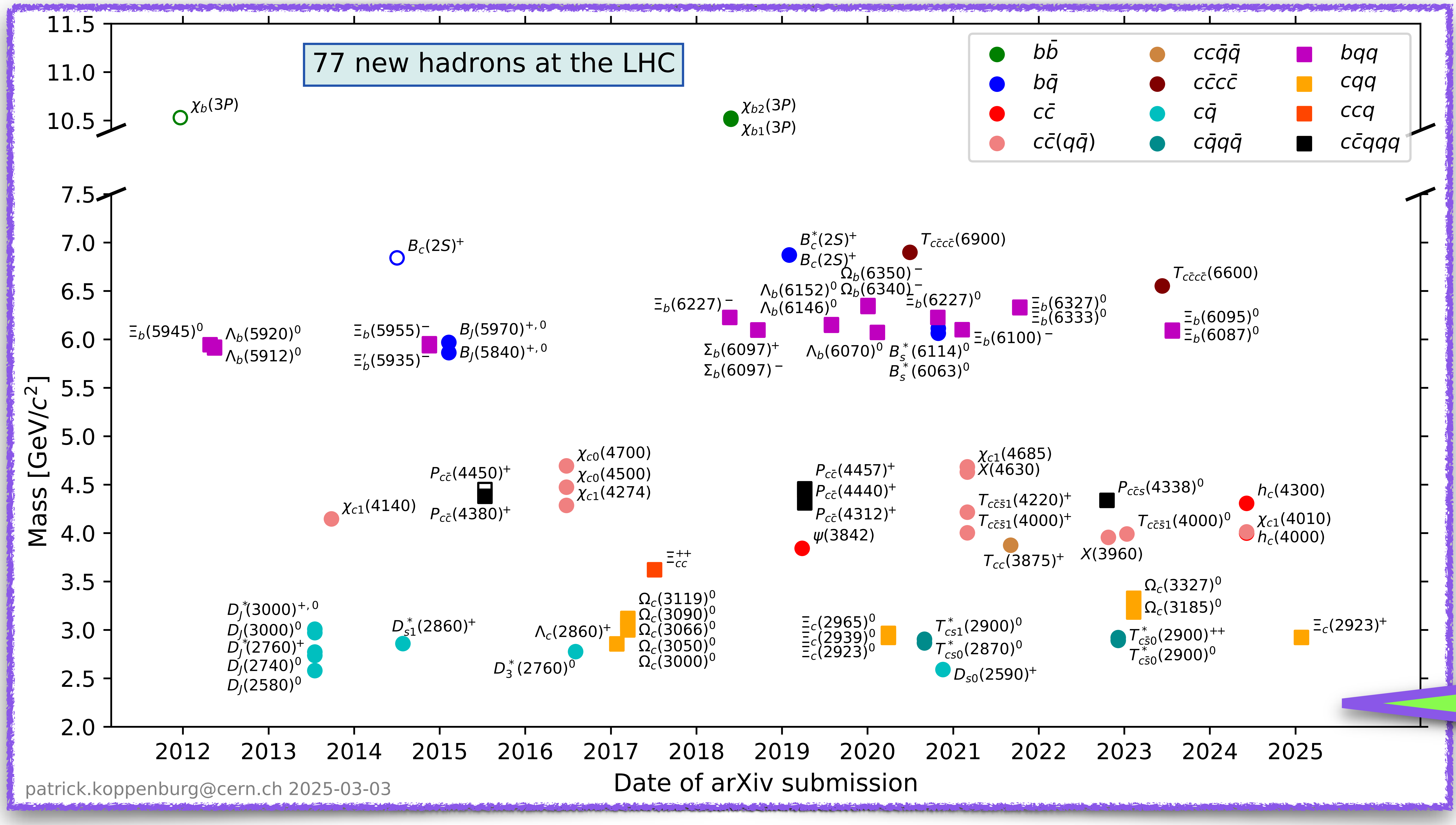

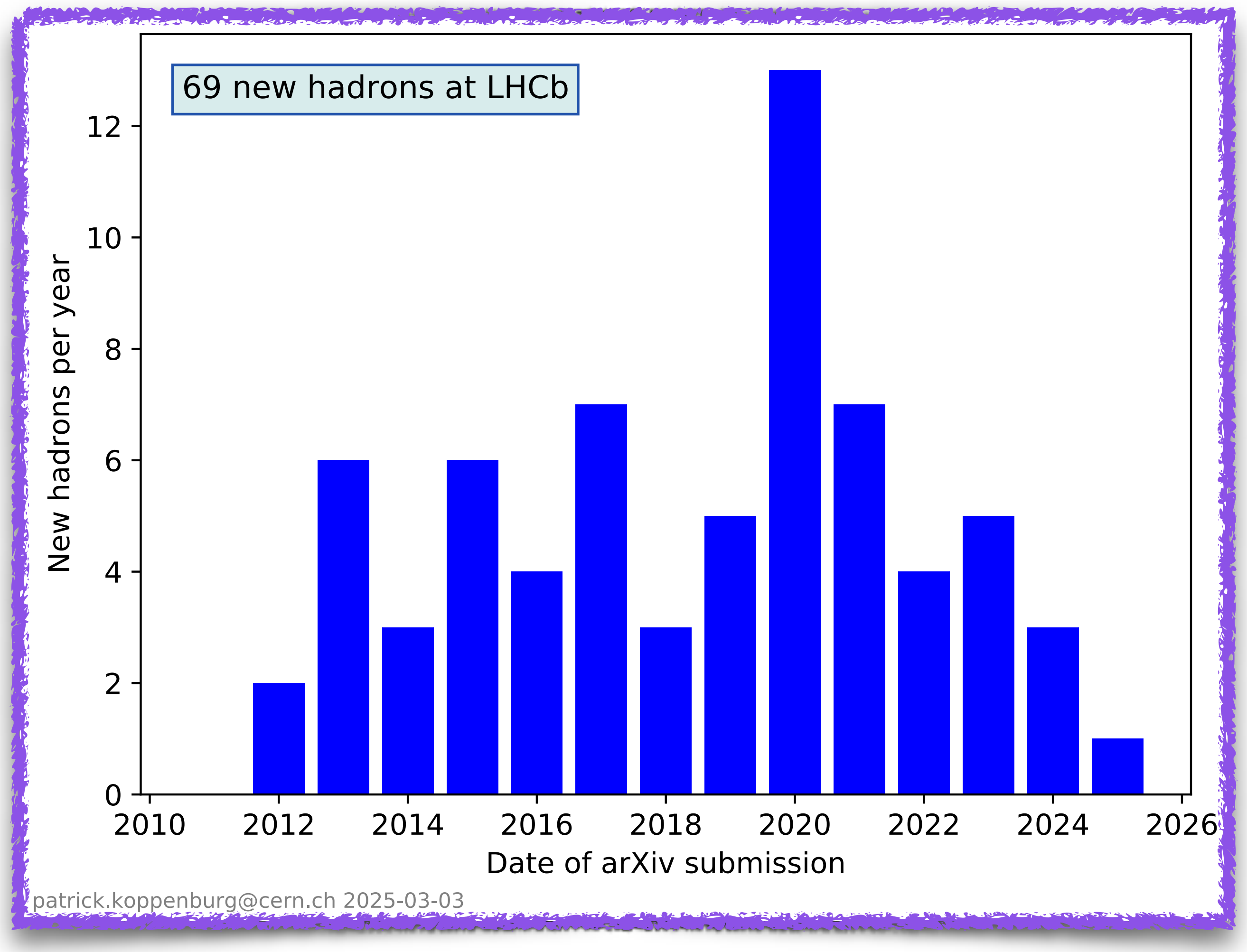

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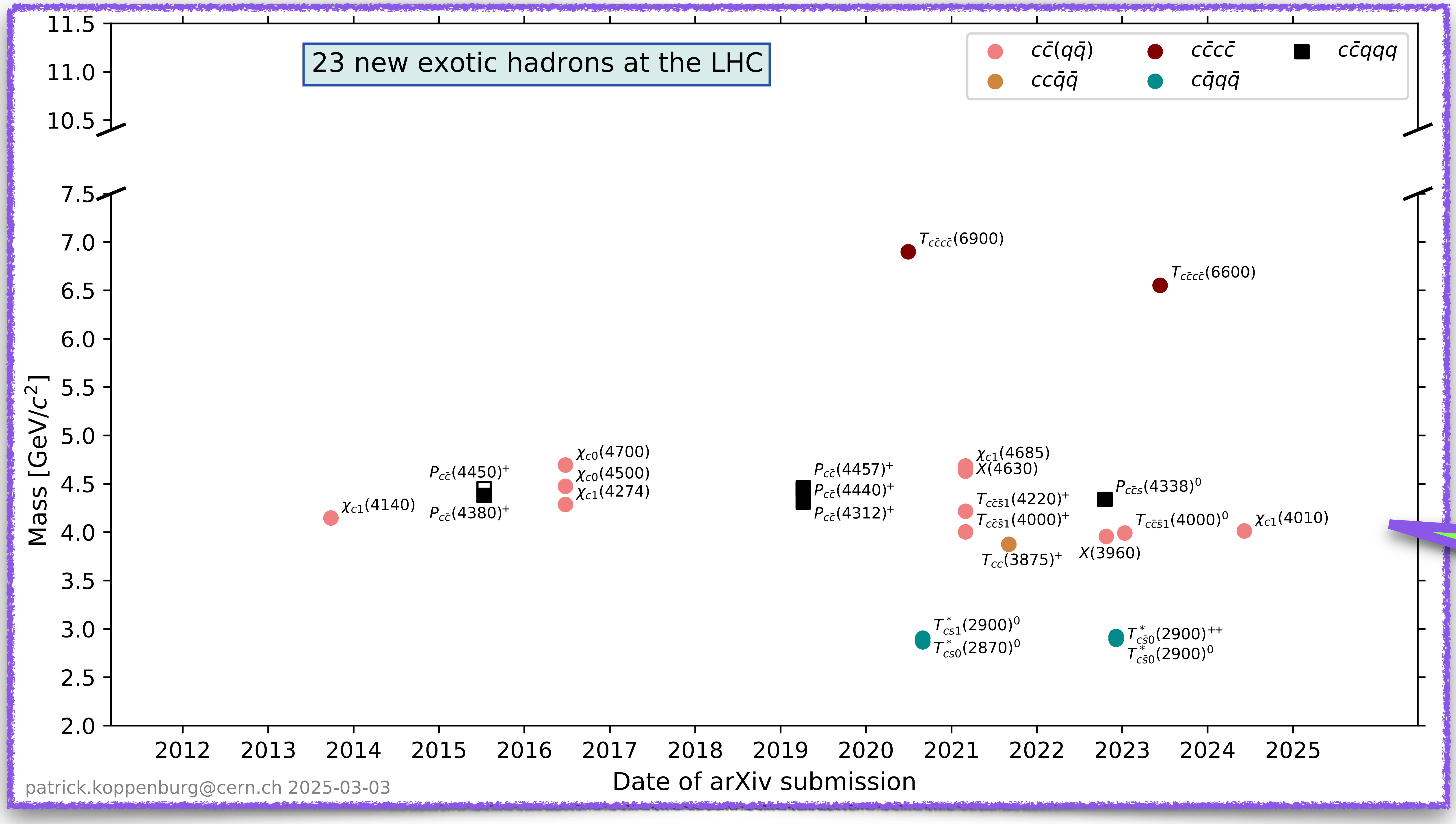

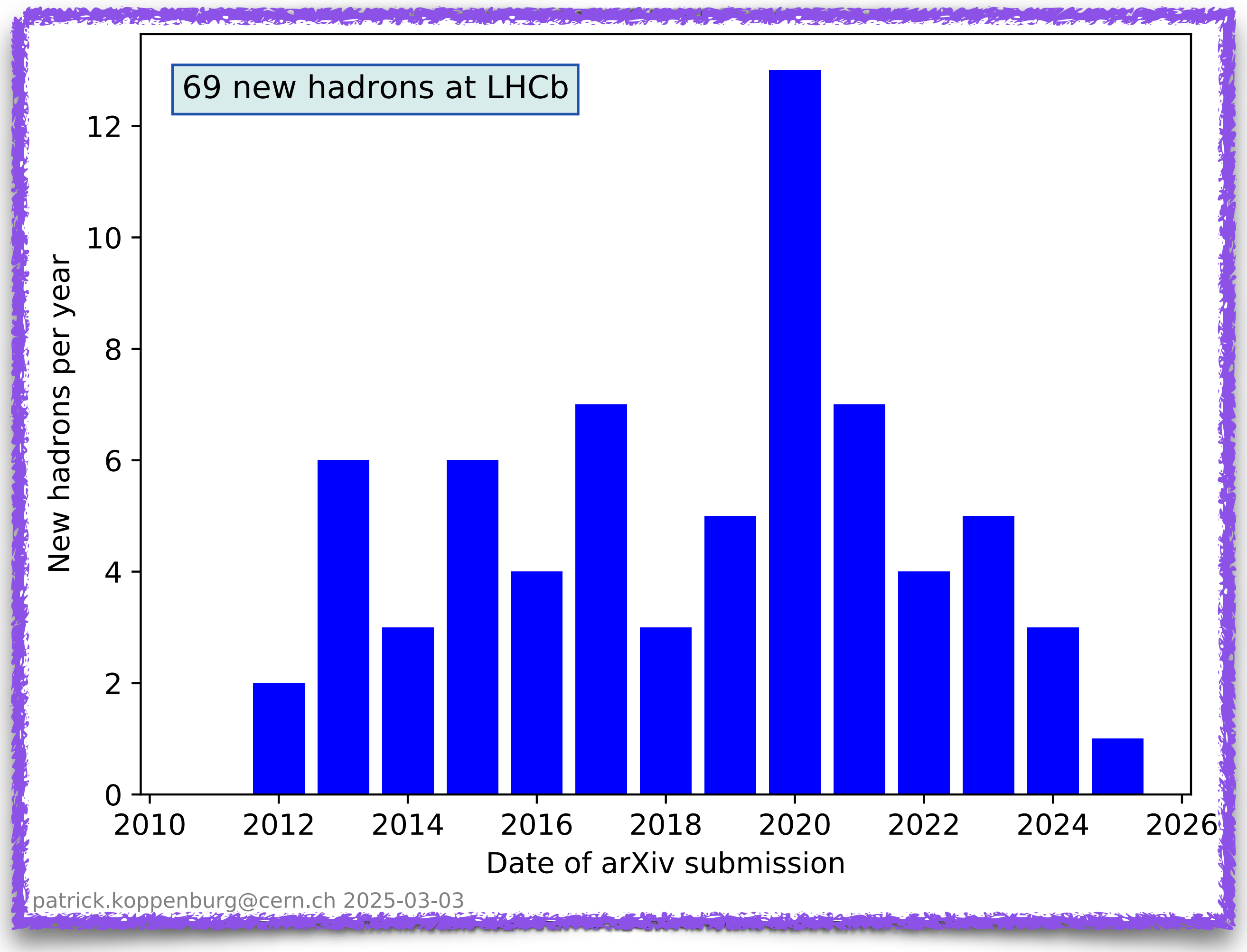

Collection of exotic particles

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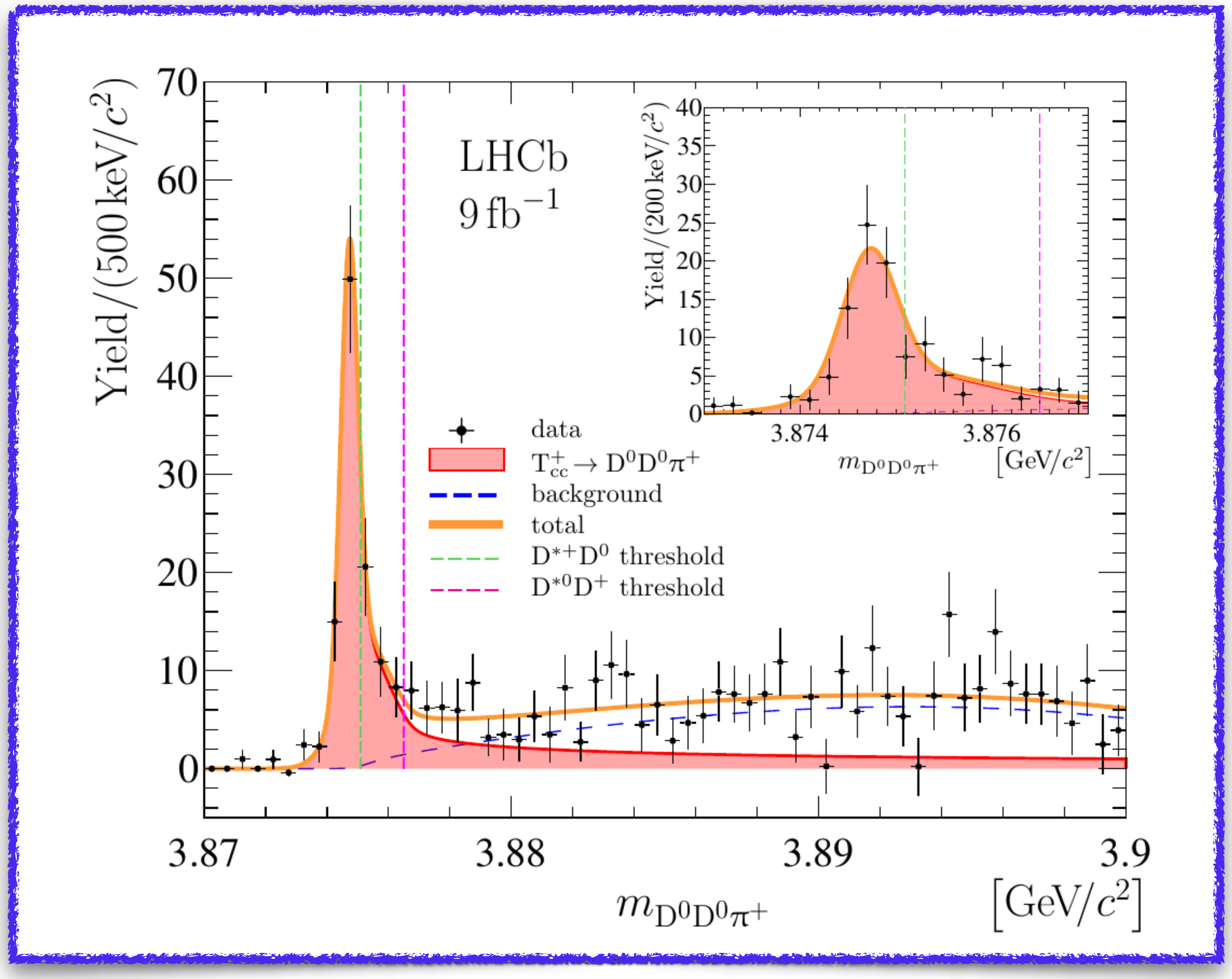
https://qwg.ph.nat.tum.de/exoticshub/

$T_{cc}(cc\bar{u}\bar{d})$ discovery at LHC



Nature phys: https://rdcu.be/dNMRV
Arxiv:2109.01038

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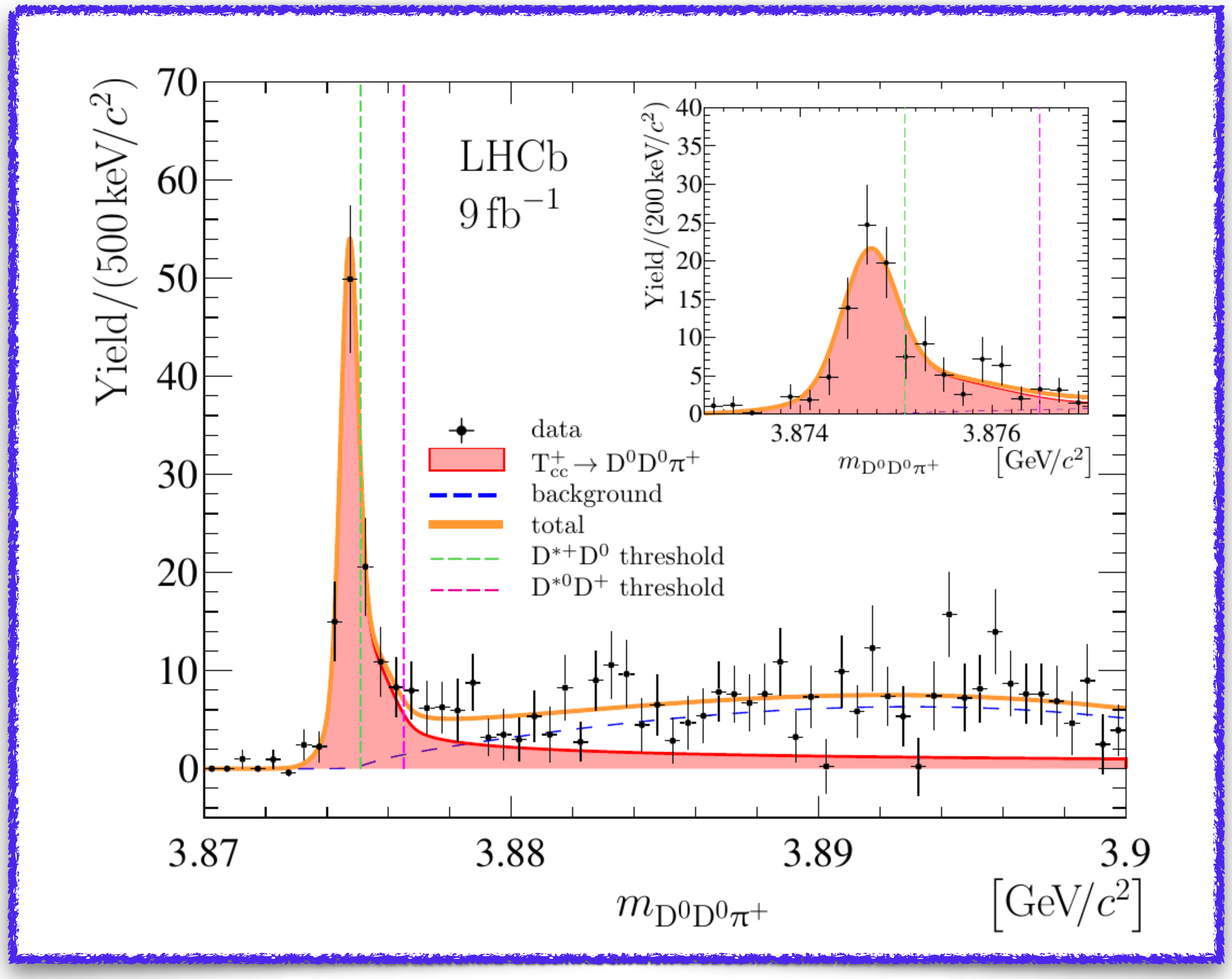

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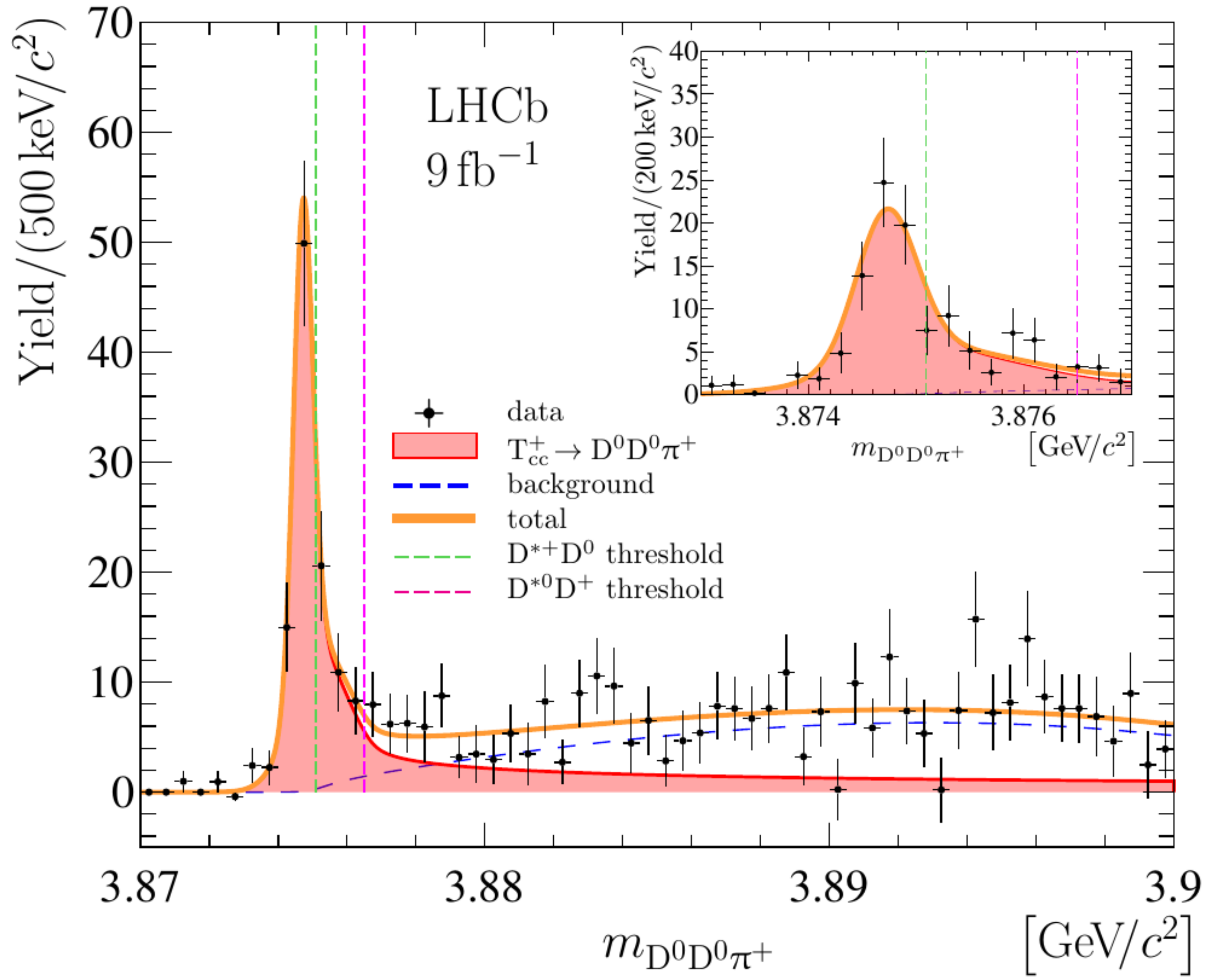

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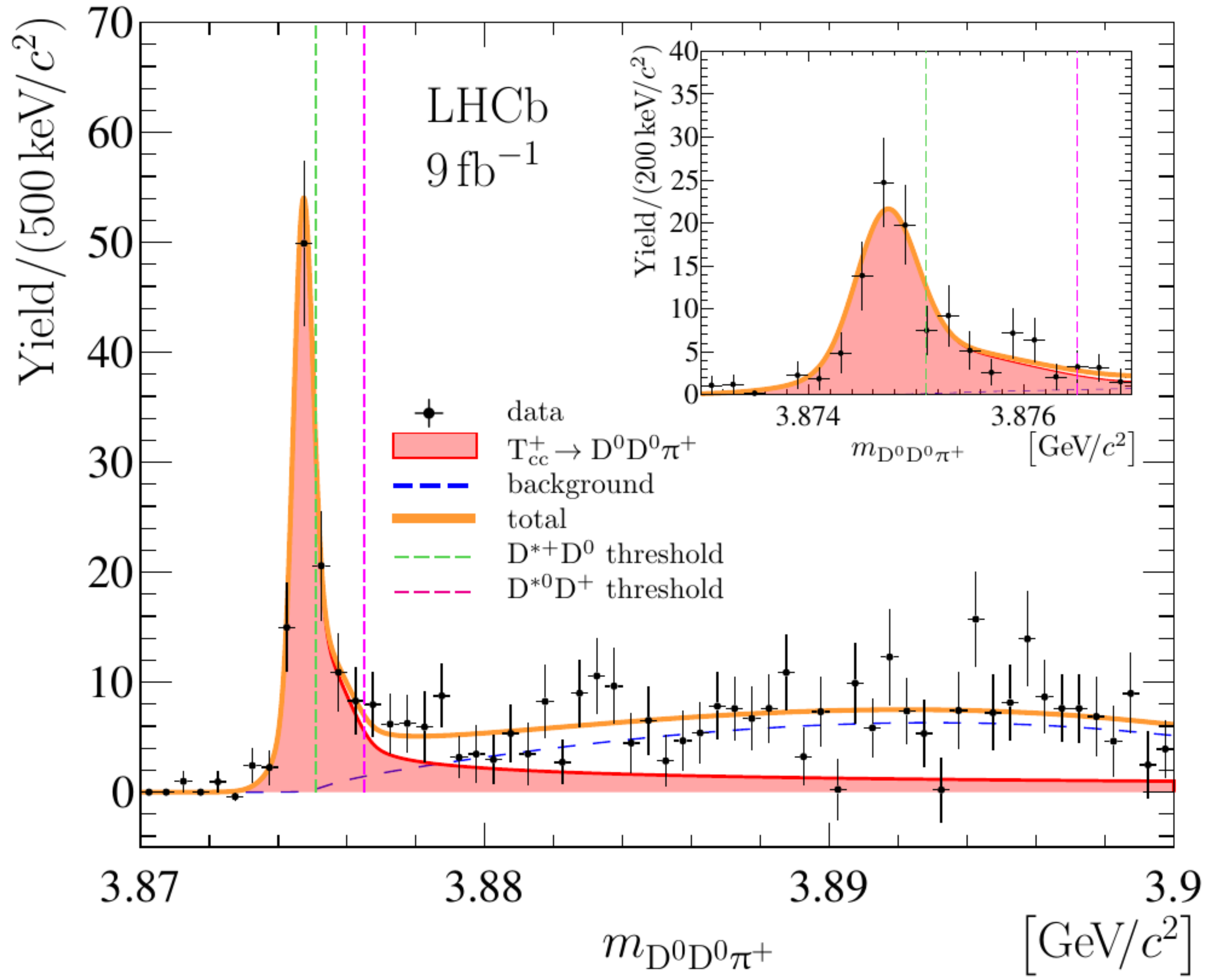
- Many more exotic tetraquark discovered recently e.g. T_{cs}, $T_{c\bar{s}}$, Z_c and so on. Scope for T_{bc}, T_{bs} in near future.



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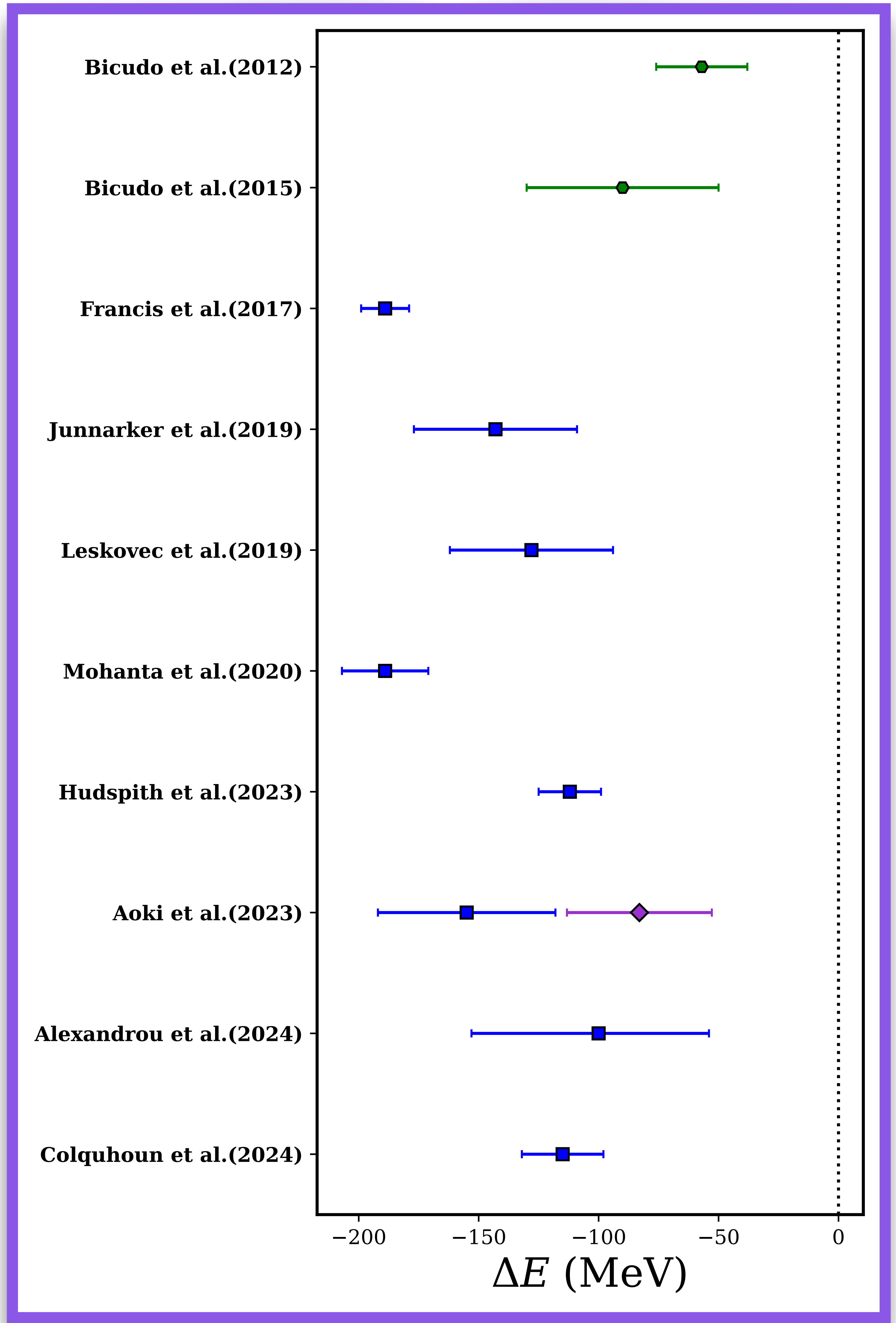

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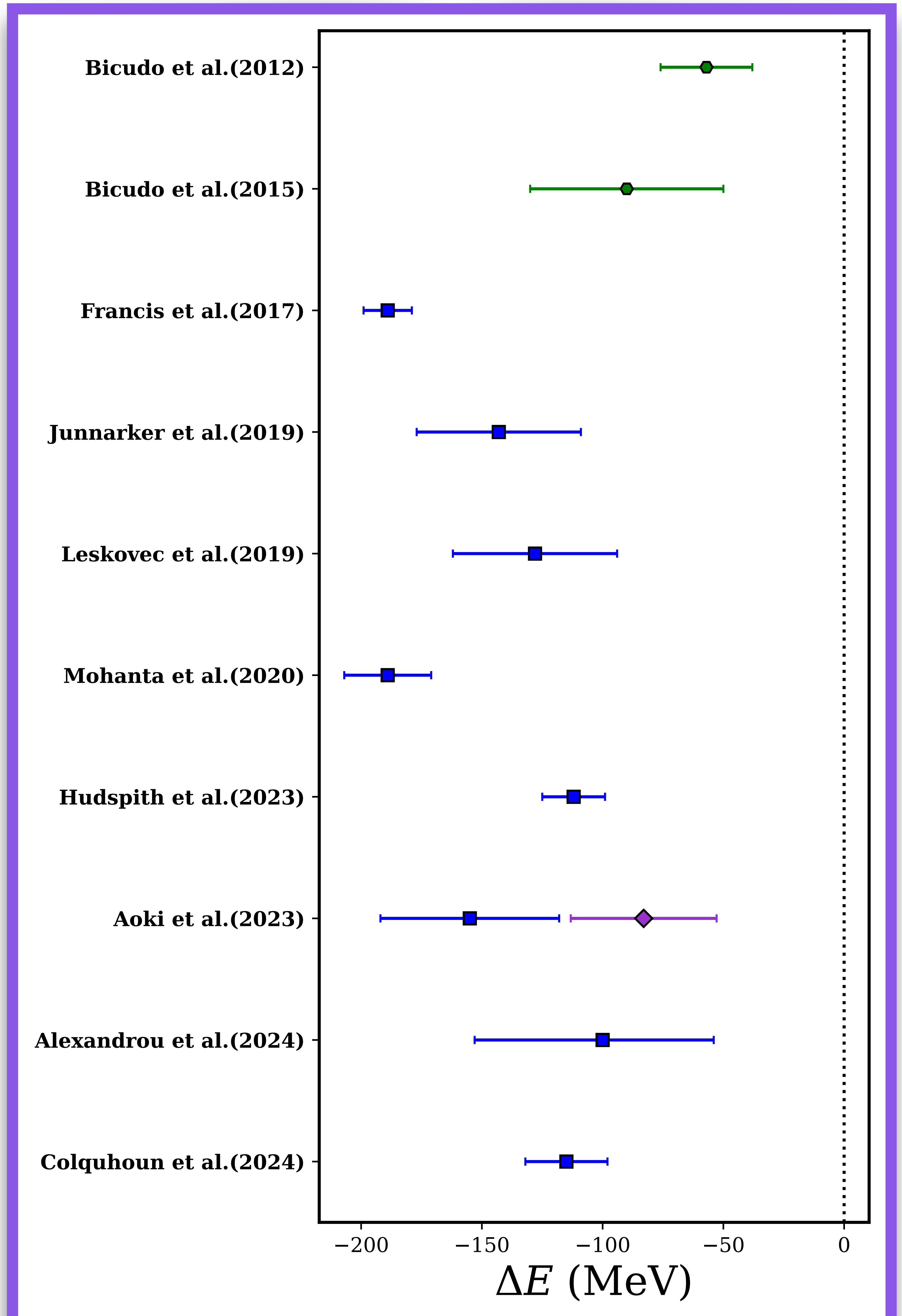

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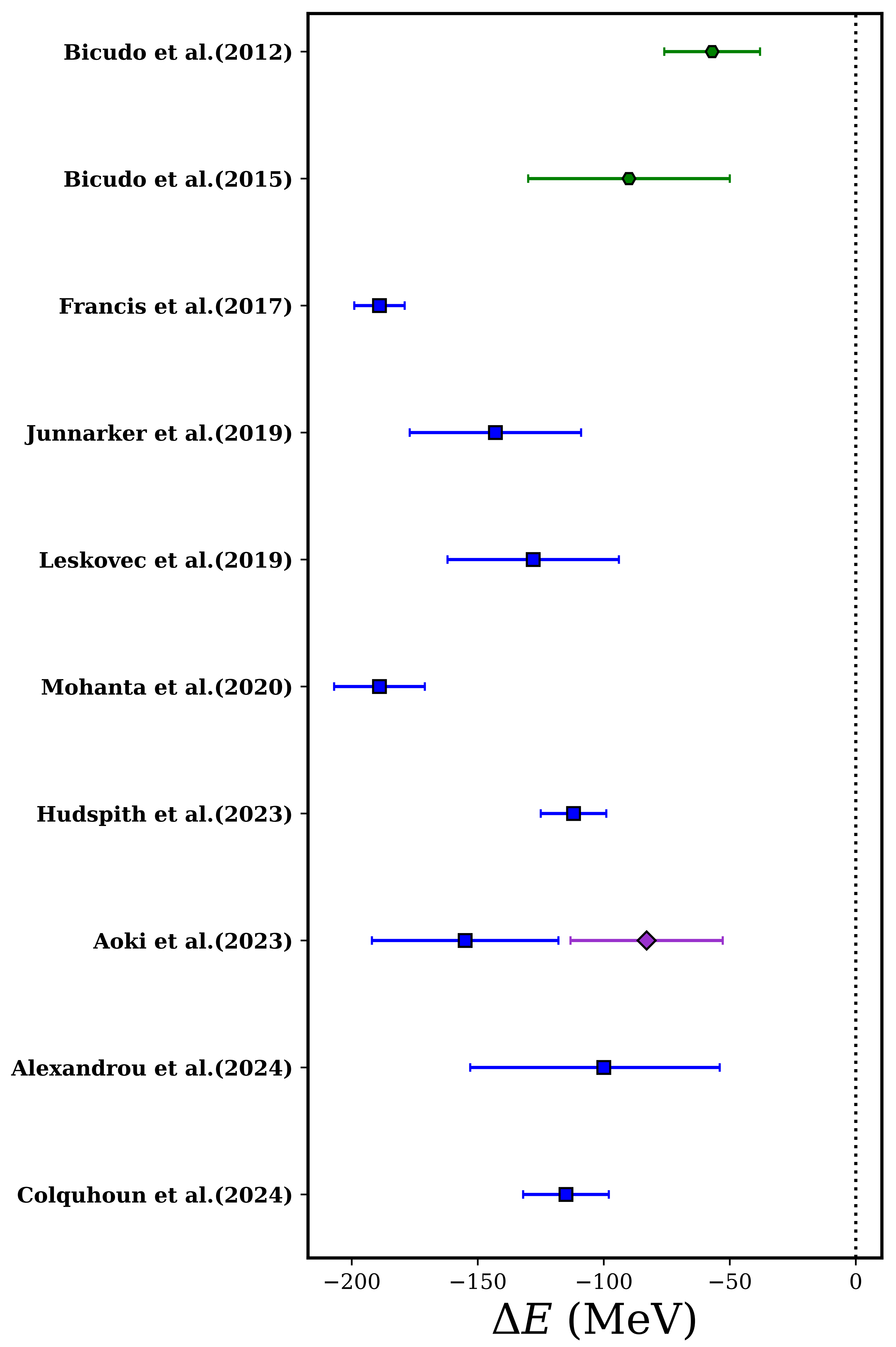

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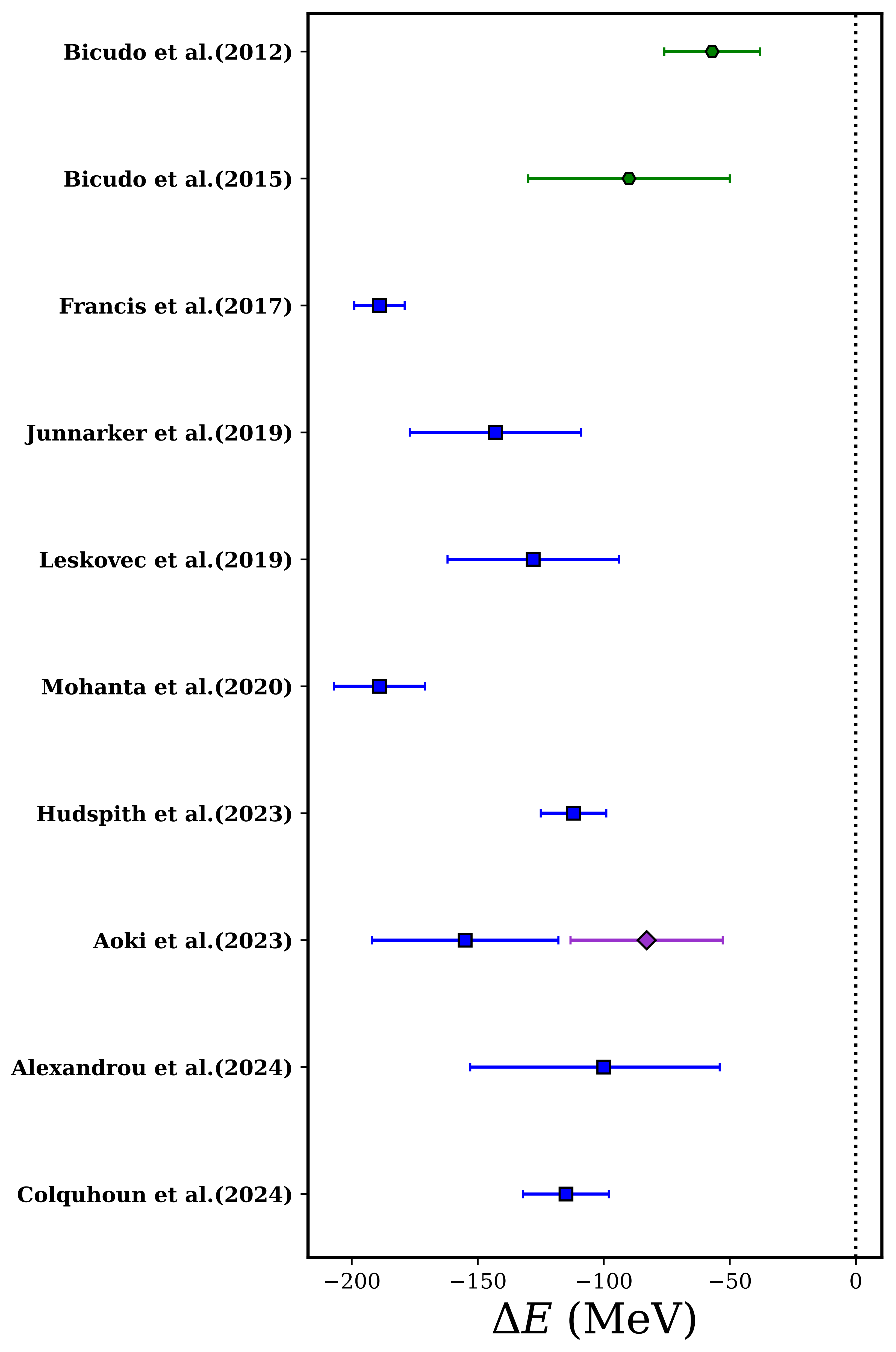

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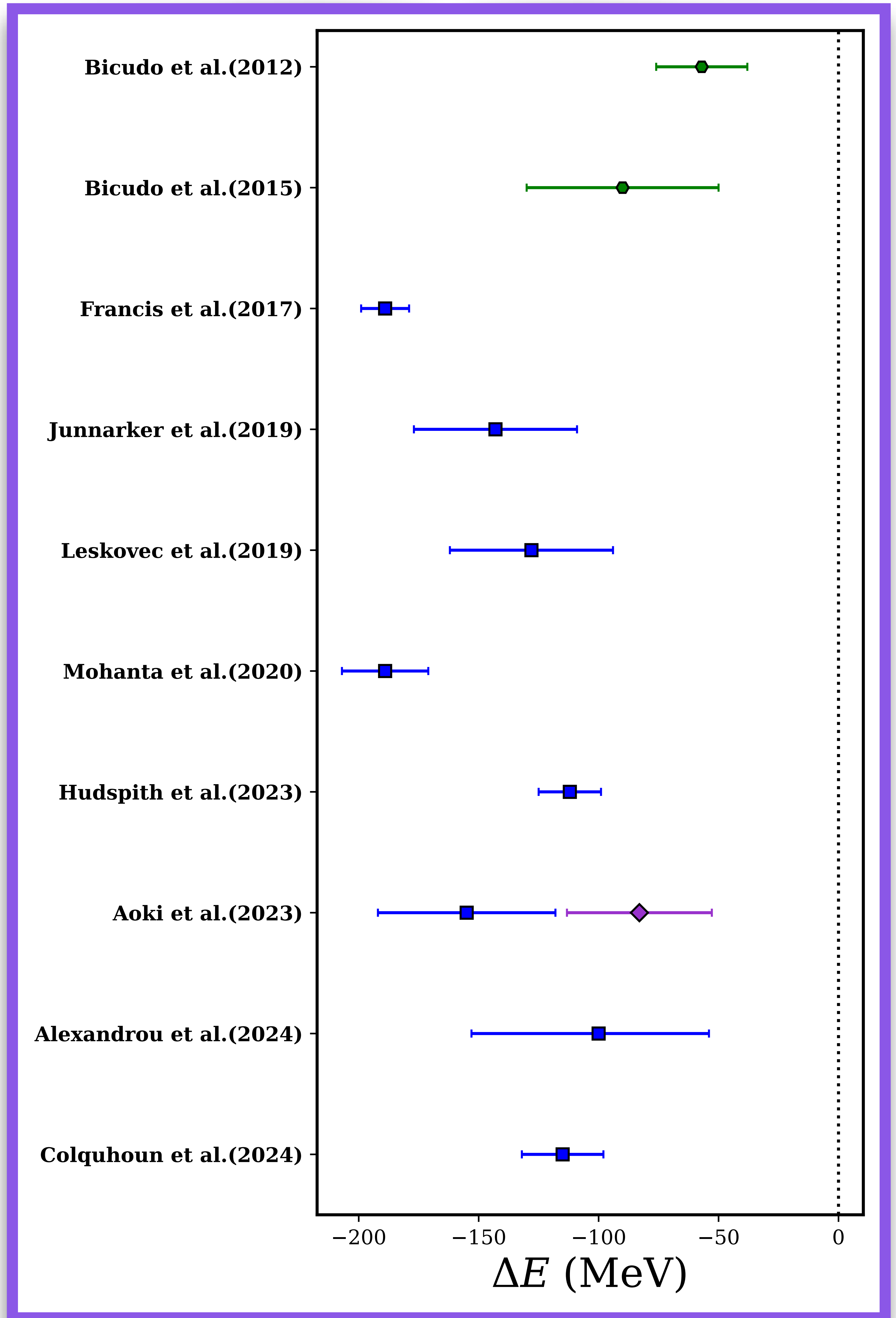

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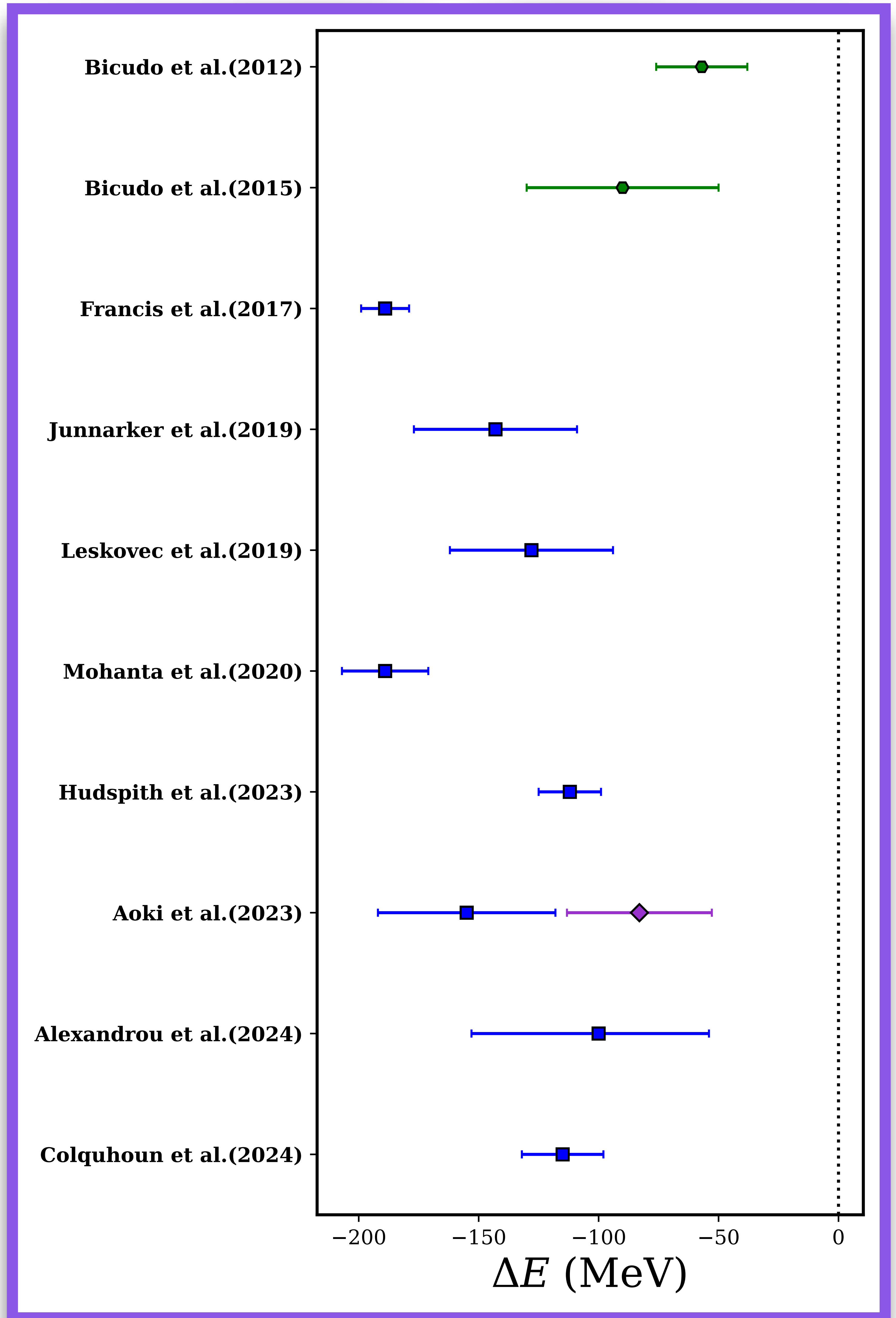

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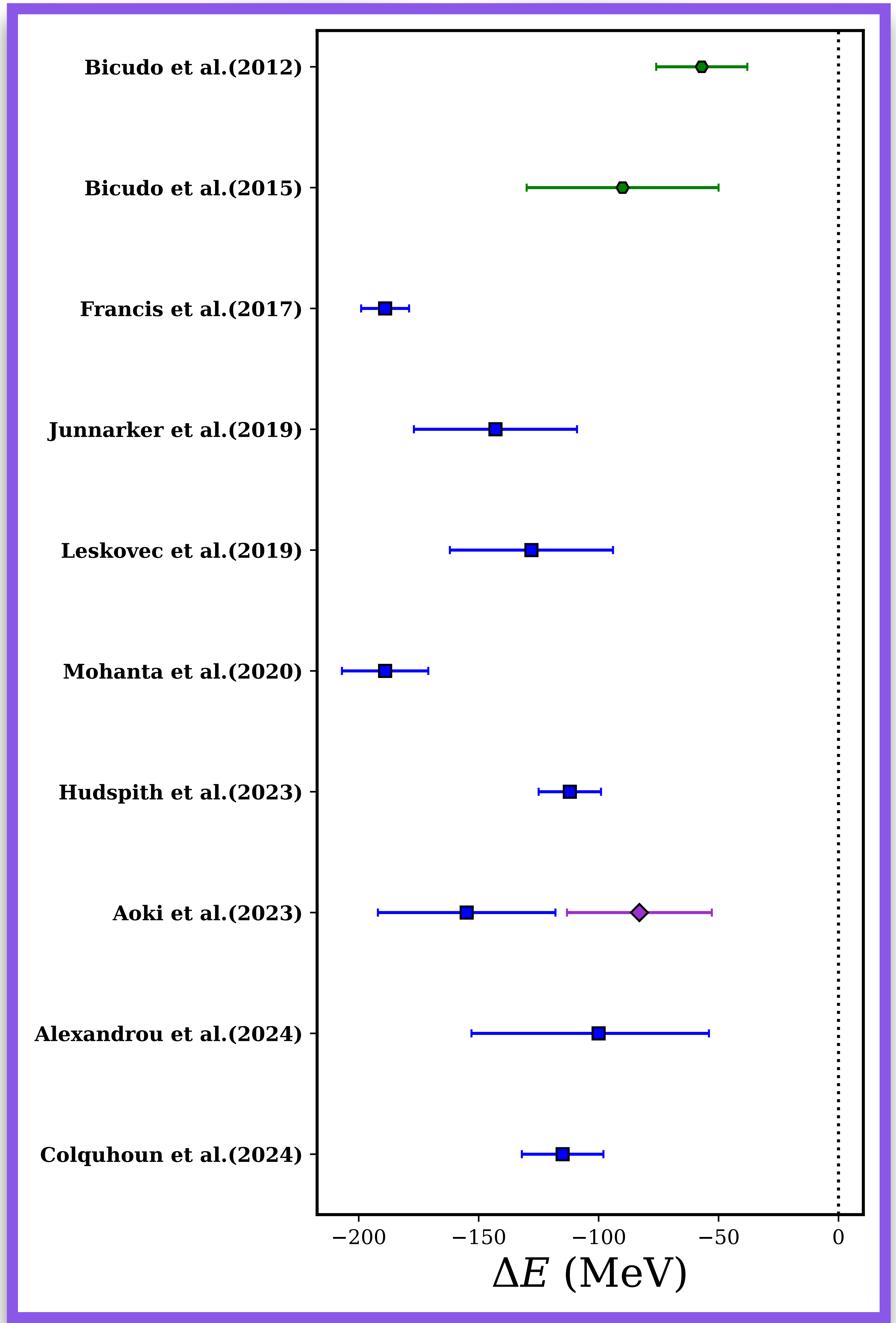

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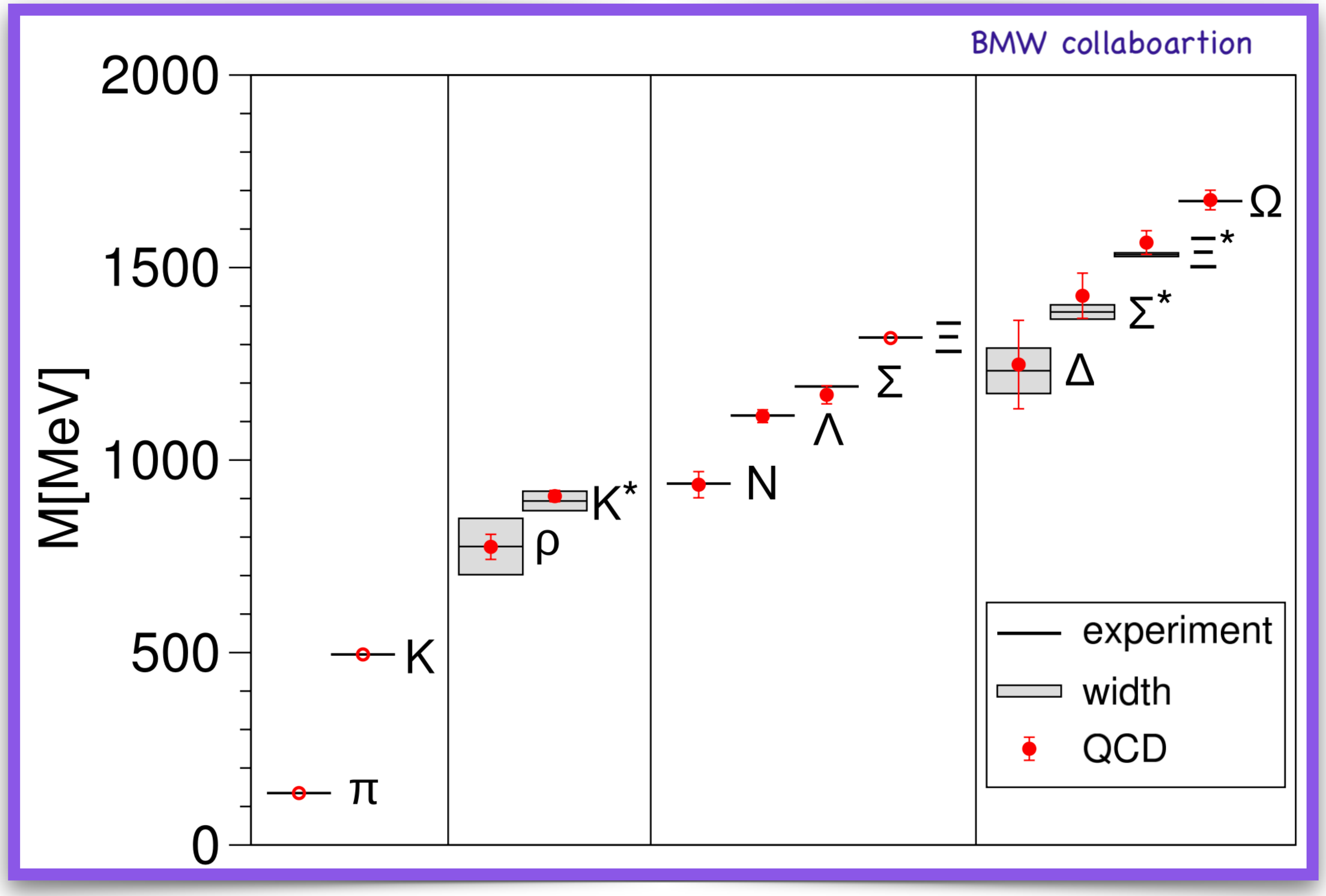
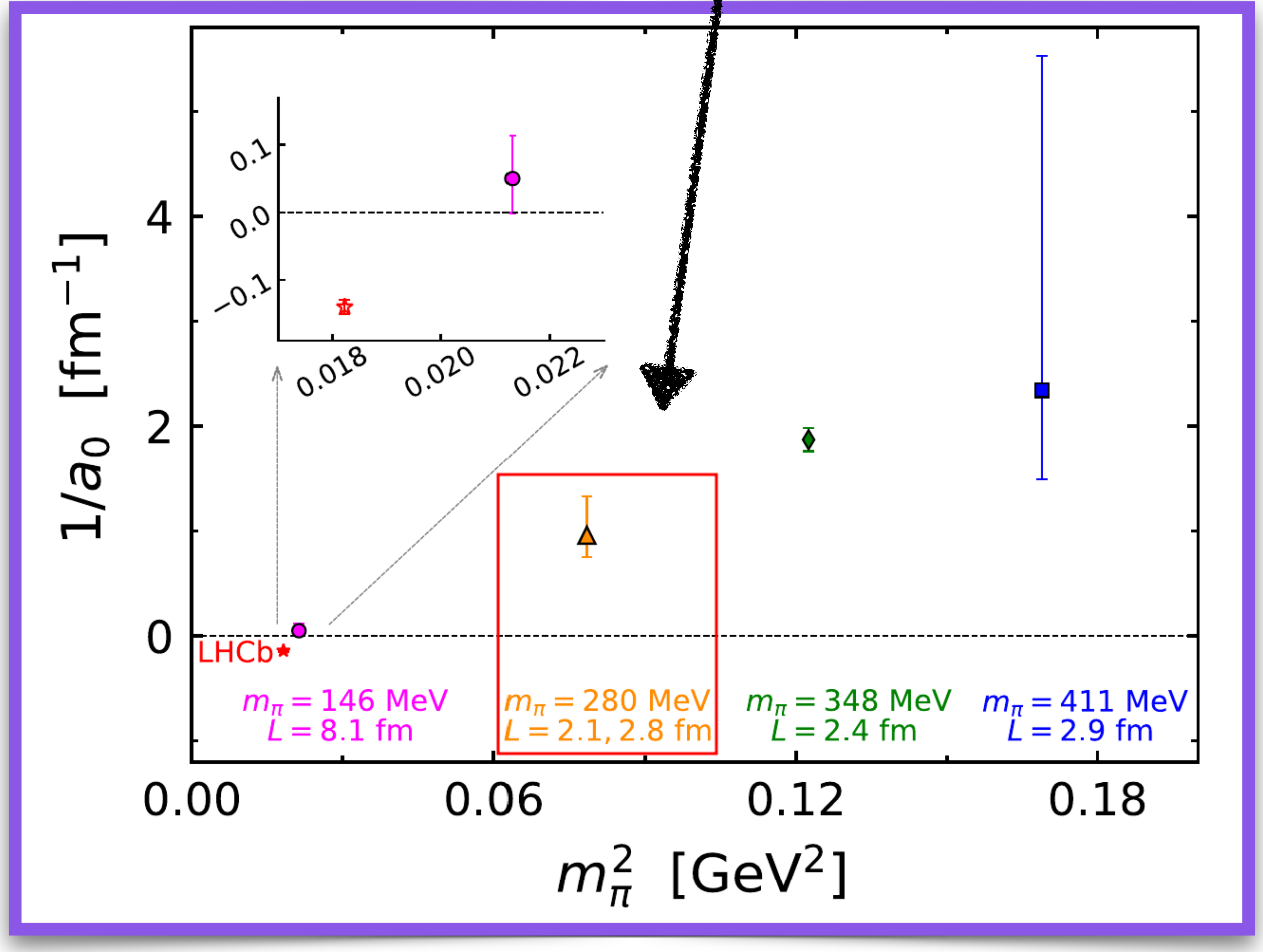


Science 322 (2008) 1224-1227

Phys. Rev. Lett. 129, 032002 Padmanath, Prelovsek



Yan Lyu et al. Phys Rev Lett.131.161901

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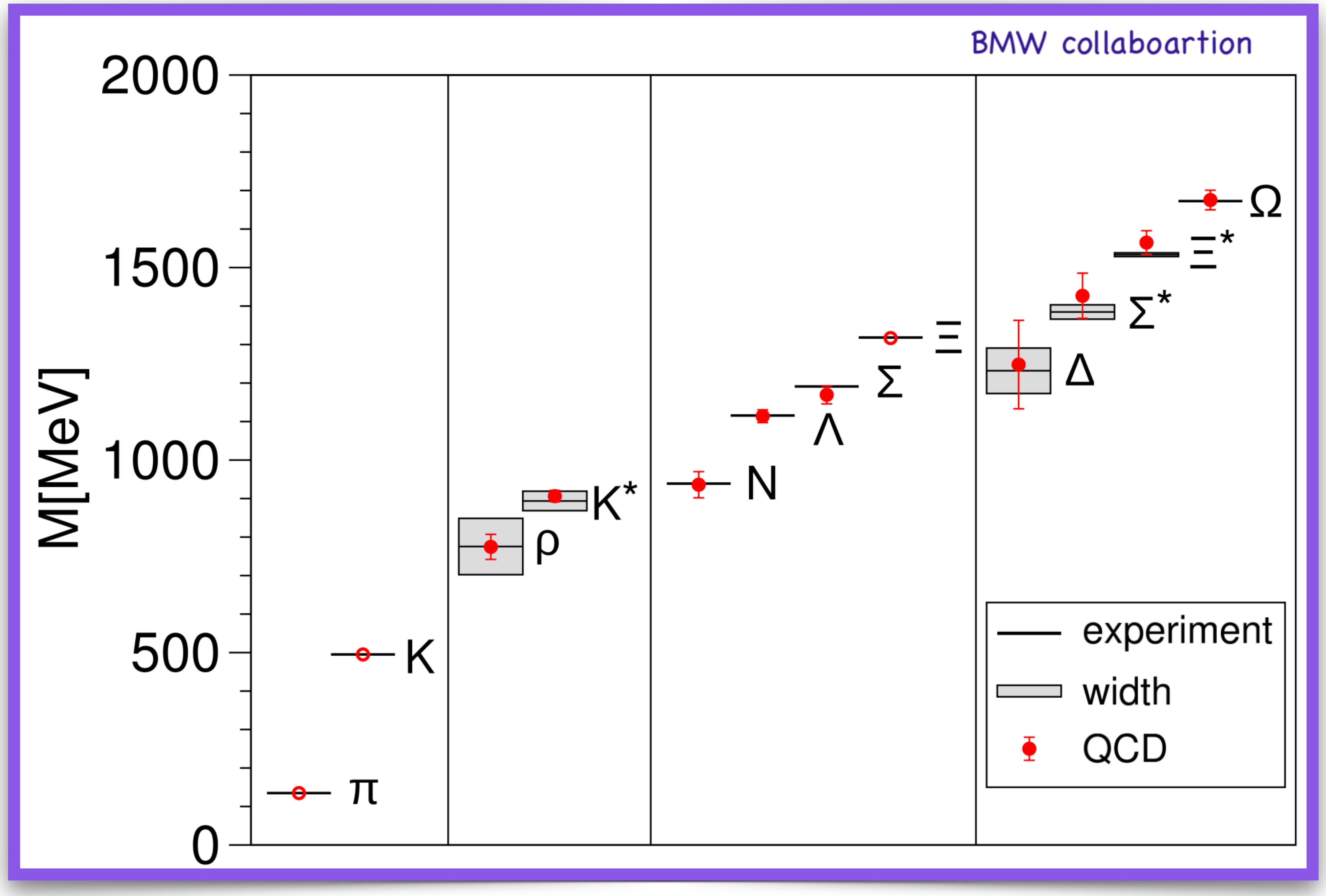

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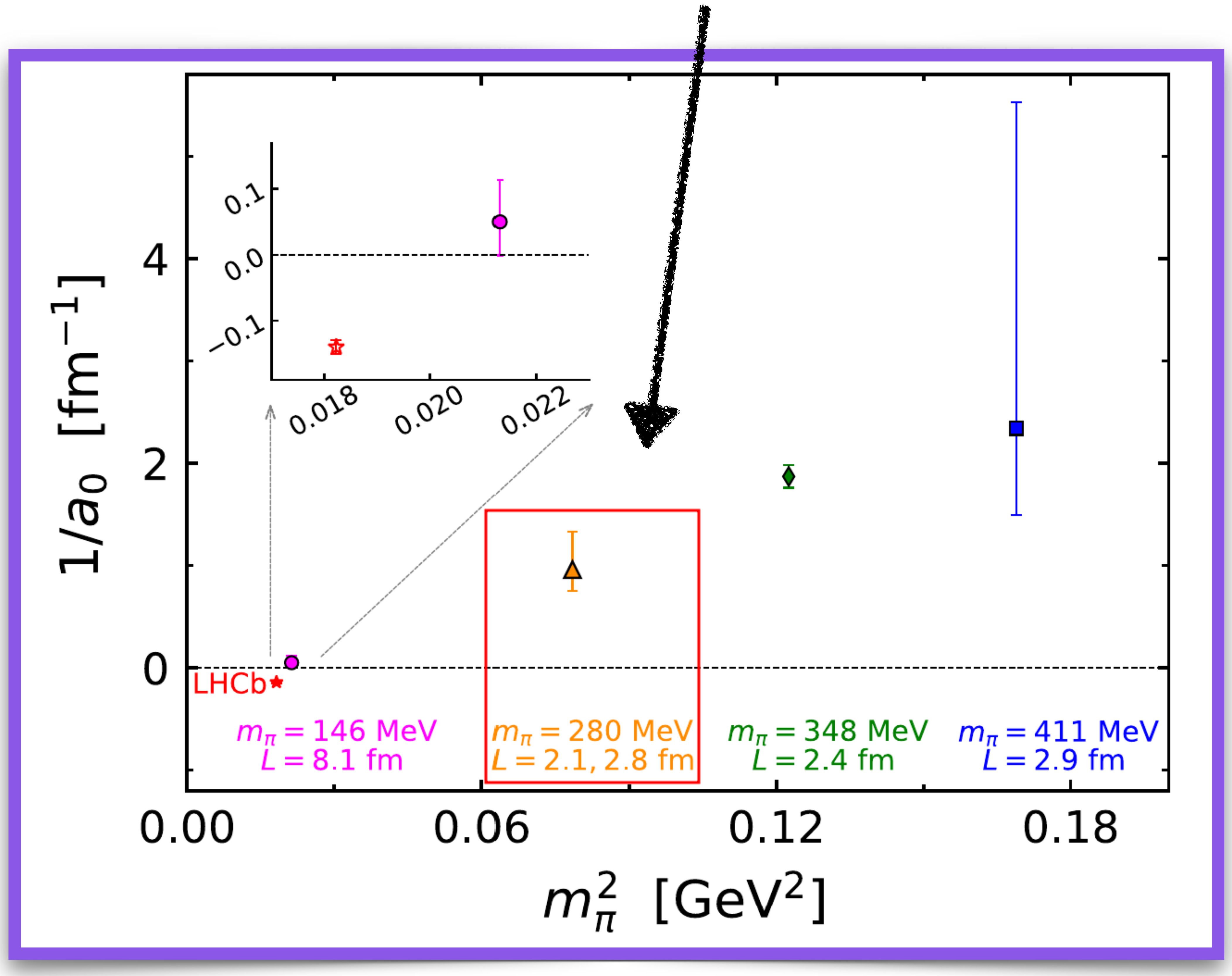

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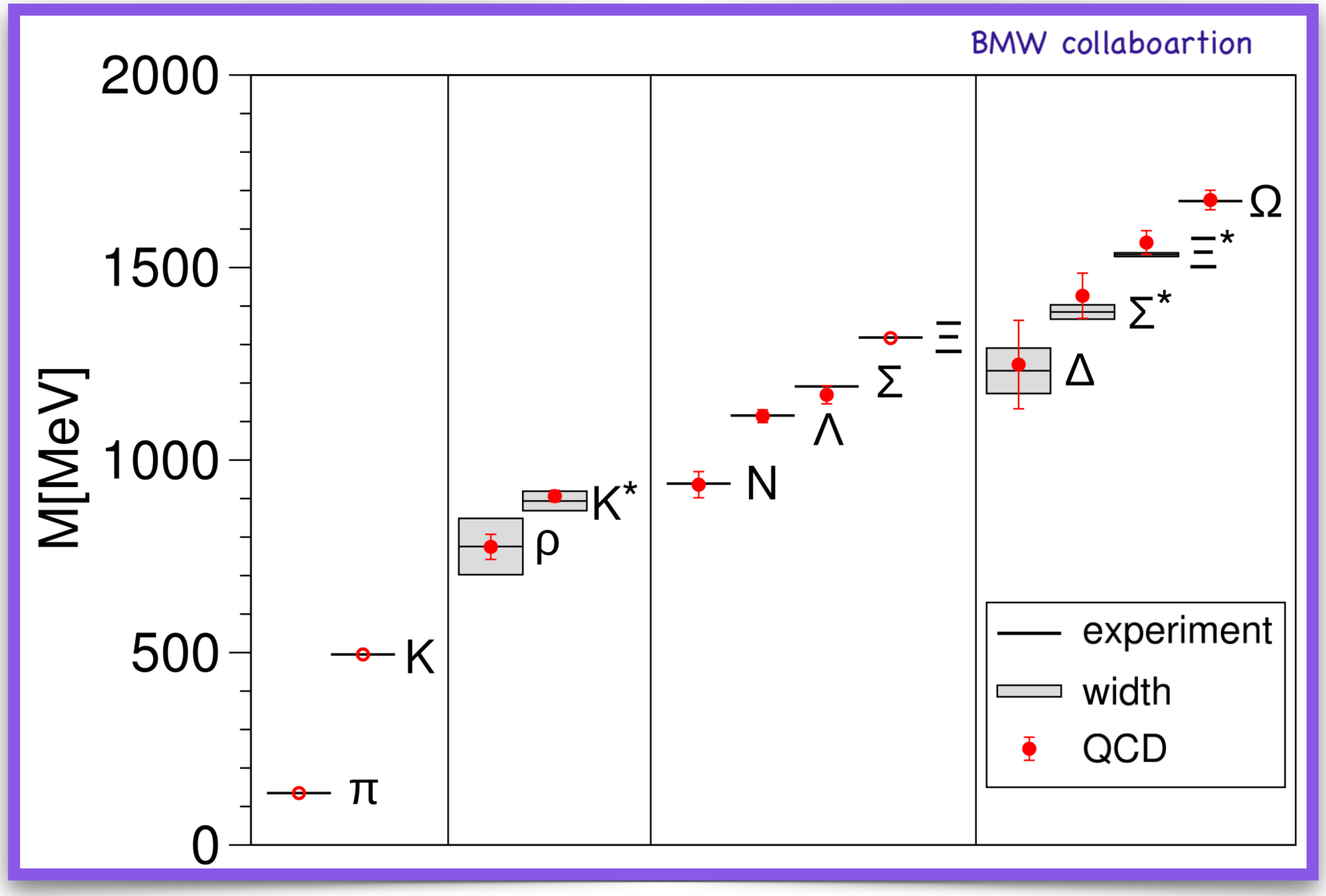

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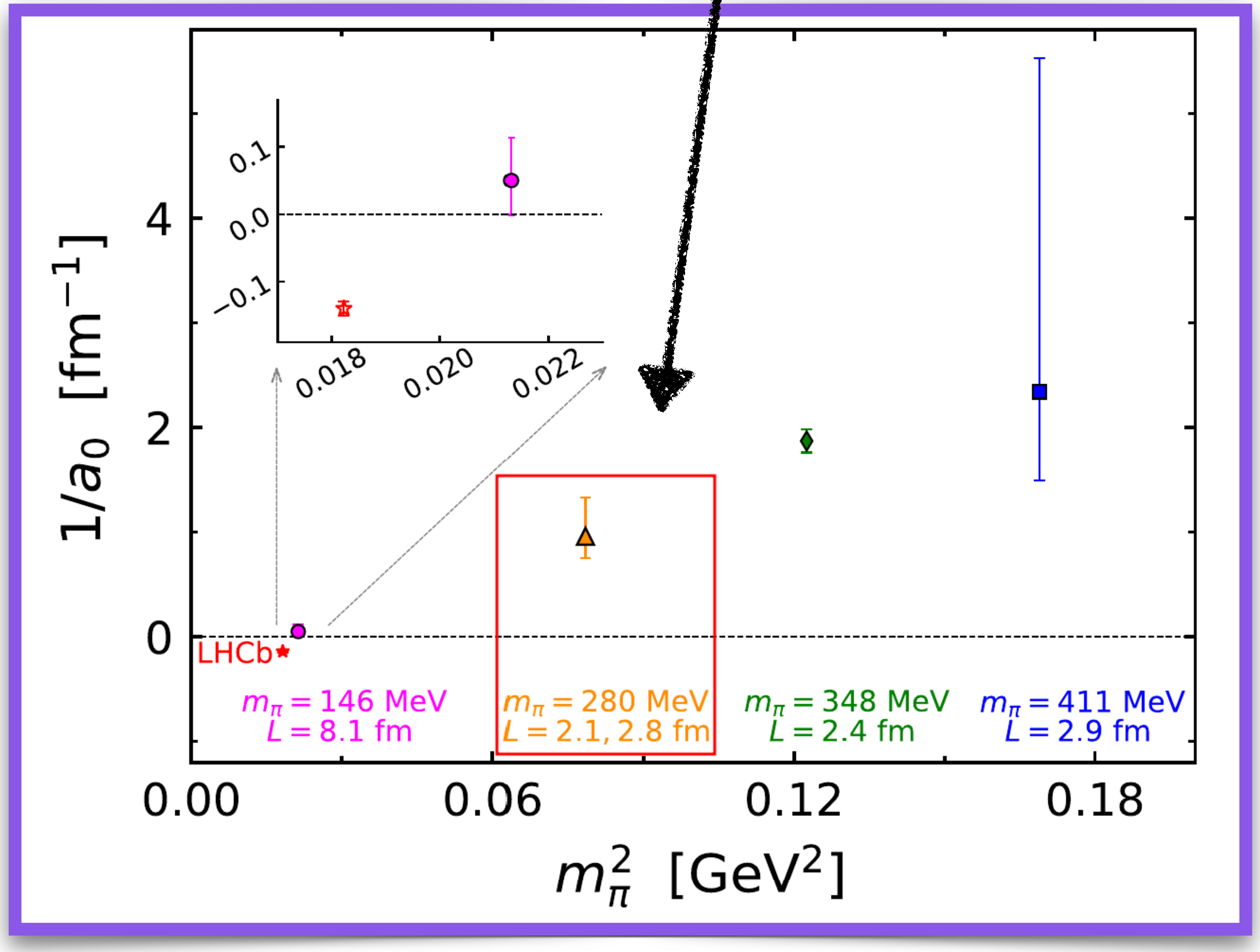

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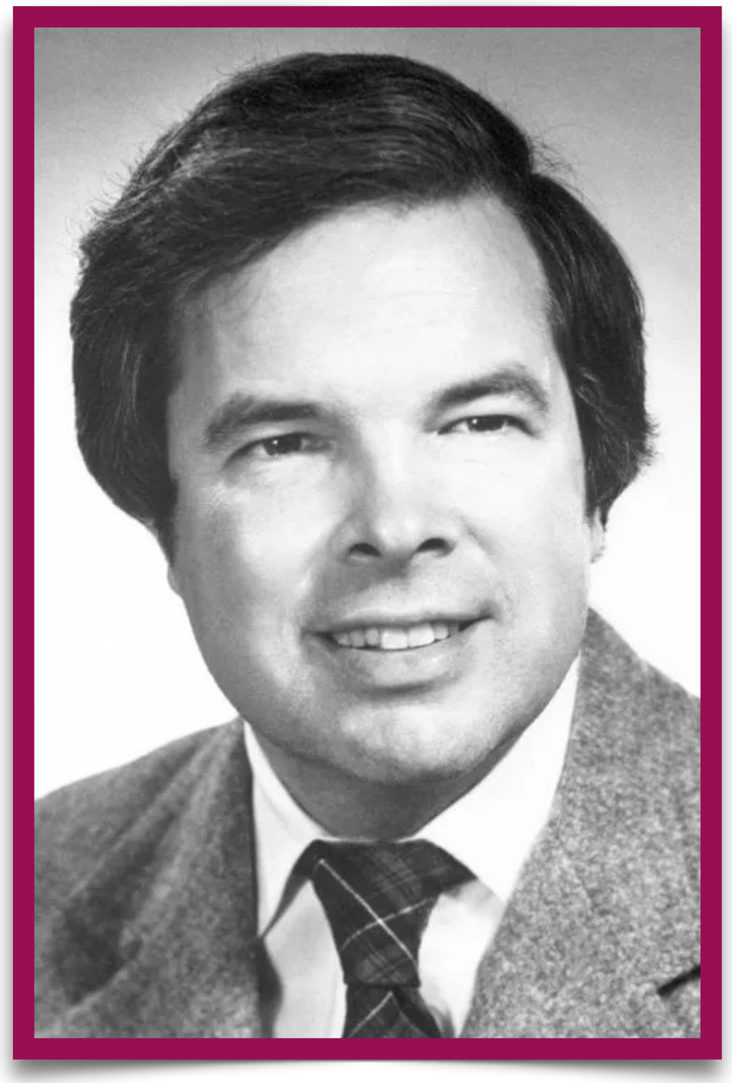

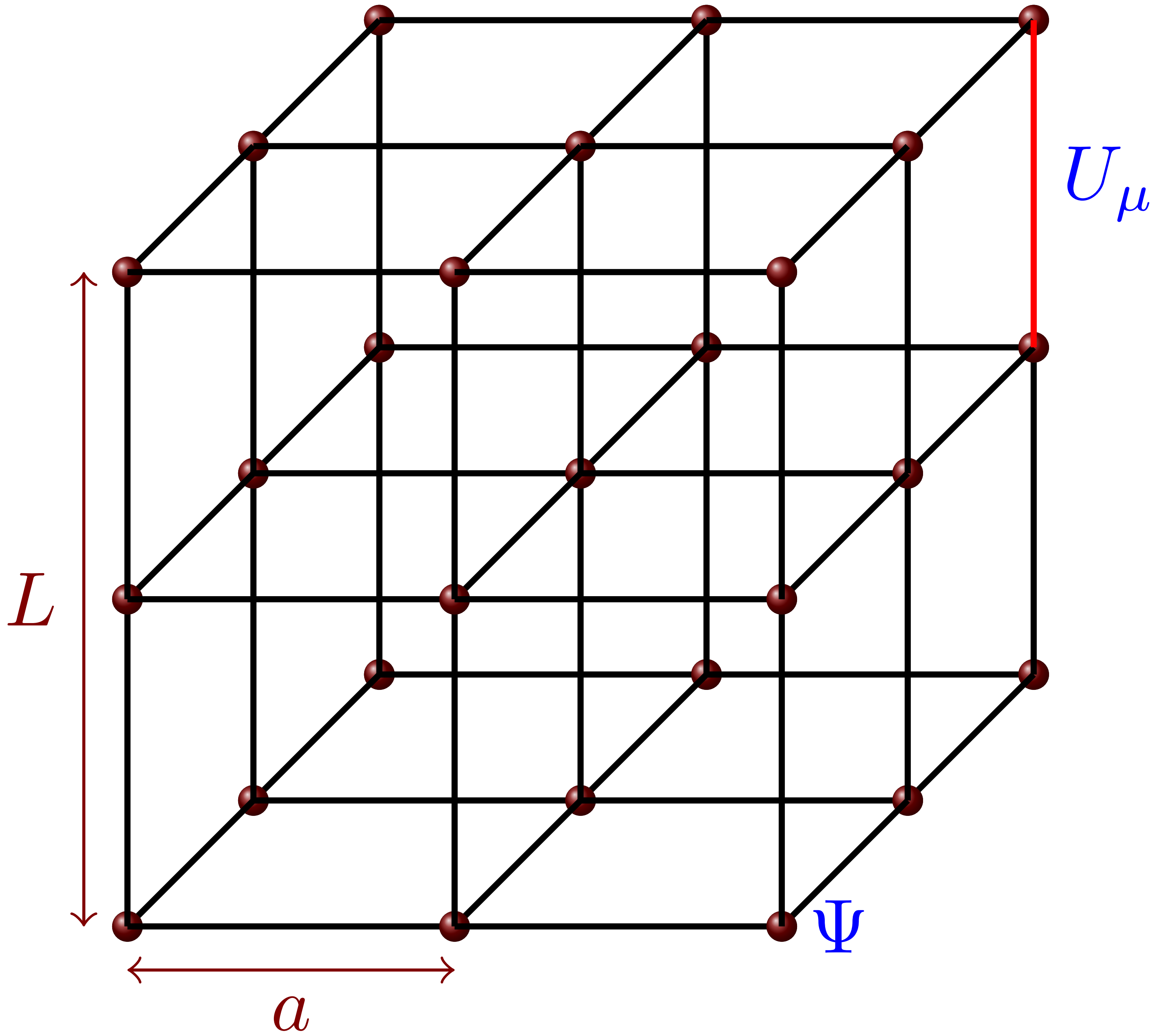

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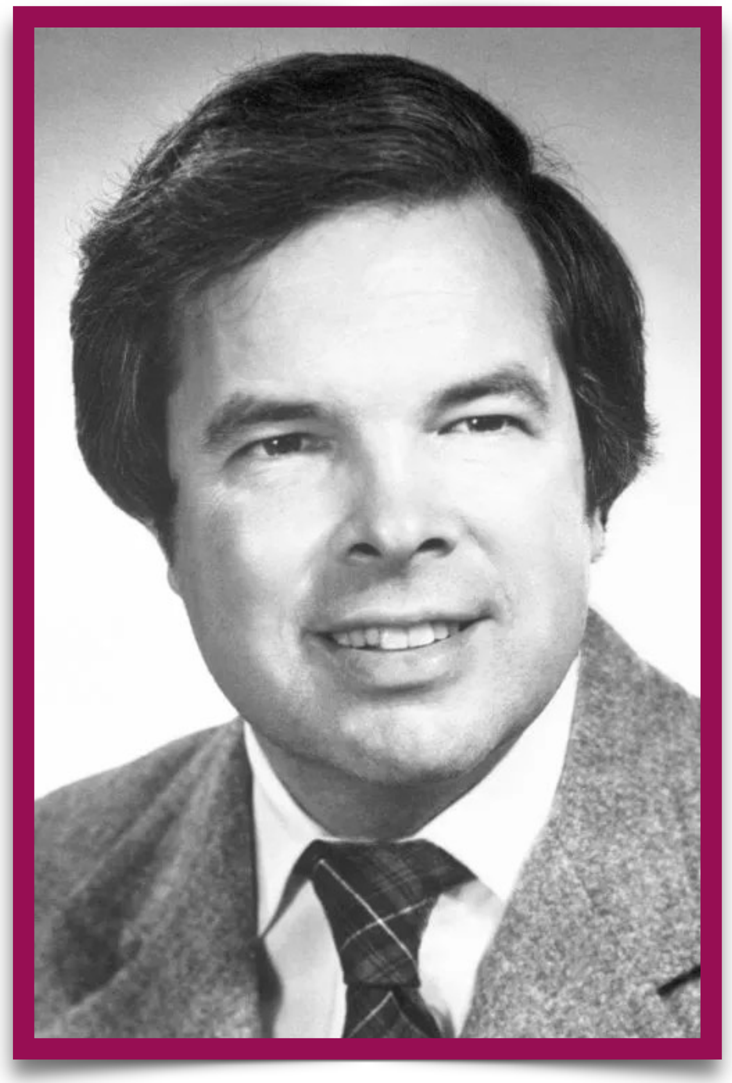

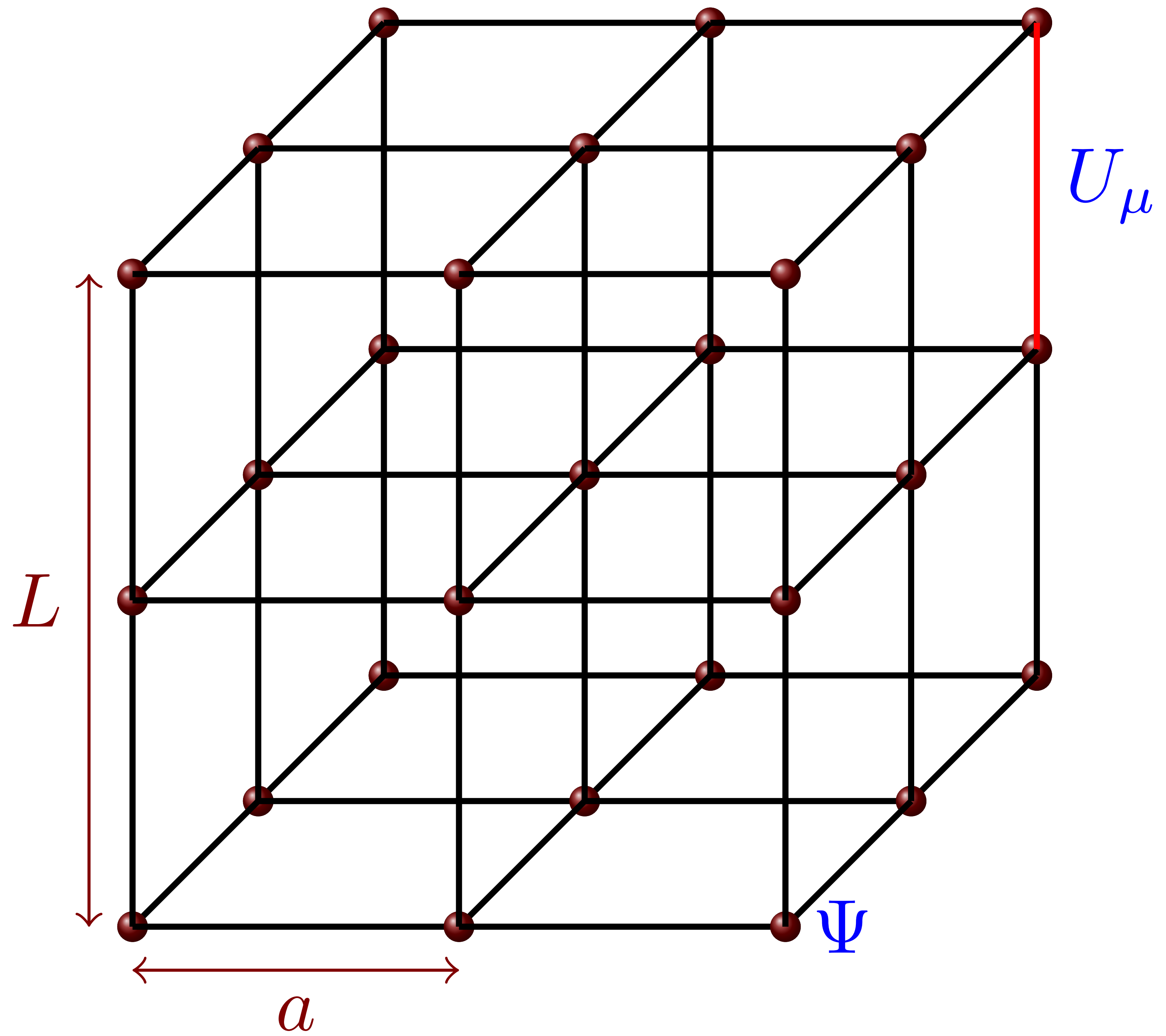

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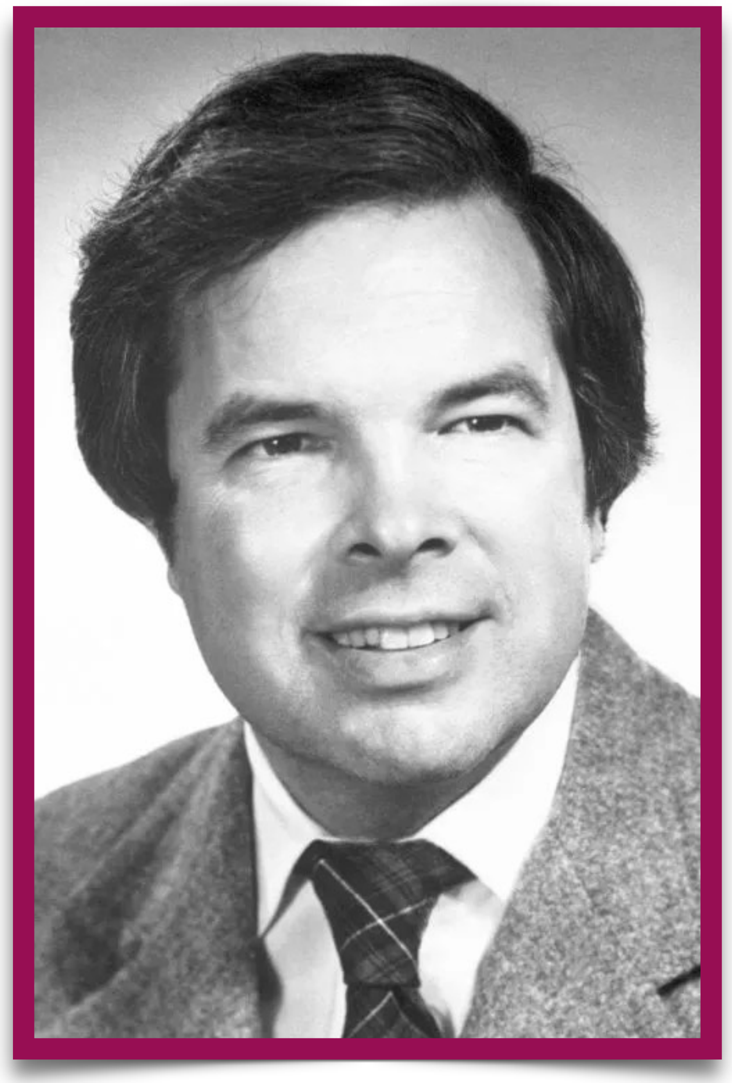

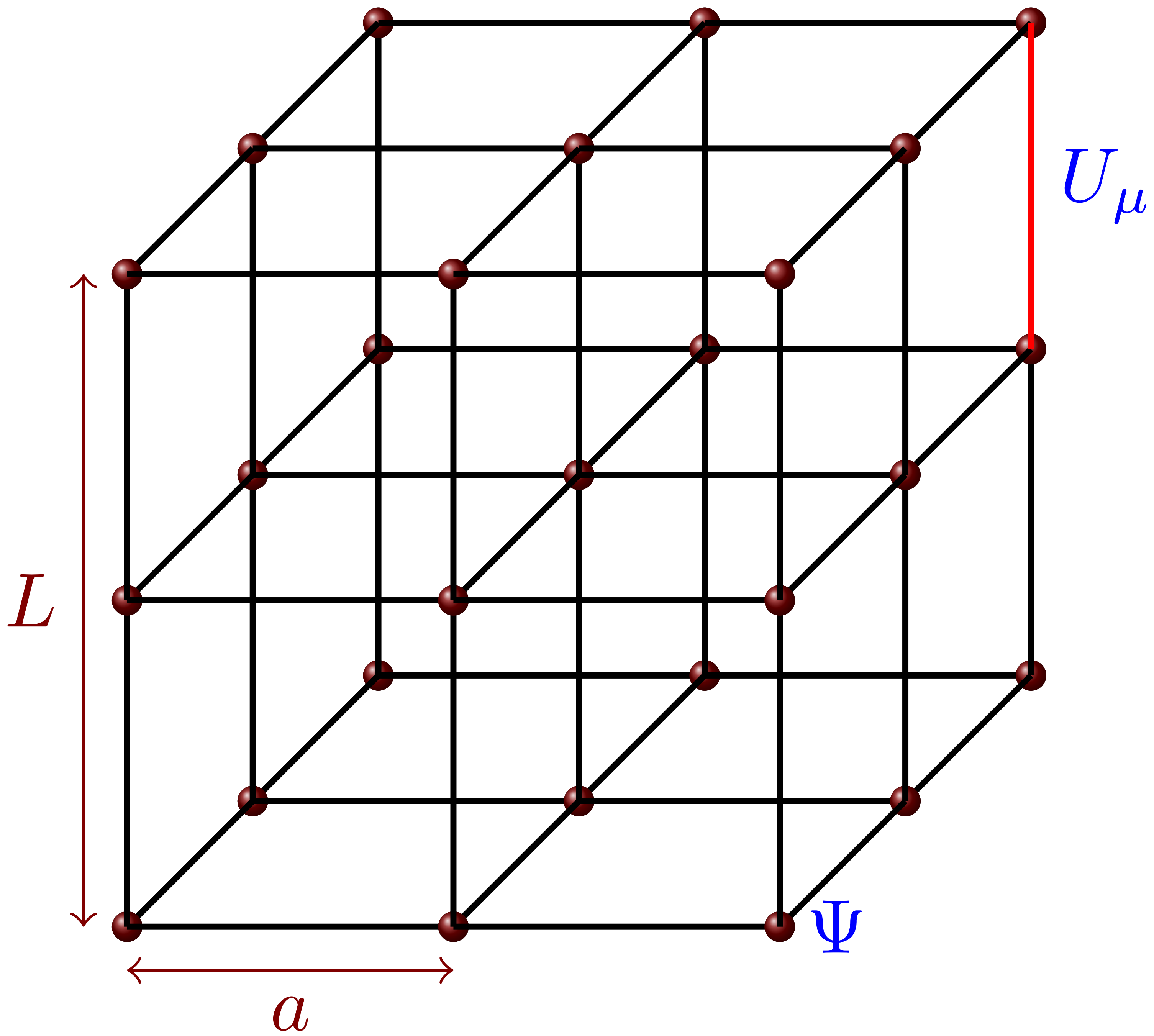

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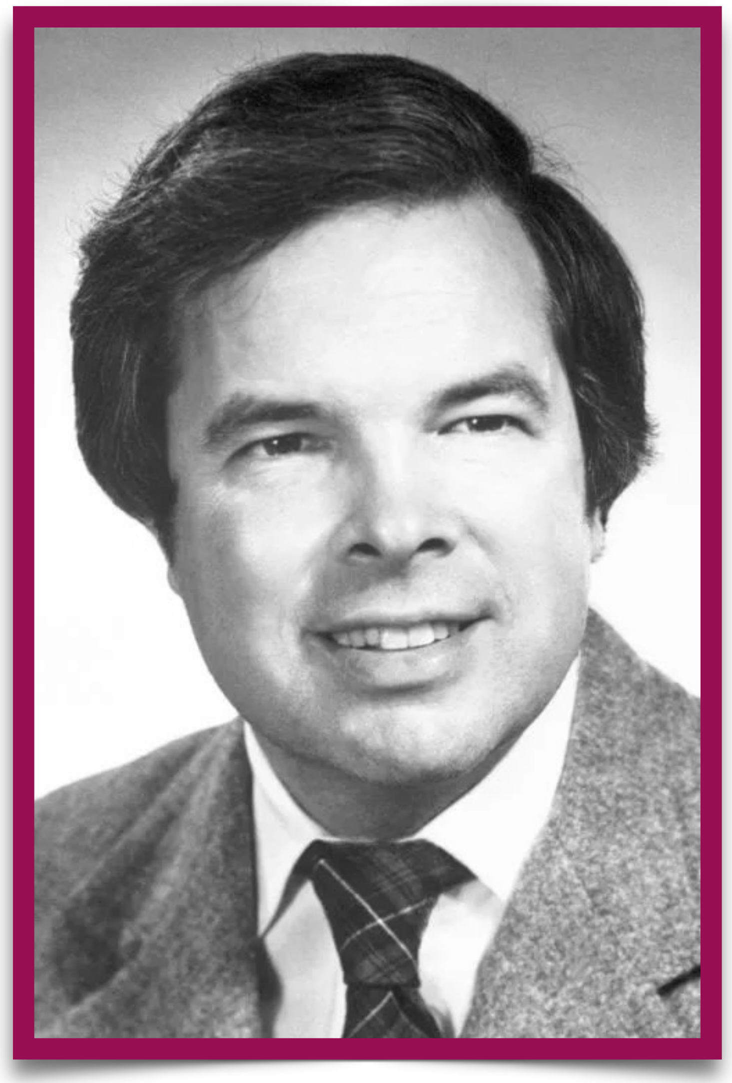

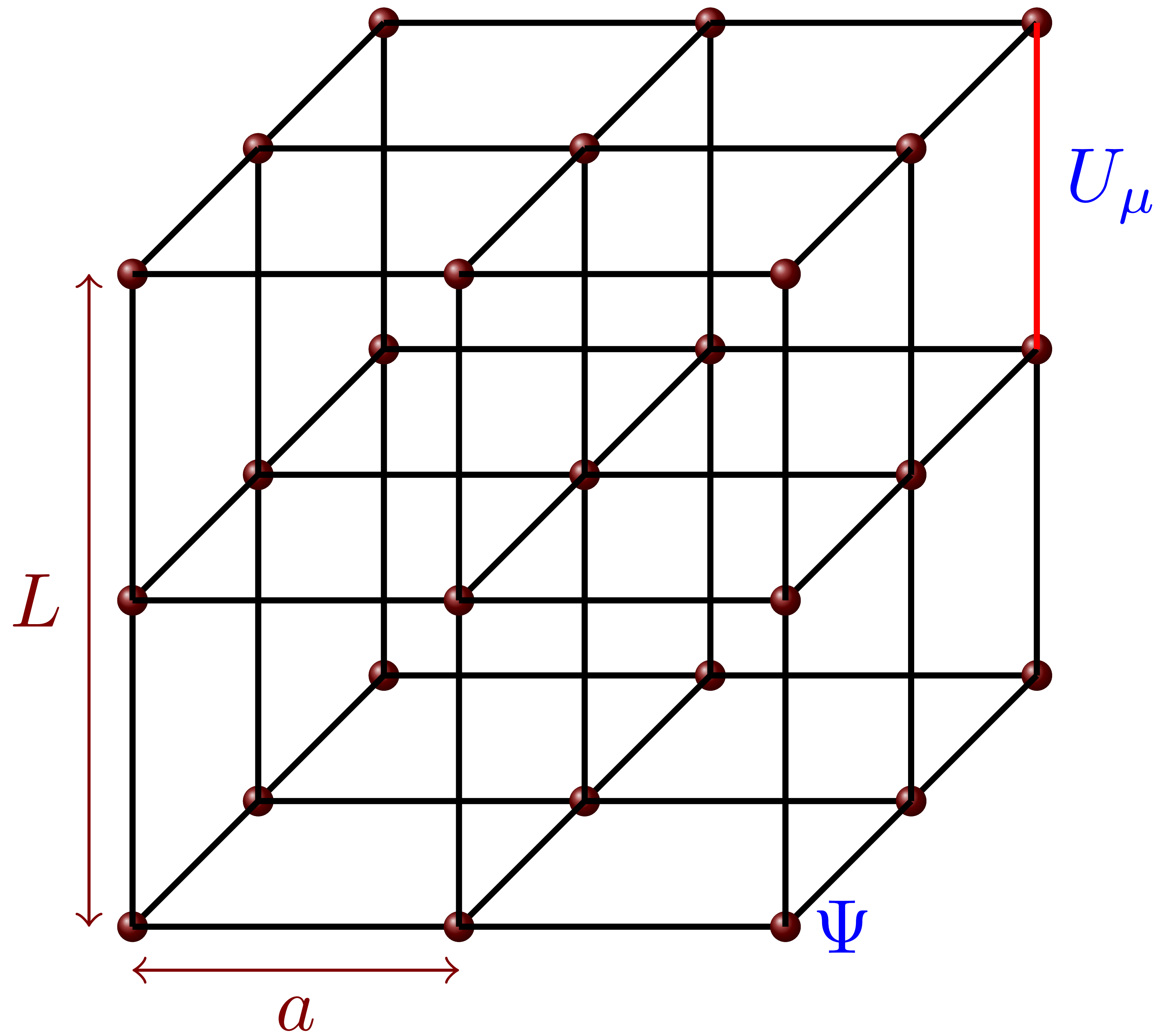

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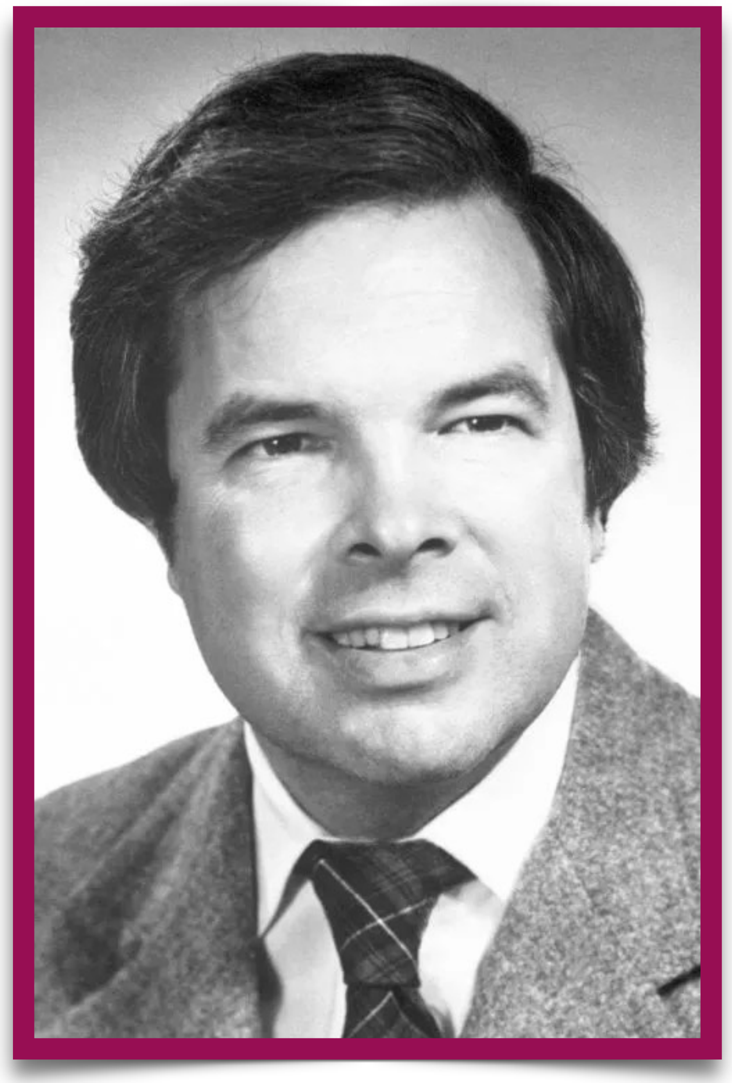

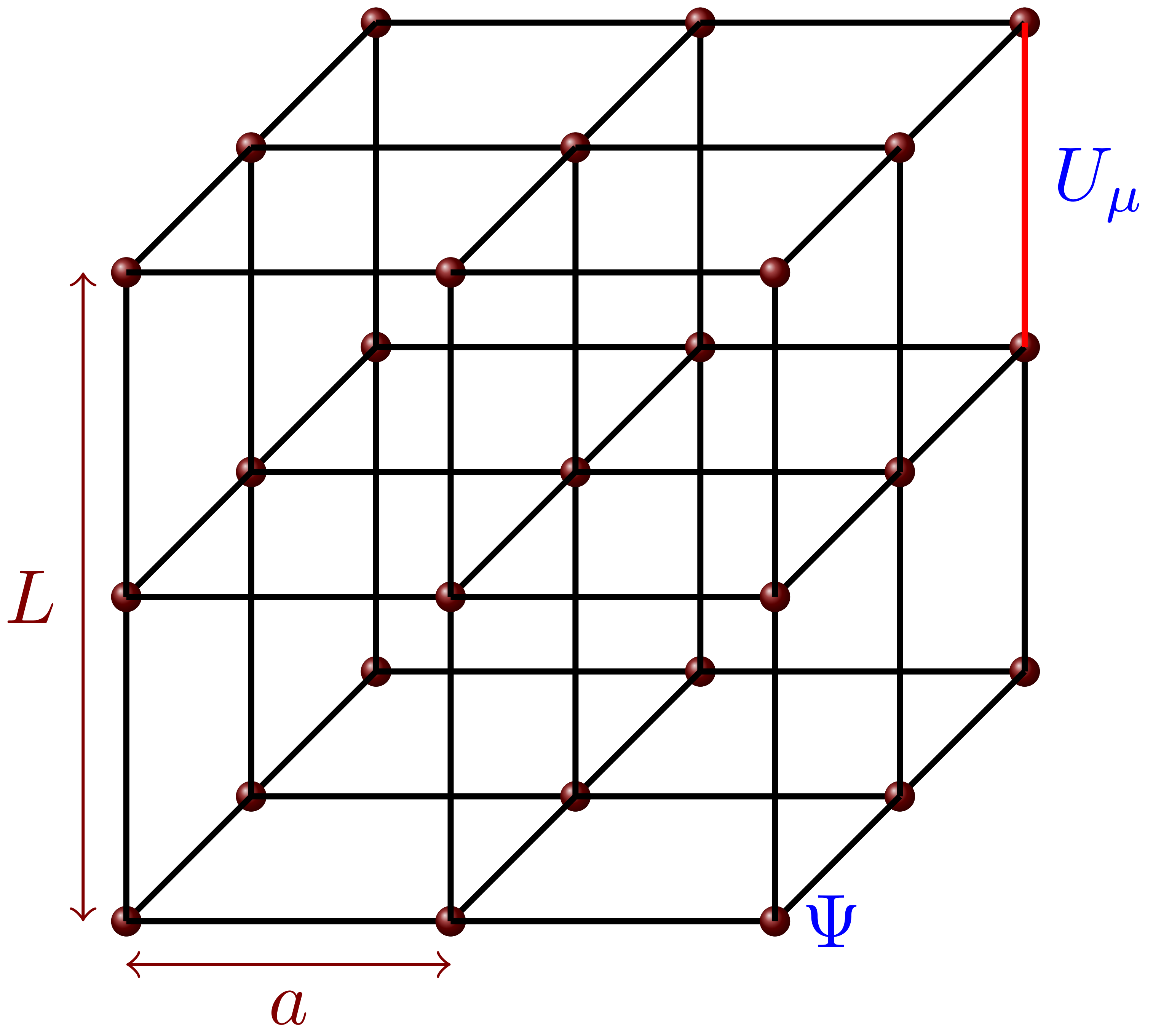

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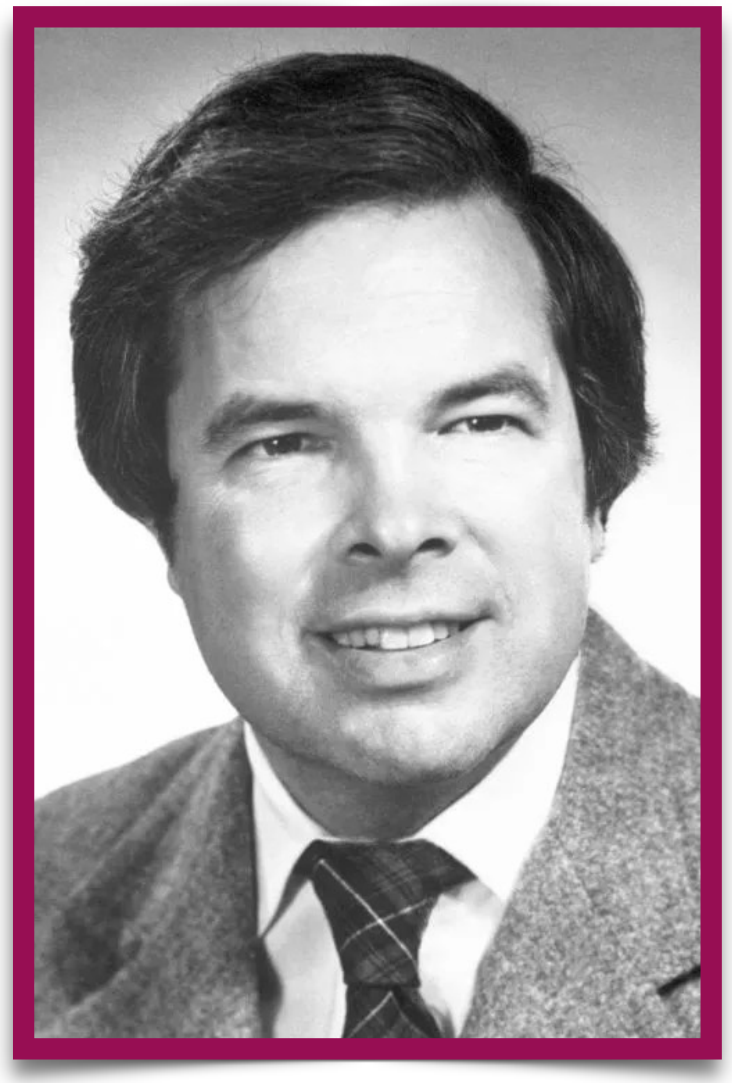

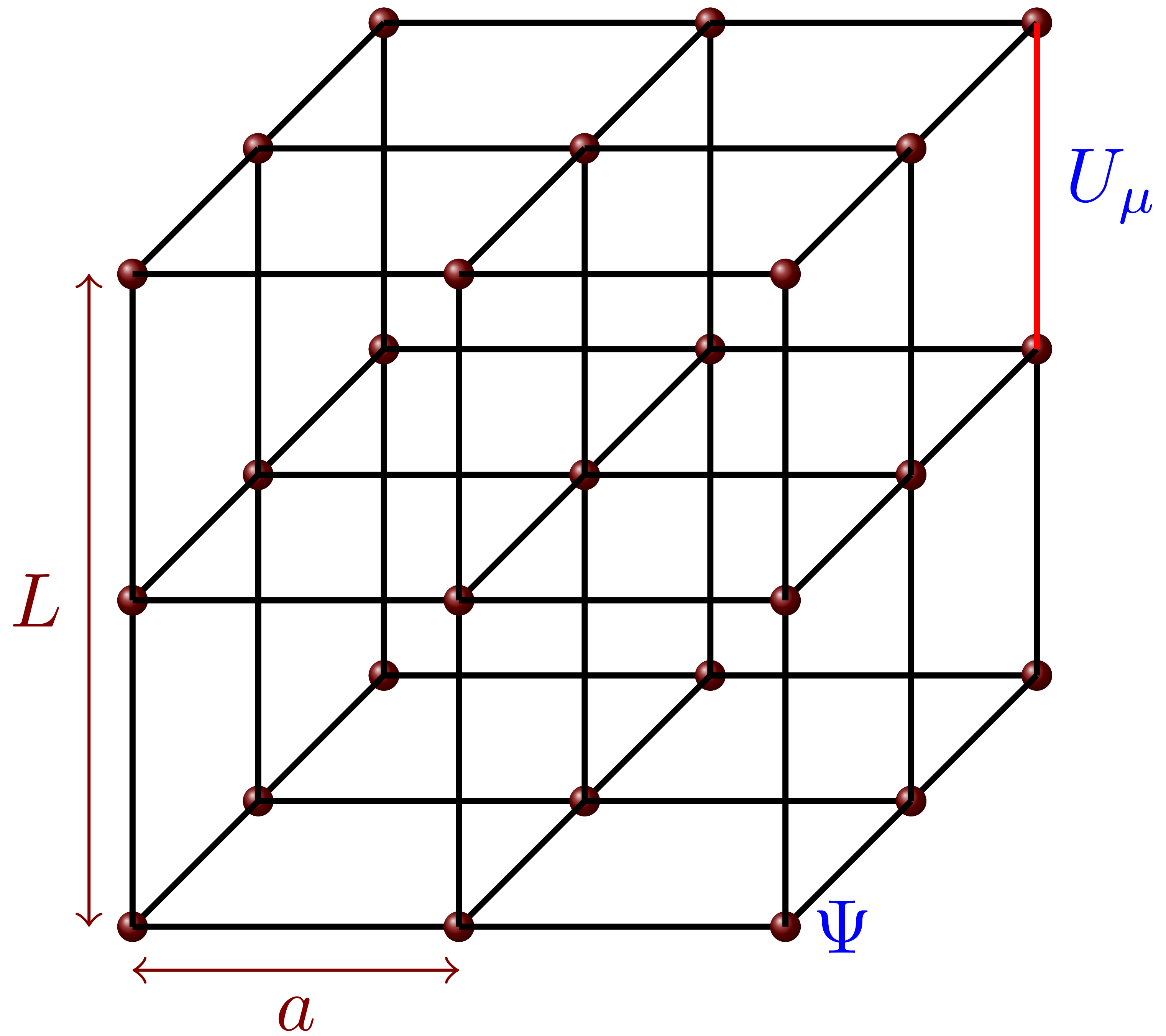

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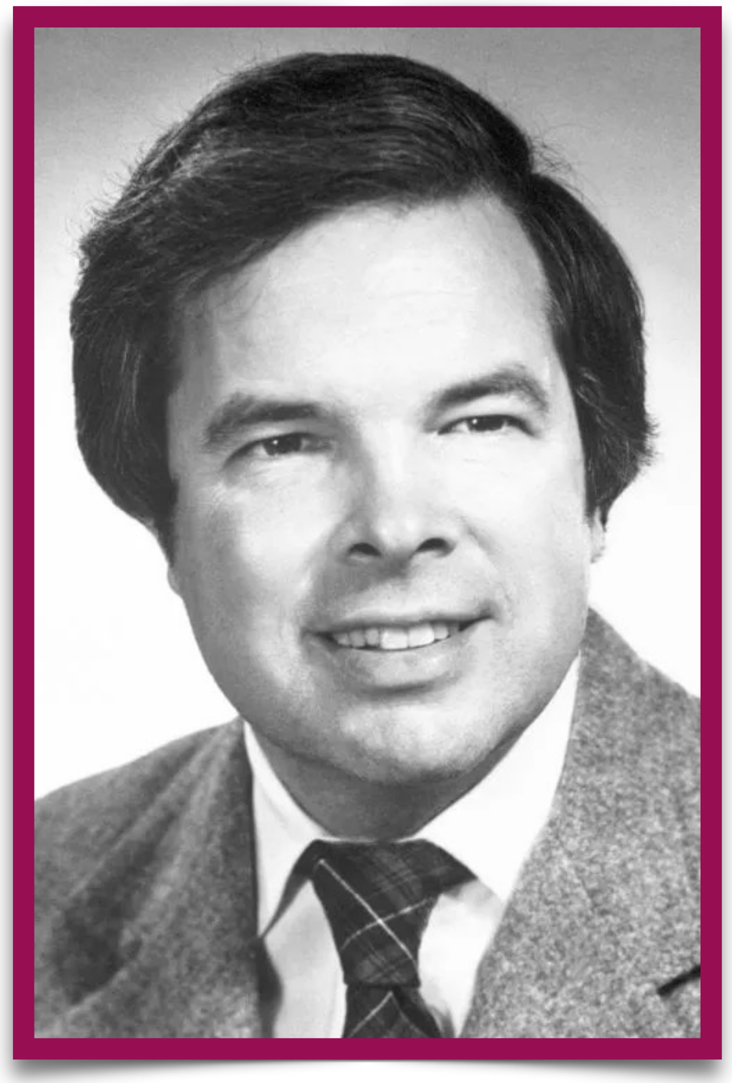

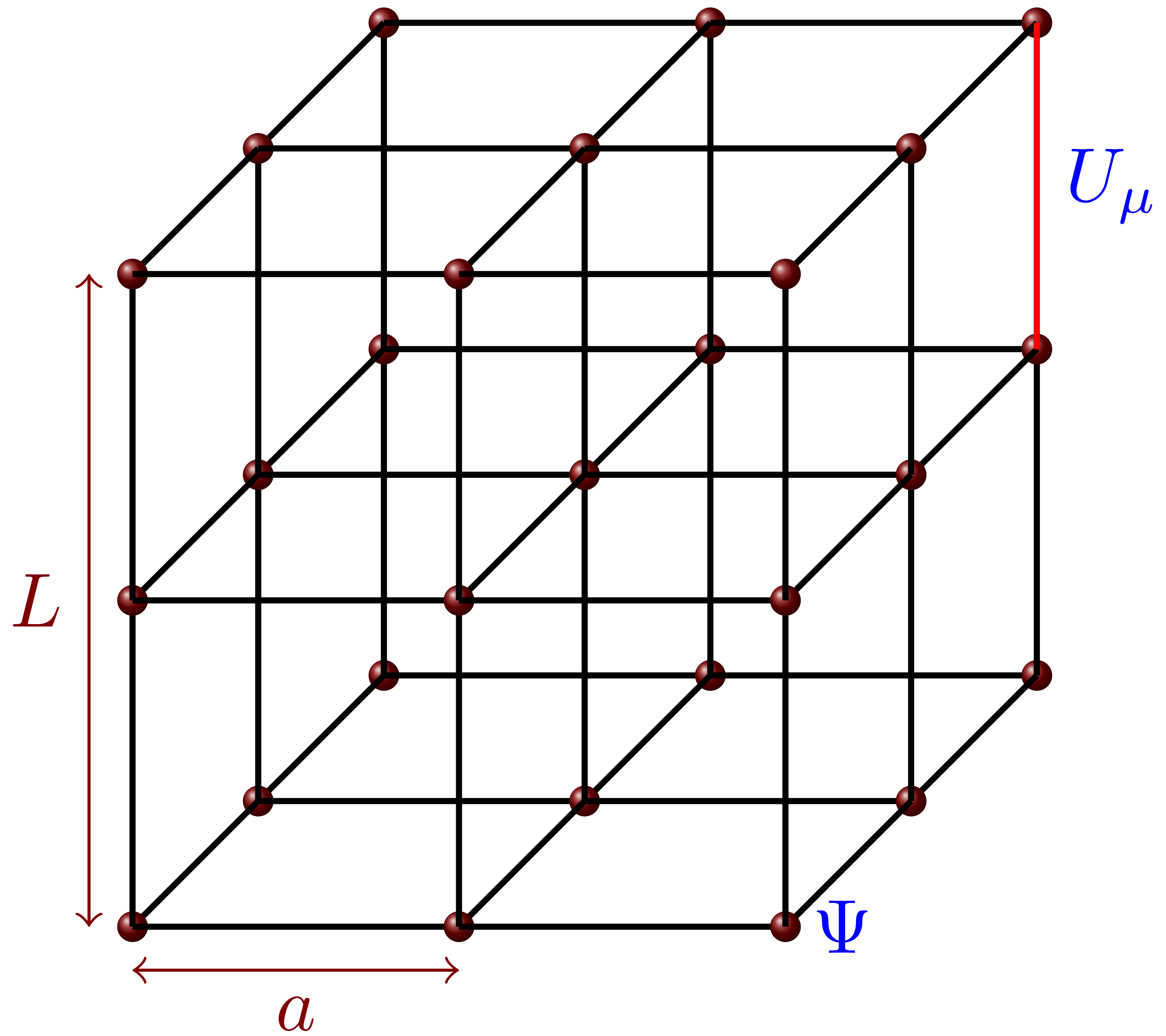

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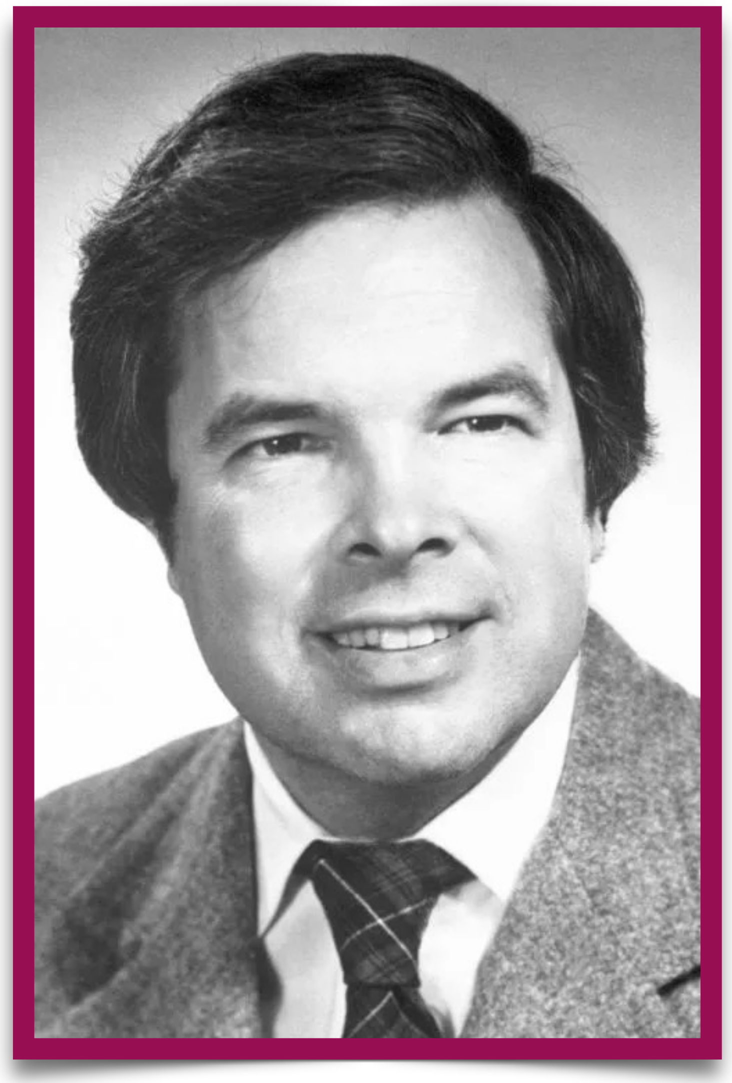
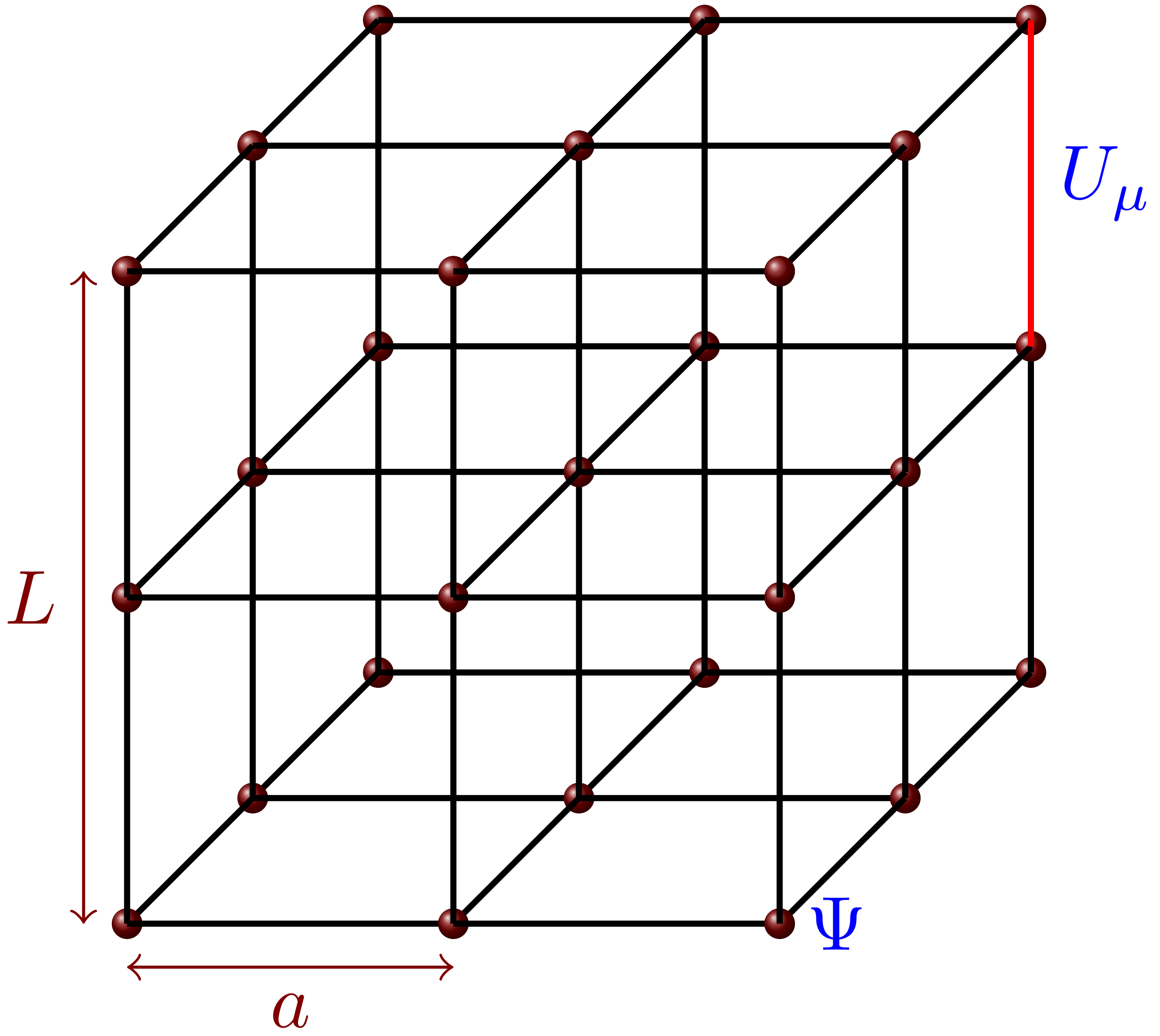
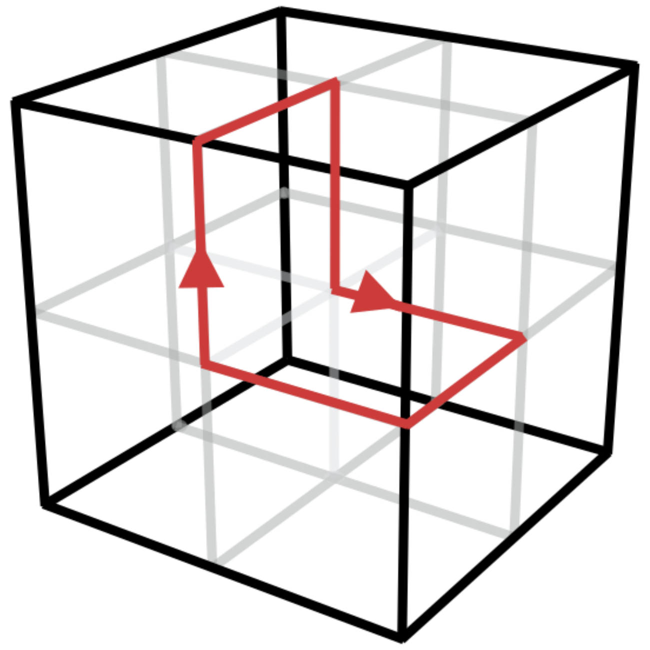
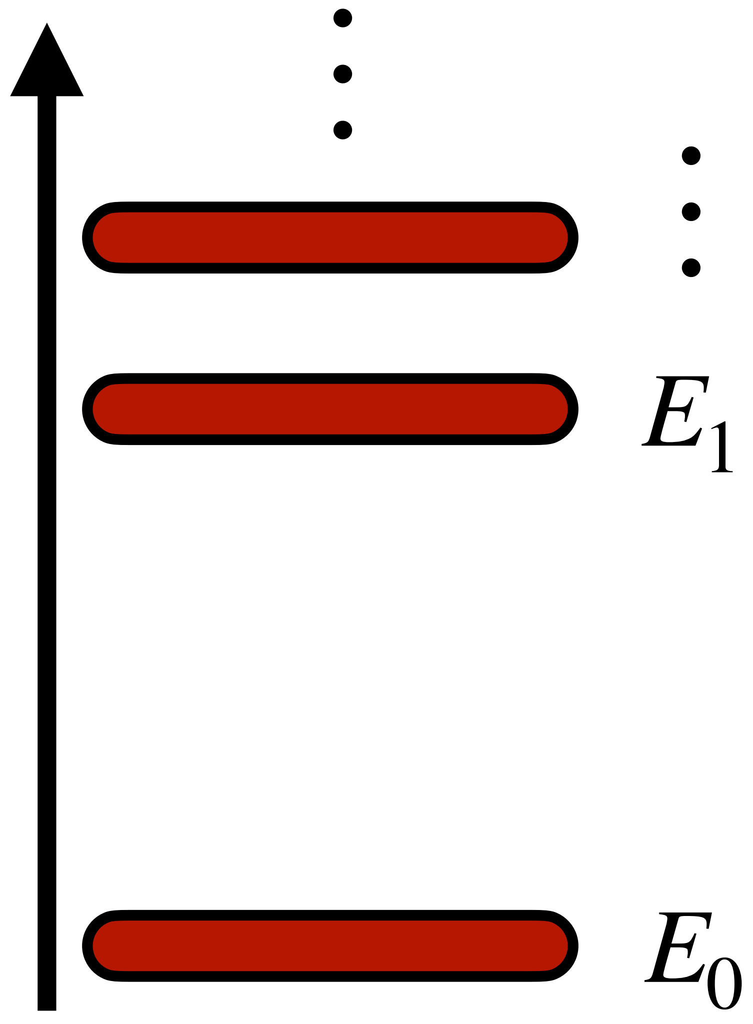




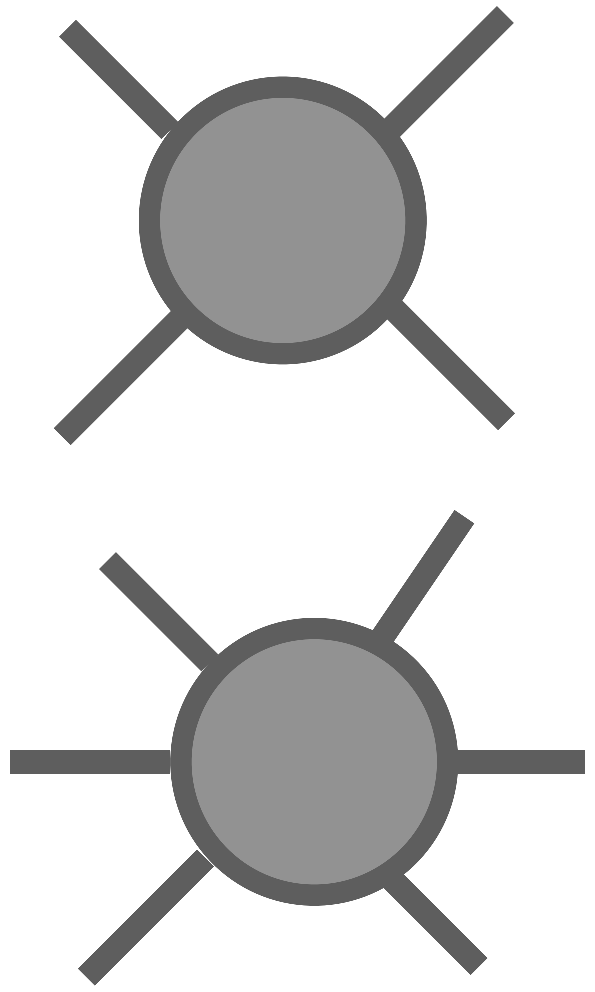
Hadron Spectroscopy in a Nutshell



Ensemble Generation



Finite Volume Spectra



Scattering Amplitude

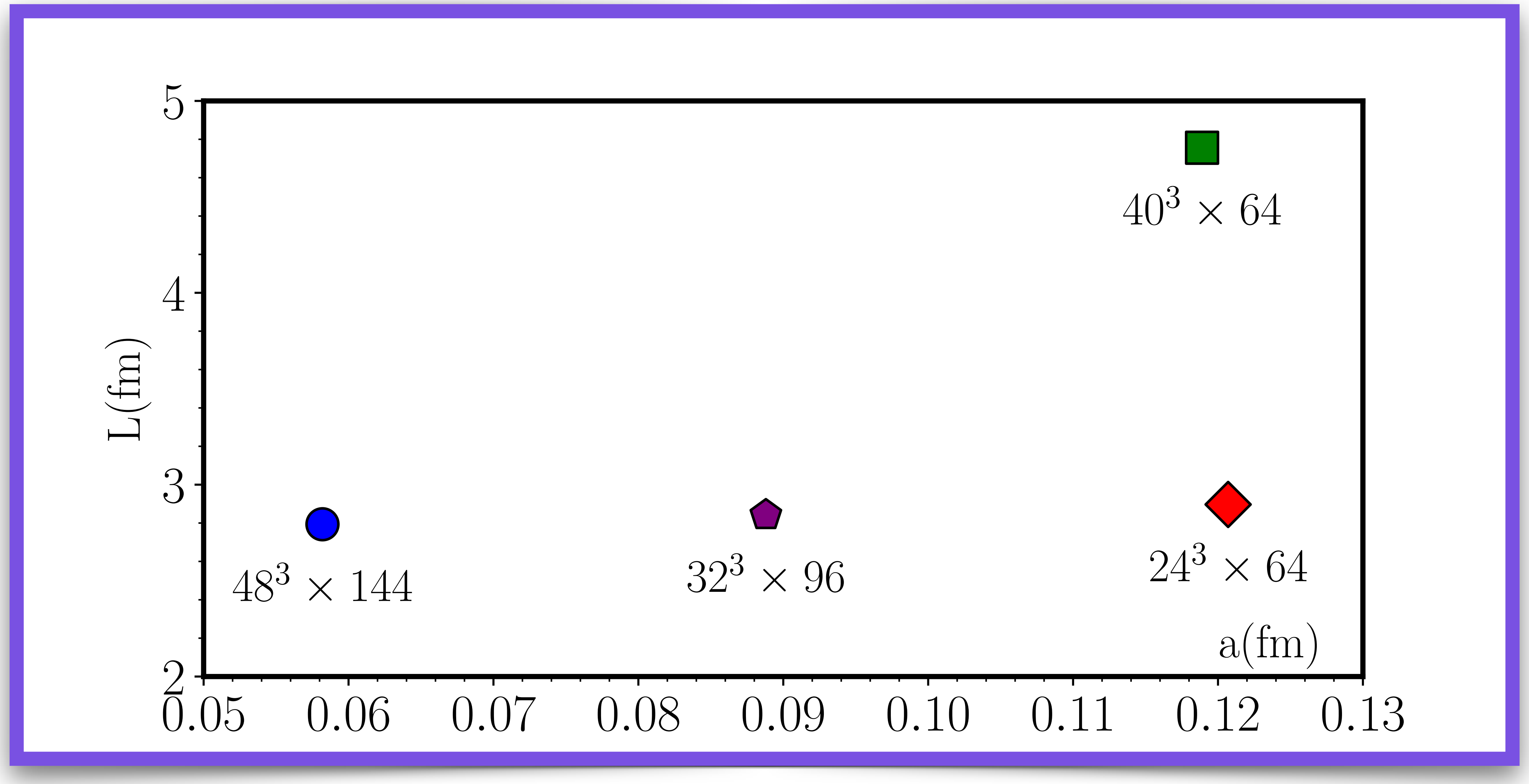
$a \to 0$

$L \to \infty$

$M_{ps} \to M_{\pi}^{phy}$

Continuum Extrapolation

Lattice Setup



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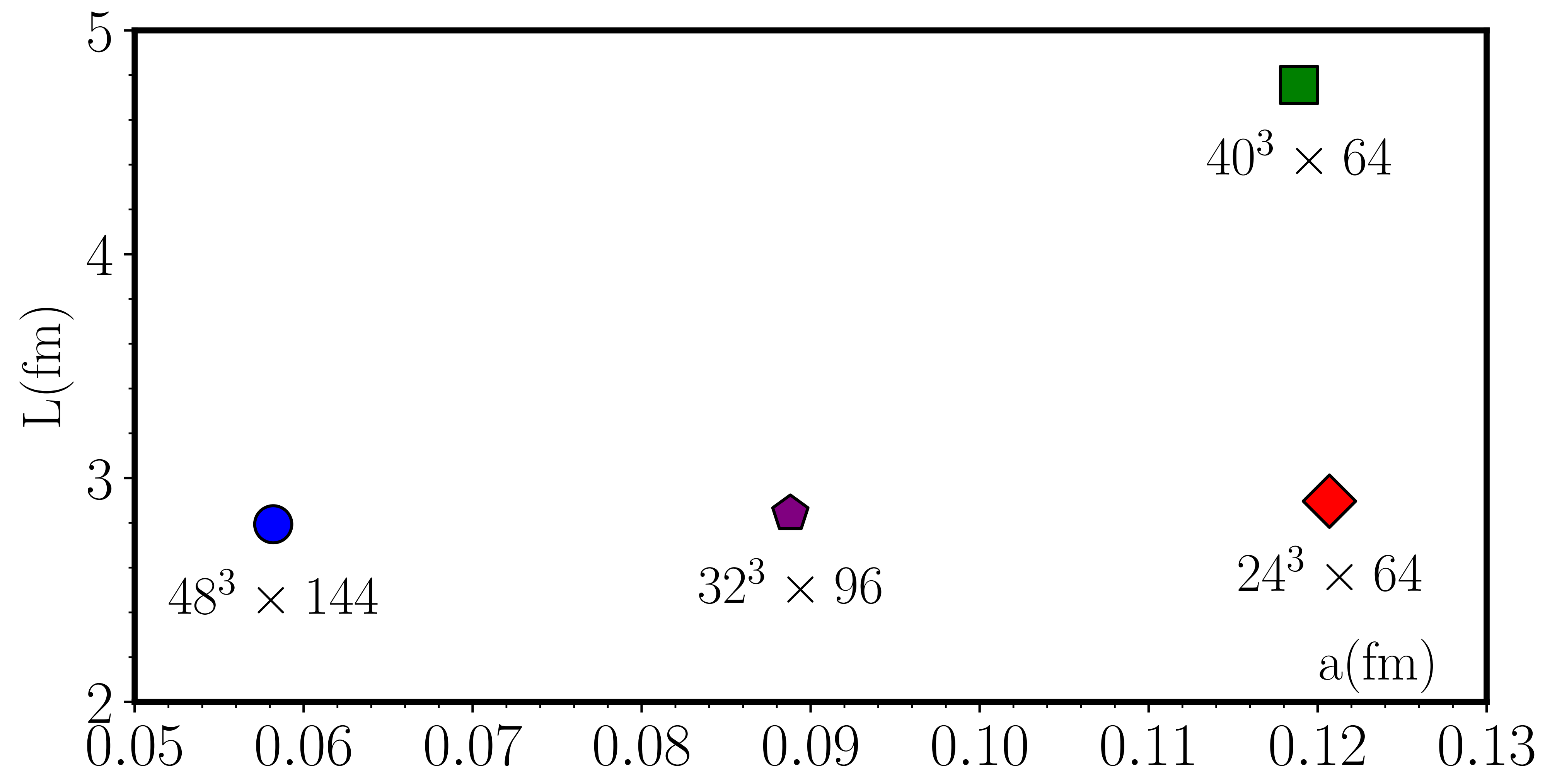

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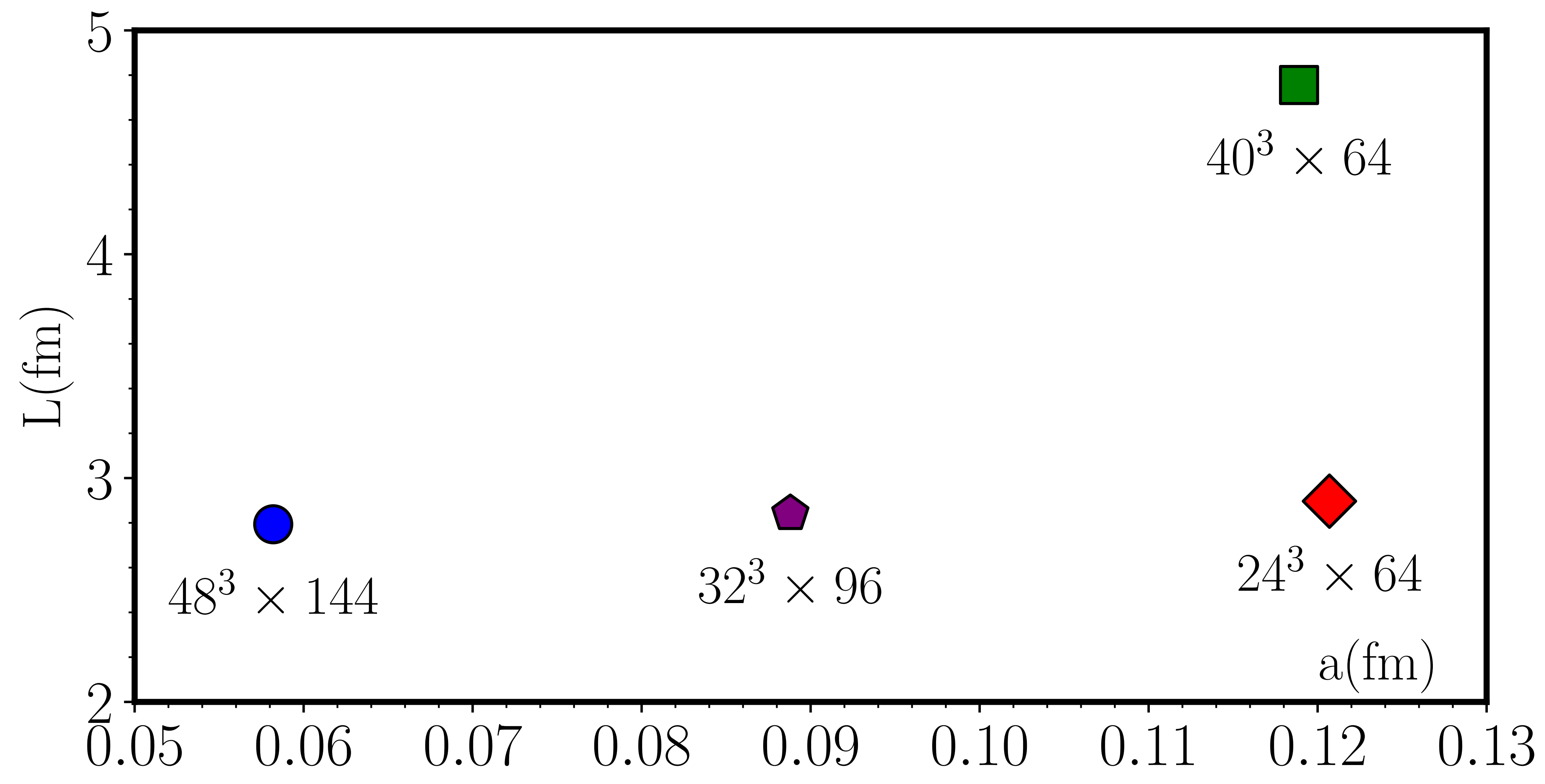

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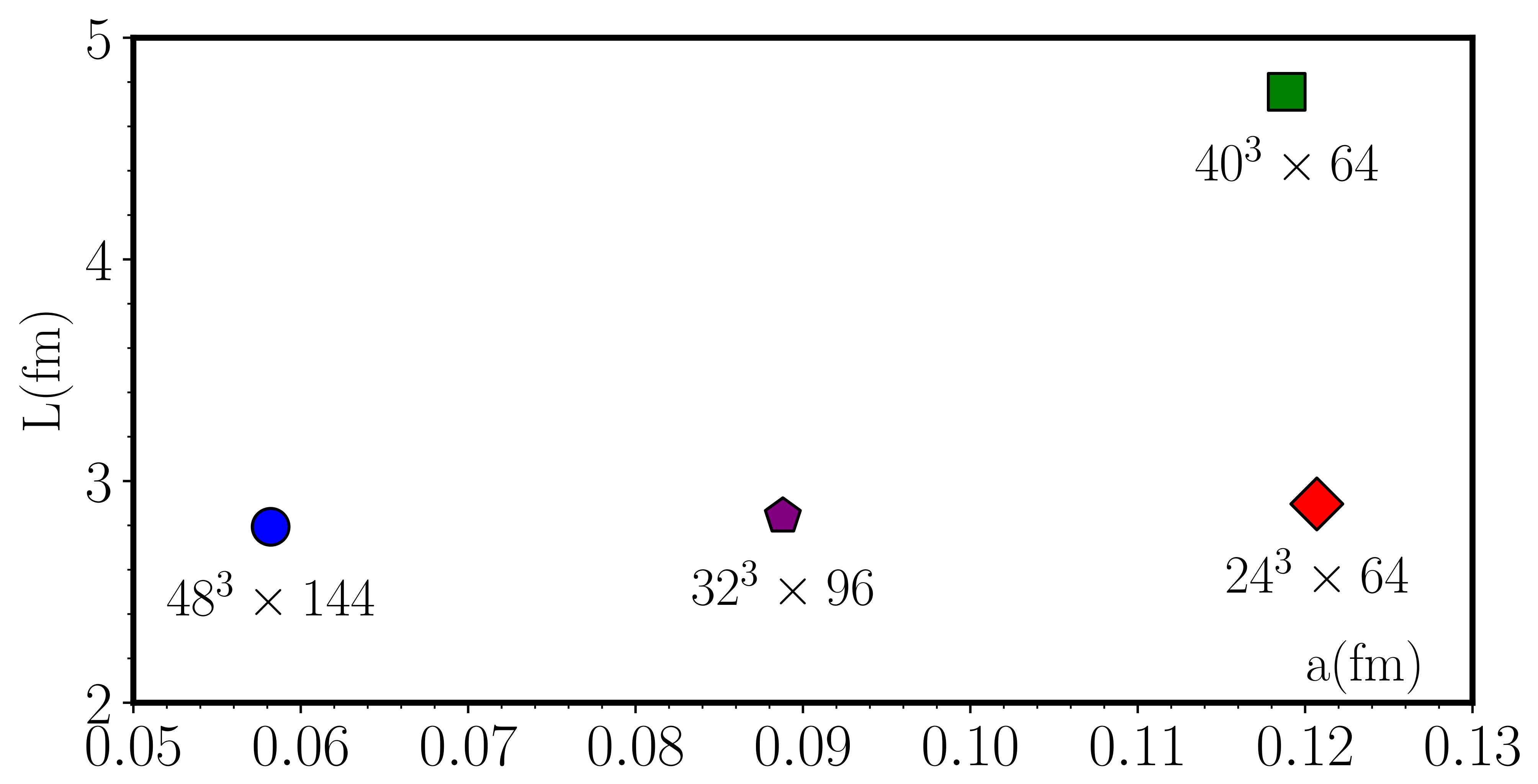

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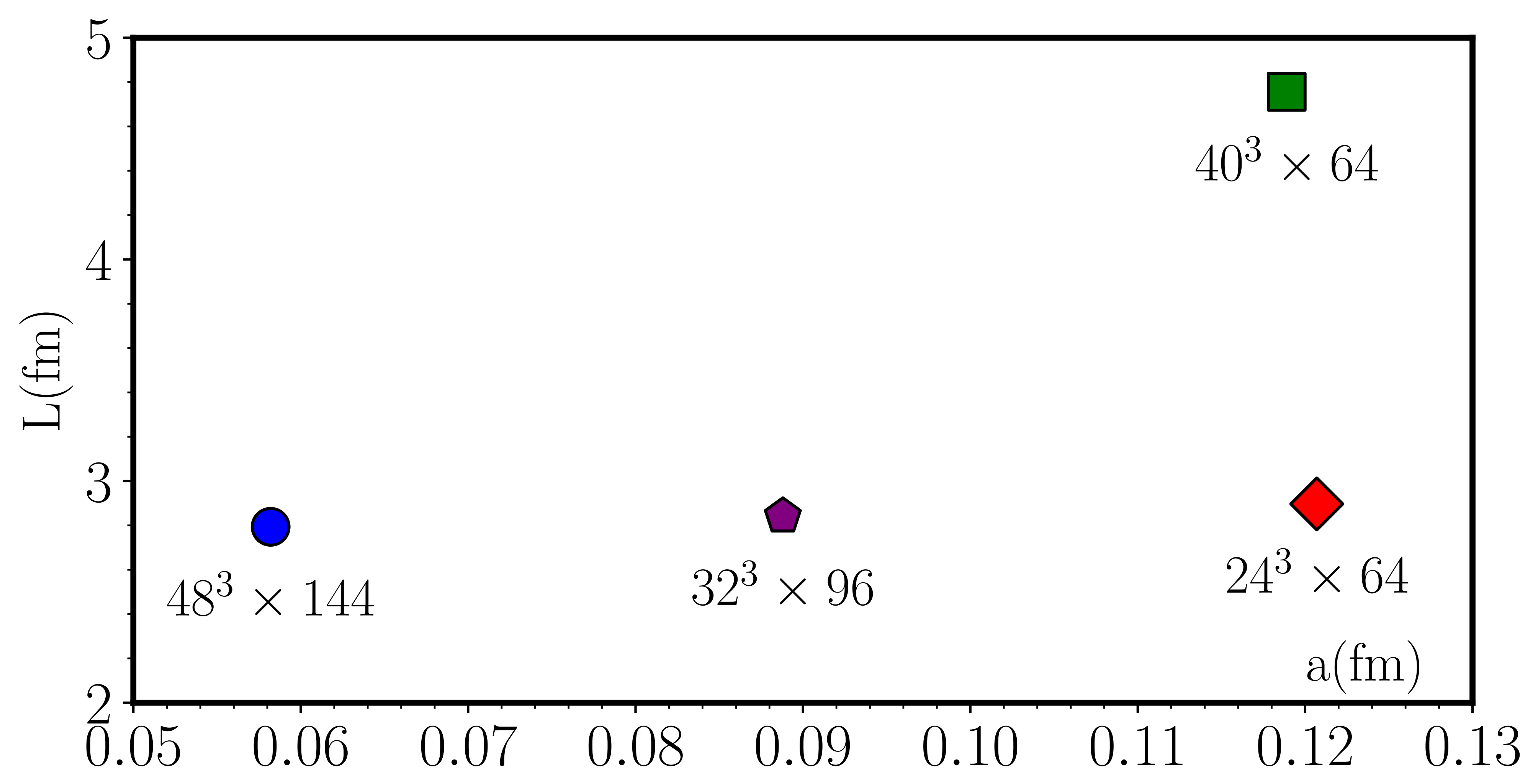

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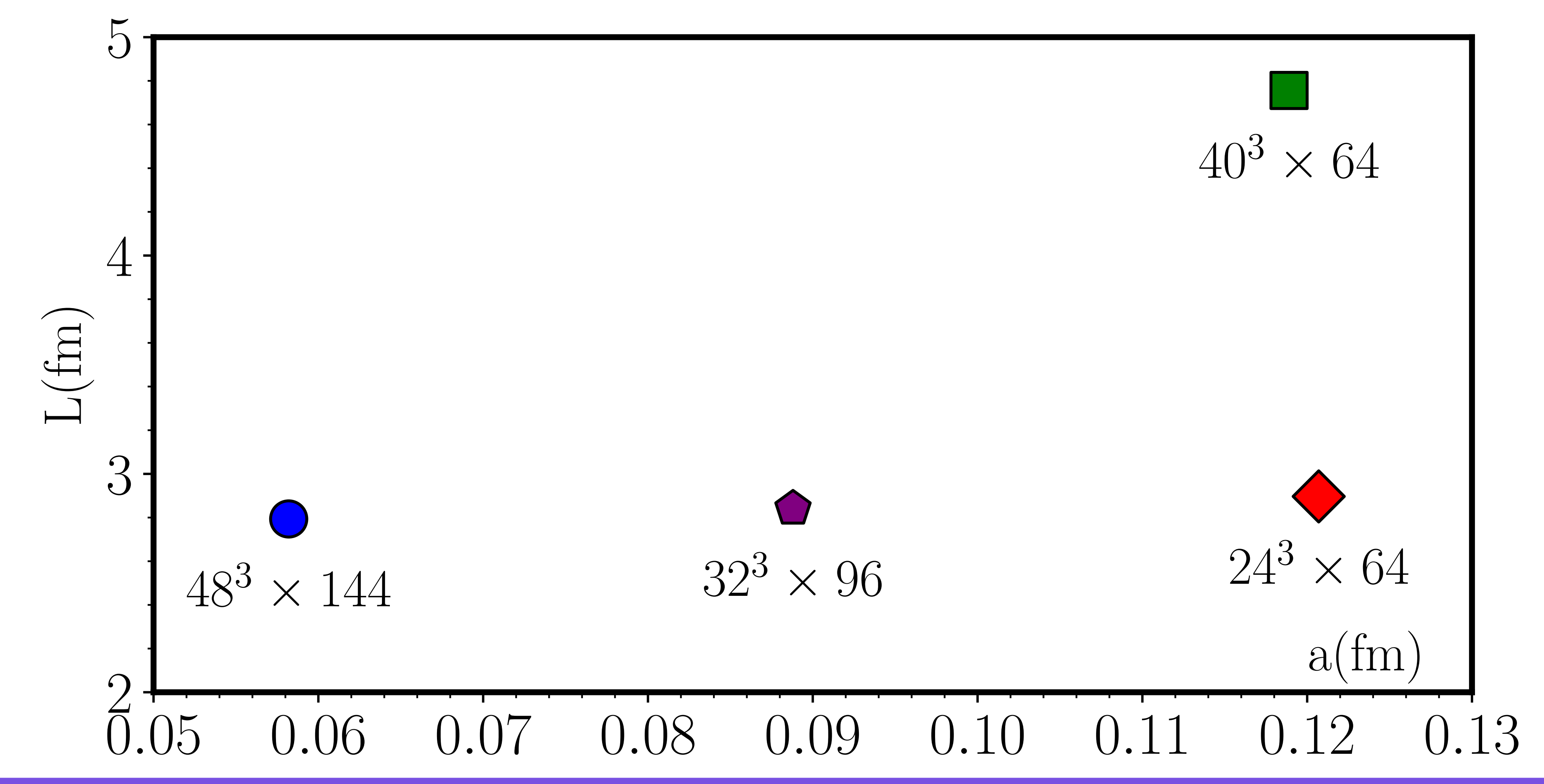

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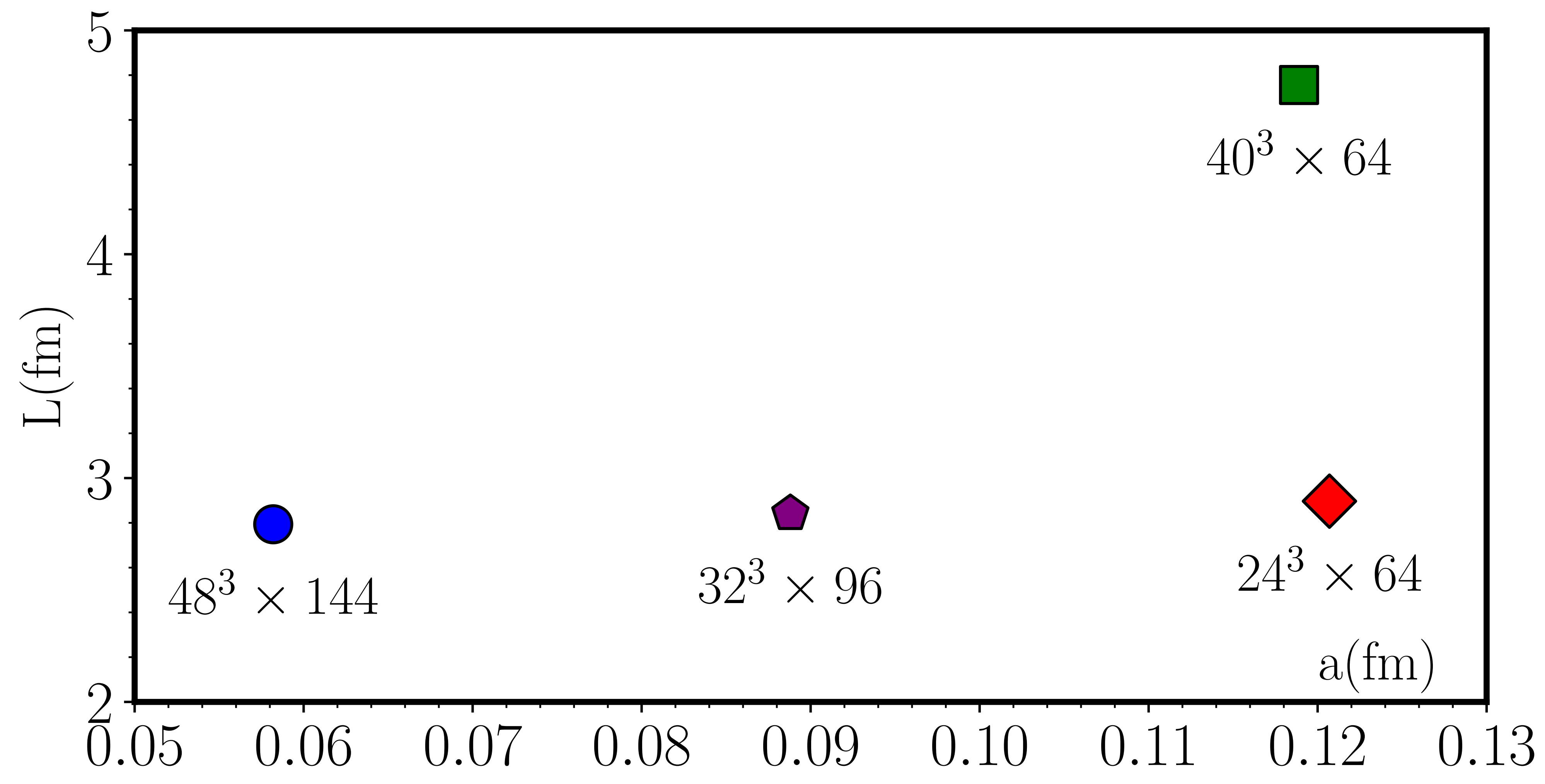

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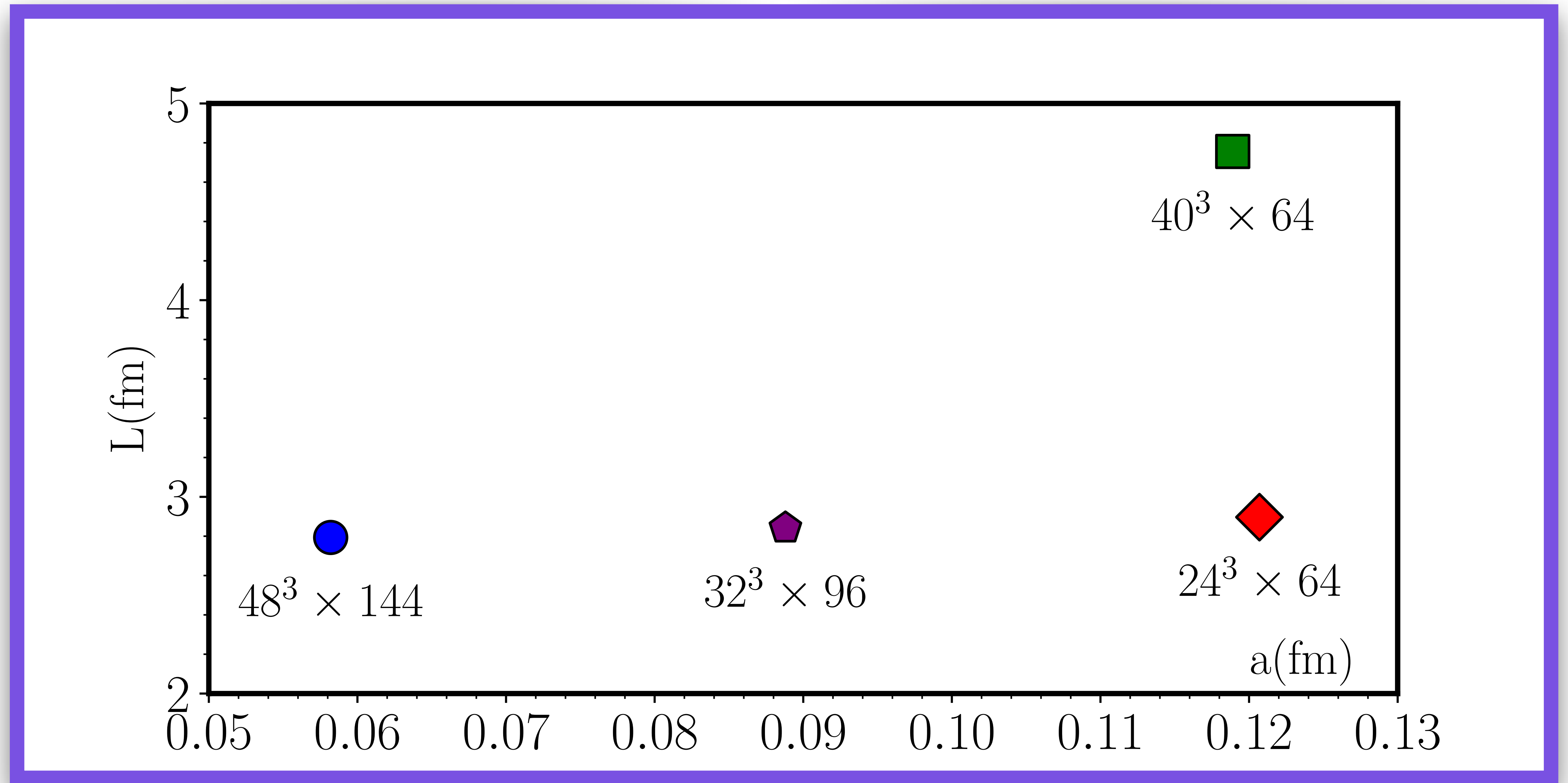

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- Offset correction accounted at the time of Analysis.

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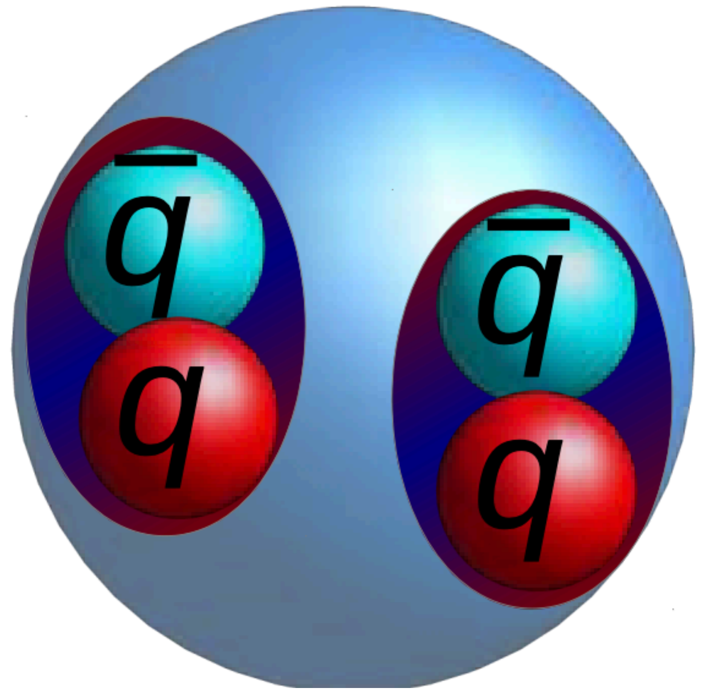
Pseudo scalar

Extracting Finite Volume Spectrum

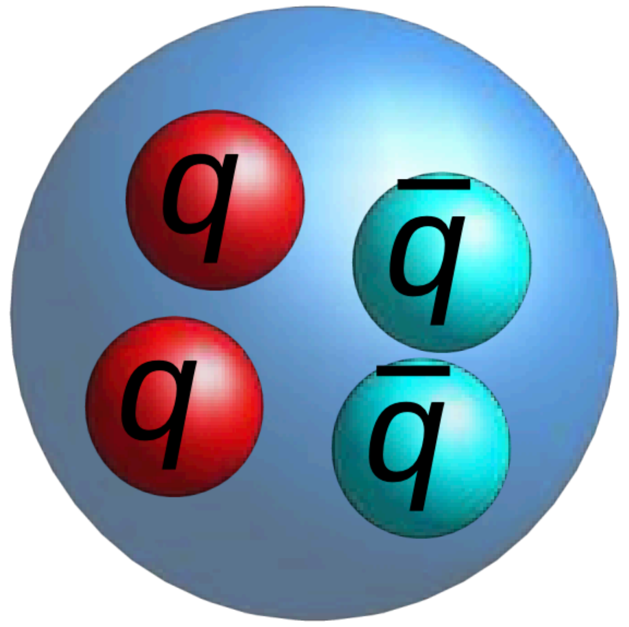
- To extract spectrum we need good interpolating operators.

- Here we are using two types of operators.



Meson-Meson :

$$\Phi_{\mathcal{M}_{BB^*}}(x) = [\bar{u}(x)\gamma_i b(x)][\bar{d}(x)\gamma_5 b(x)] - [\bar{u}(x)\gamma_5 b(x)][\bar{d}(x)\gamma_i b(x)]$$



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$$\Phi_{\mathcal{D}}(x) = [((\bar{u}(x)^T C\gamma_5 \bar{d}(x)) - (\bar{d}(x)^T C\gamma_5 \bar{u}(x))) \times (b^T(x) C\gamma_i b(x))]$$

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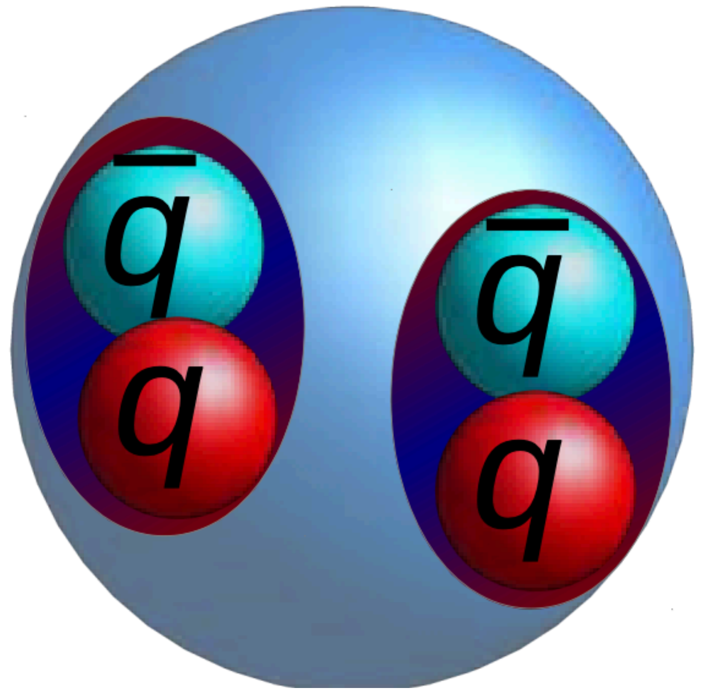

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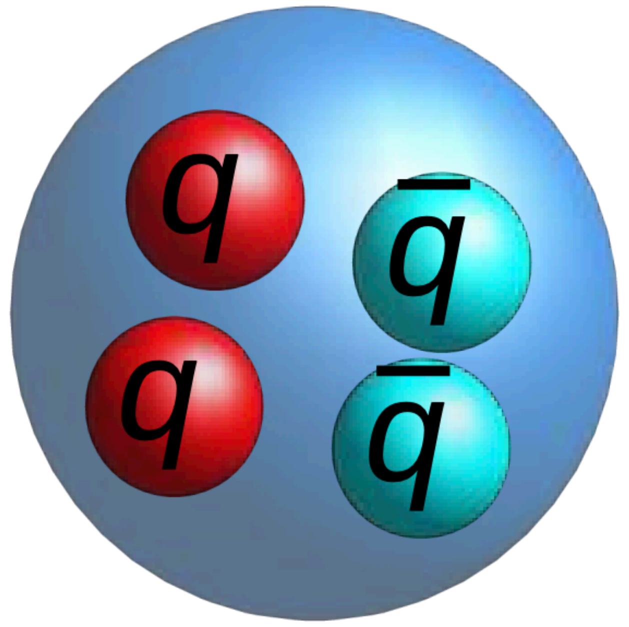

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Operators for T_{bs}

- For axial-vector $I(J^P) = 0(1^+)$, we use three operators,

$$\Phi_{\mathscr{M}_{KB^*}}(x) = \left[\bar{u}(x)\gamma_i b(x)\right]\left[\bar{d}(x)\gamma_5 s(x)\right] - \left[\bar{u}(x)\gamma_5 s(x)\right]\left[\bar{d}(x)\gamma_i b(x)\right]$$

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- For scalar $I(J^P) = 0(0^+)$, we use two operators,

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Wall Sources Point Sink

- Instead of using a single point source, unique source is placed at every spatial point on the source time slice.

$$Q(\bar{x}, t; t') = \sum_{\bar{x}'} Q(\bar{x}, t; \bar{x}', t')$$

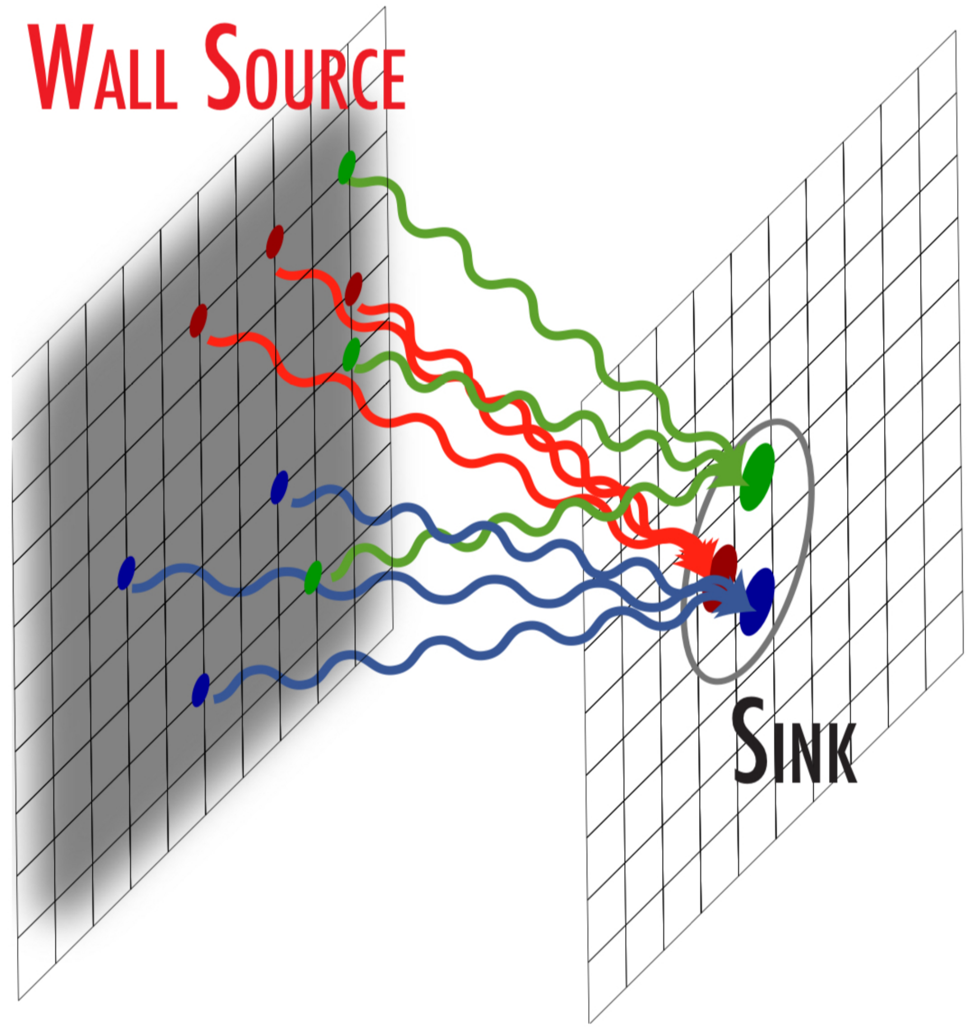

Image Credit:- S. Aoki

- ADVANTAGES:- Better signals in the ground state.
- DISADVANTAGE:-
  1. Asymmetric Correlation Function, Non-Hermitian GEVP Needed.
  2. False Plateau encounter, careful with fitting time window.
- Why not Wall source wall sink correlator? -> Very Noisy signal.

Wall source- Box Sink Correlator

- Instead of wall-sink, we build box-sink correlator.

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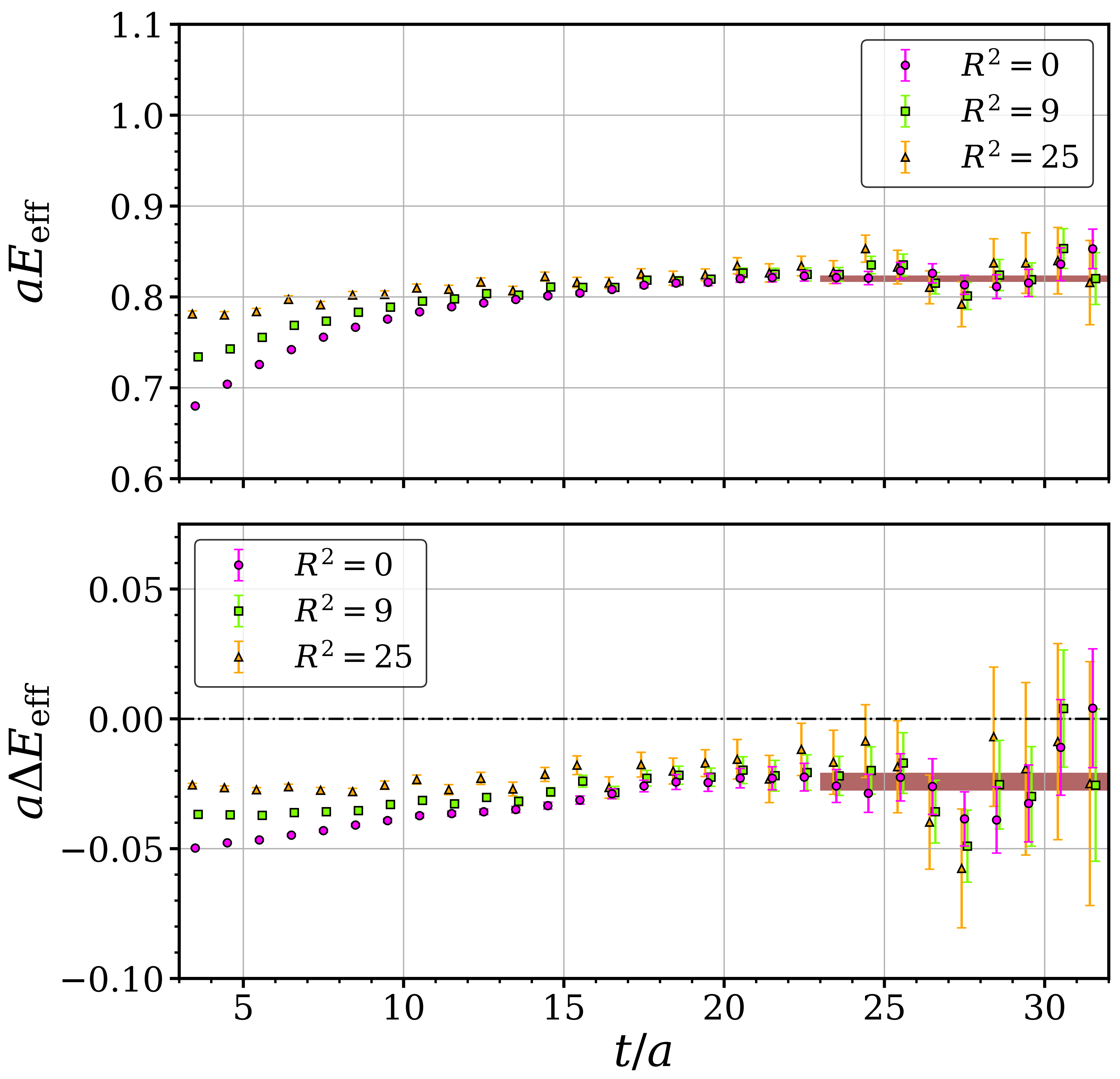
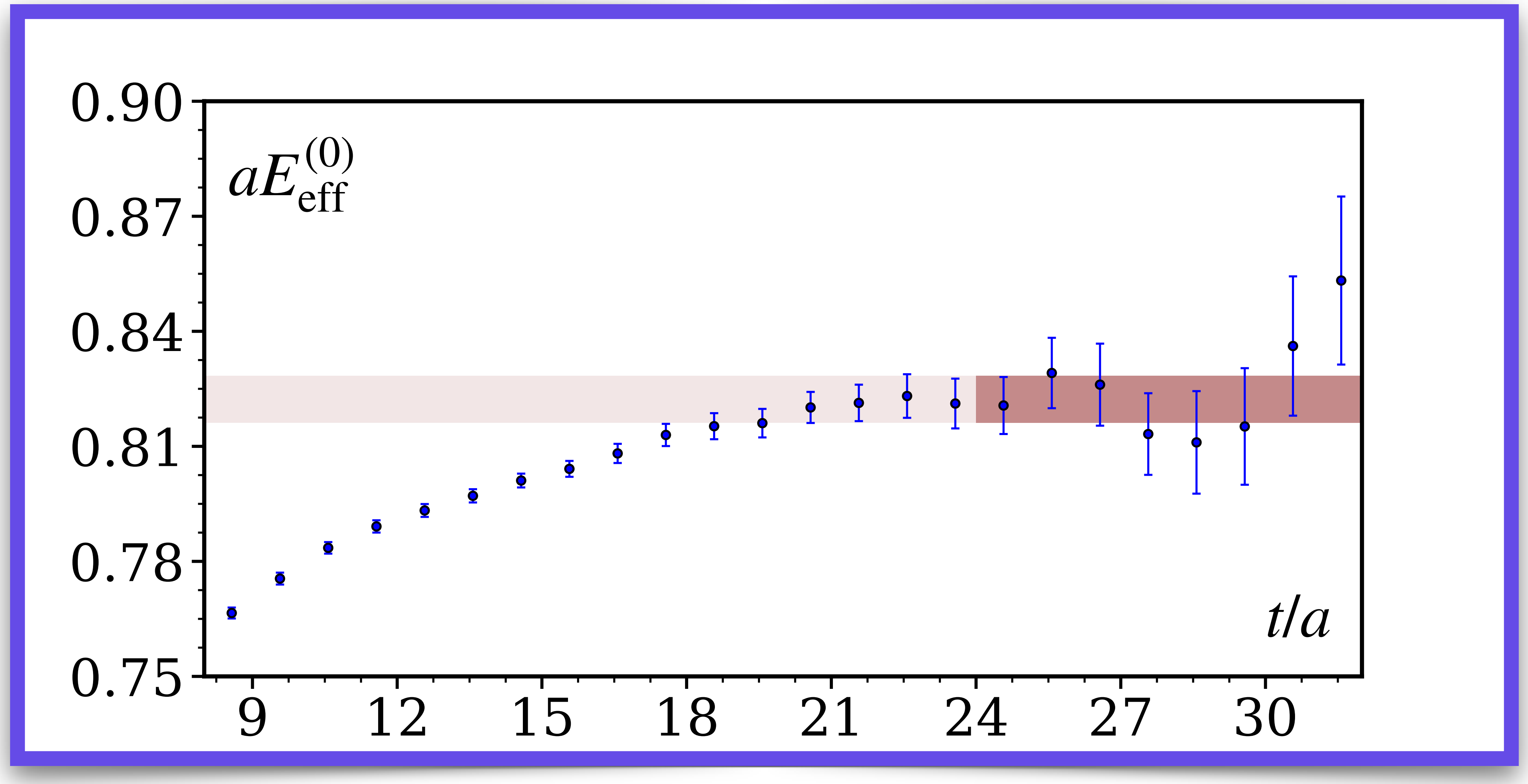
$$Q(\bar{x}, t; t') = \sum_{|\bar{y}-\bar{x}|<R} Q(\bar{y}, t; , t')$$

- As we increase box radius R, it approaches to symmetric correlator.
- Used to make comparative study of the asymptotic signals.
- Validates our energy plateau identification.



arXiv:2503.09760 BST, Mathur, Padmanath

Finite Volume Spectrum cont.



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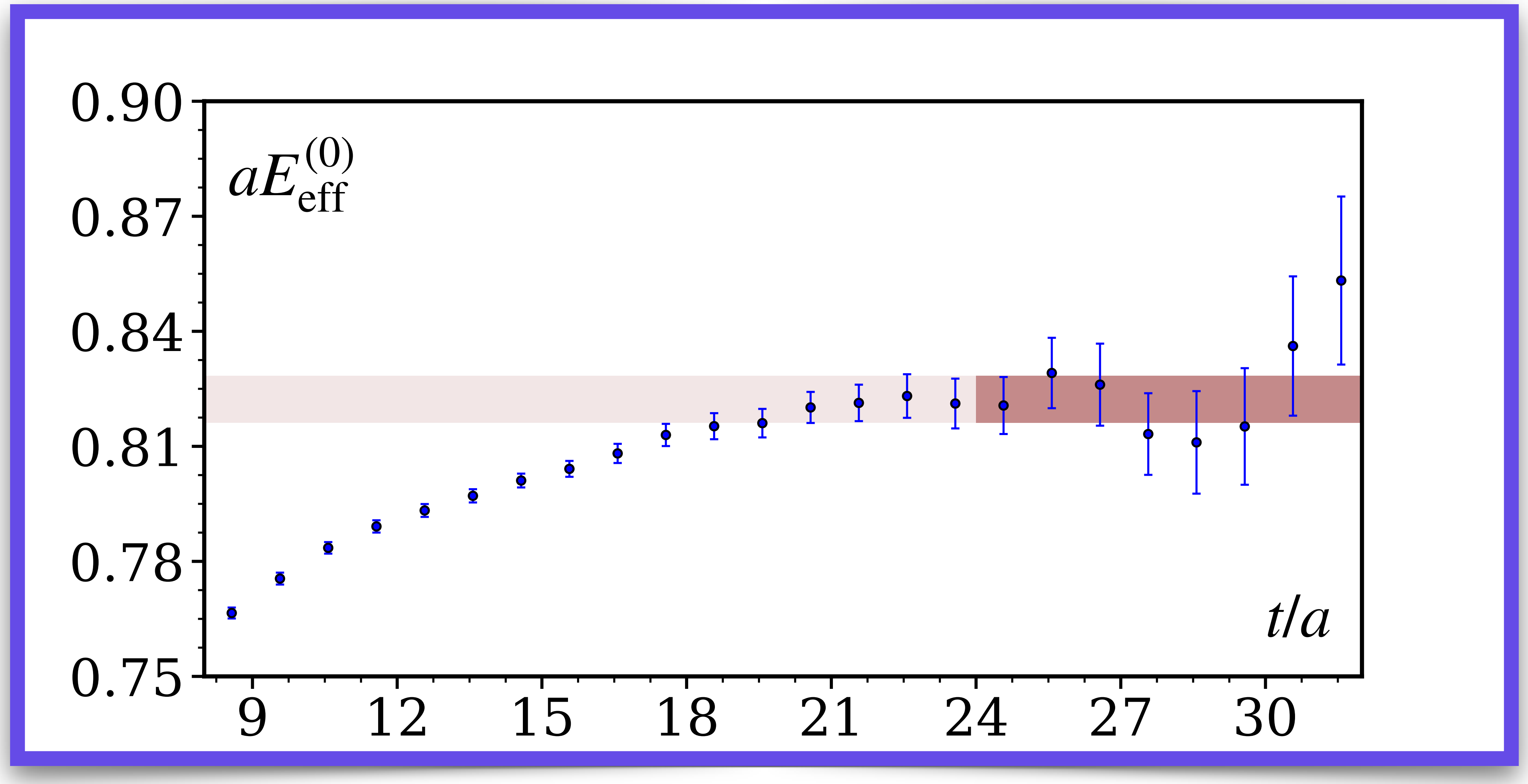

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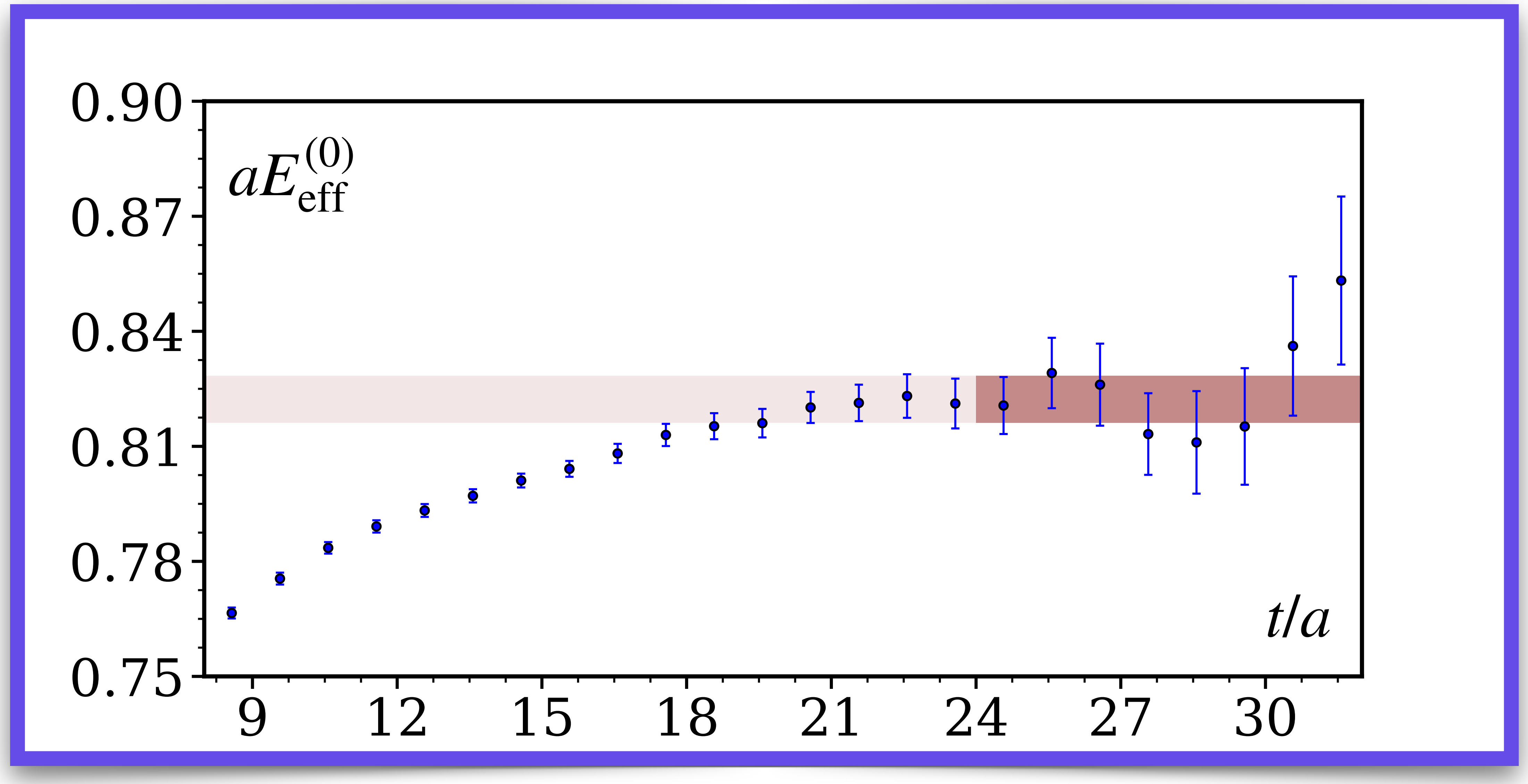

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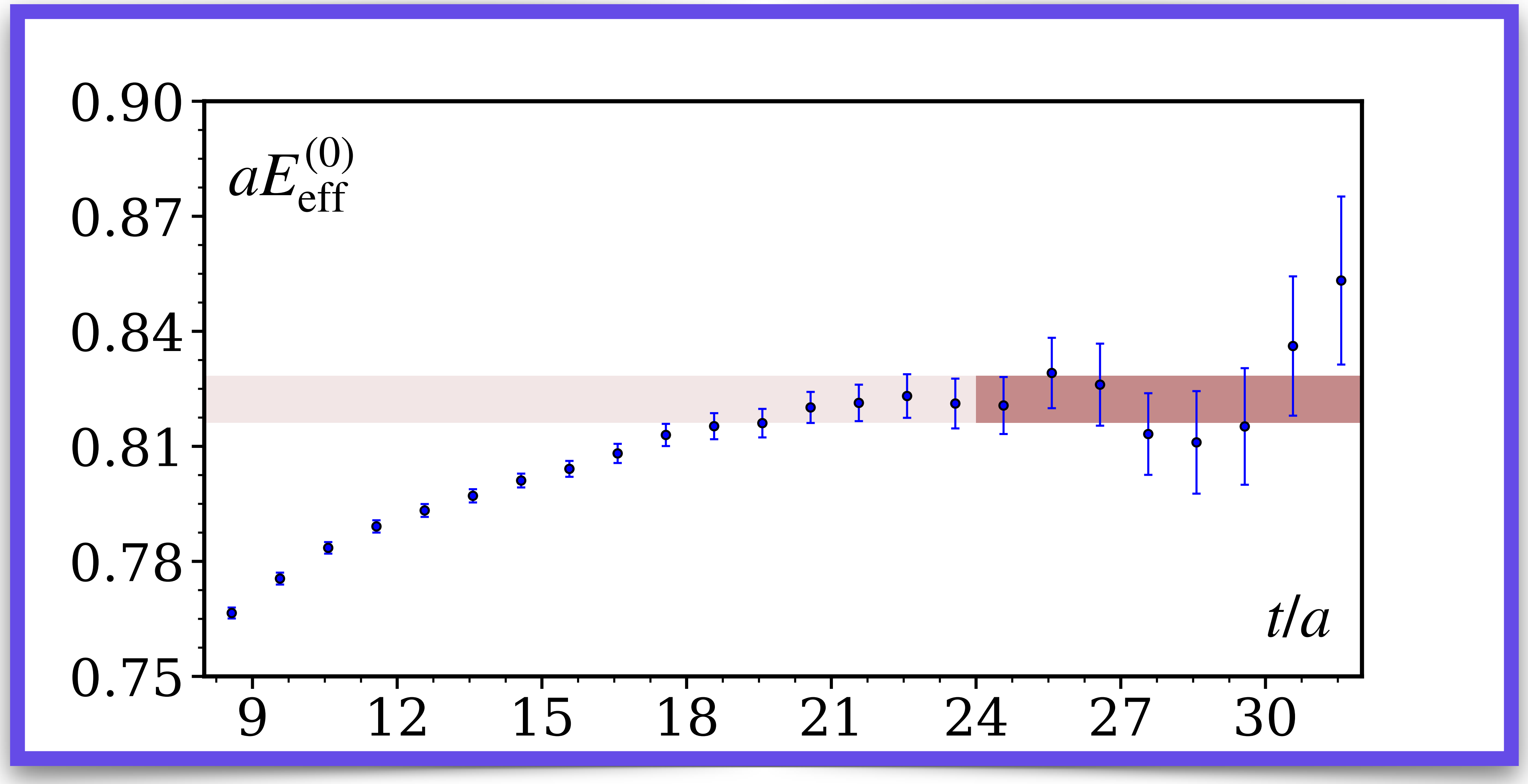

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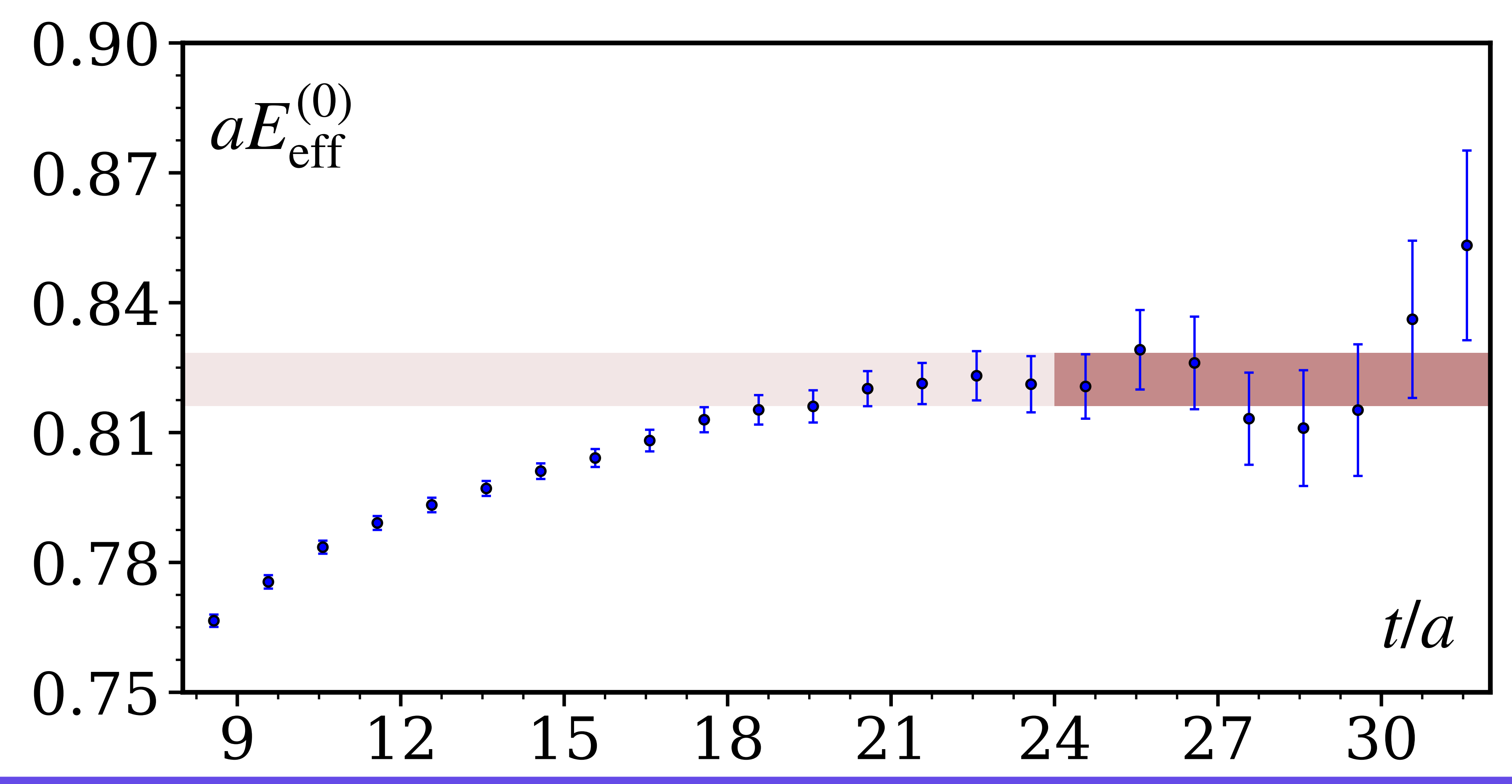

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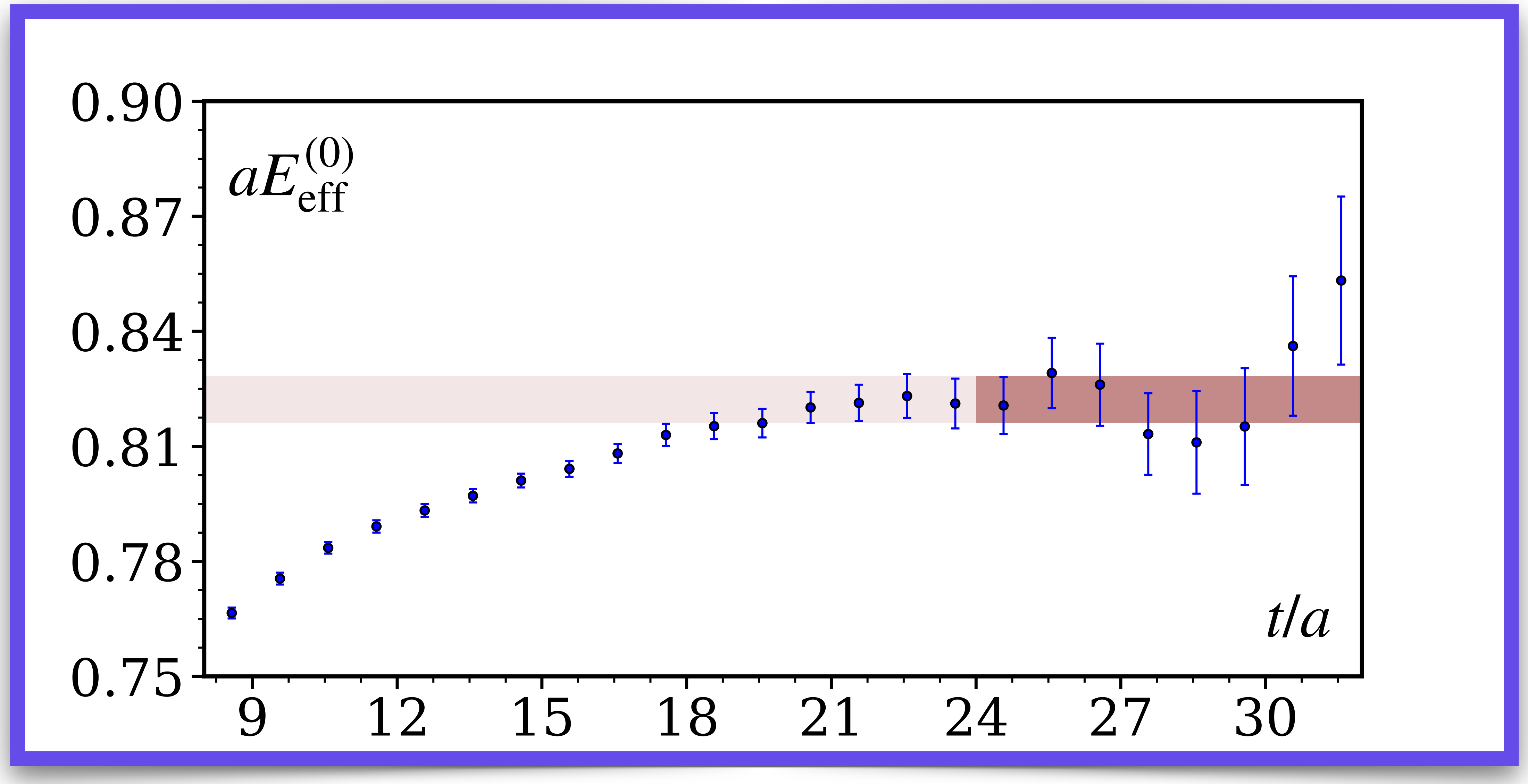

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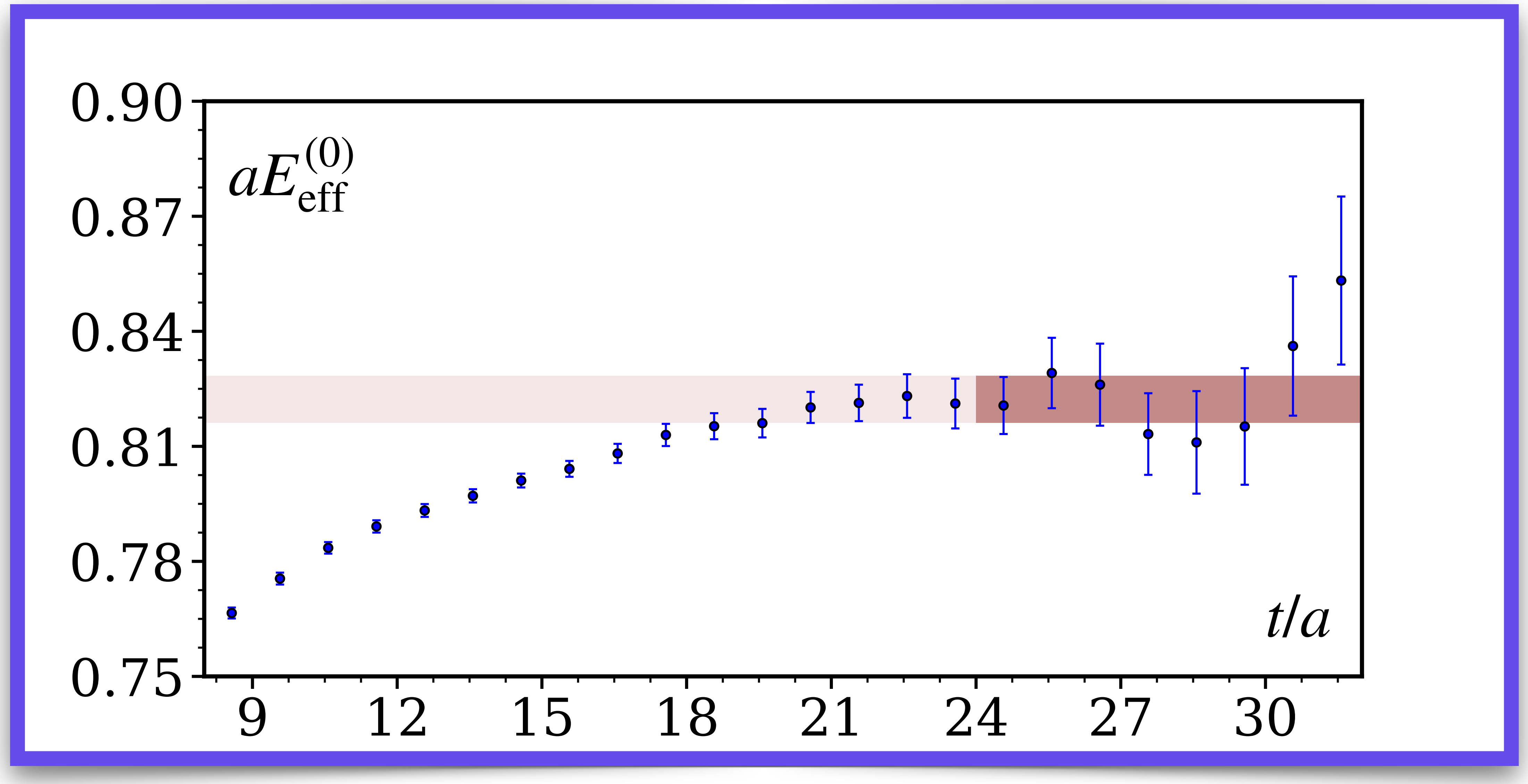

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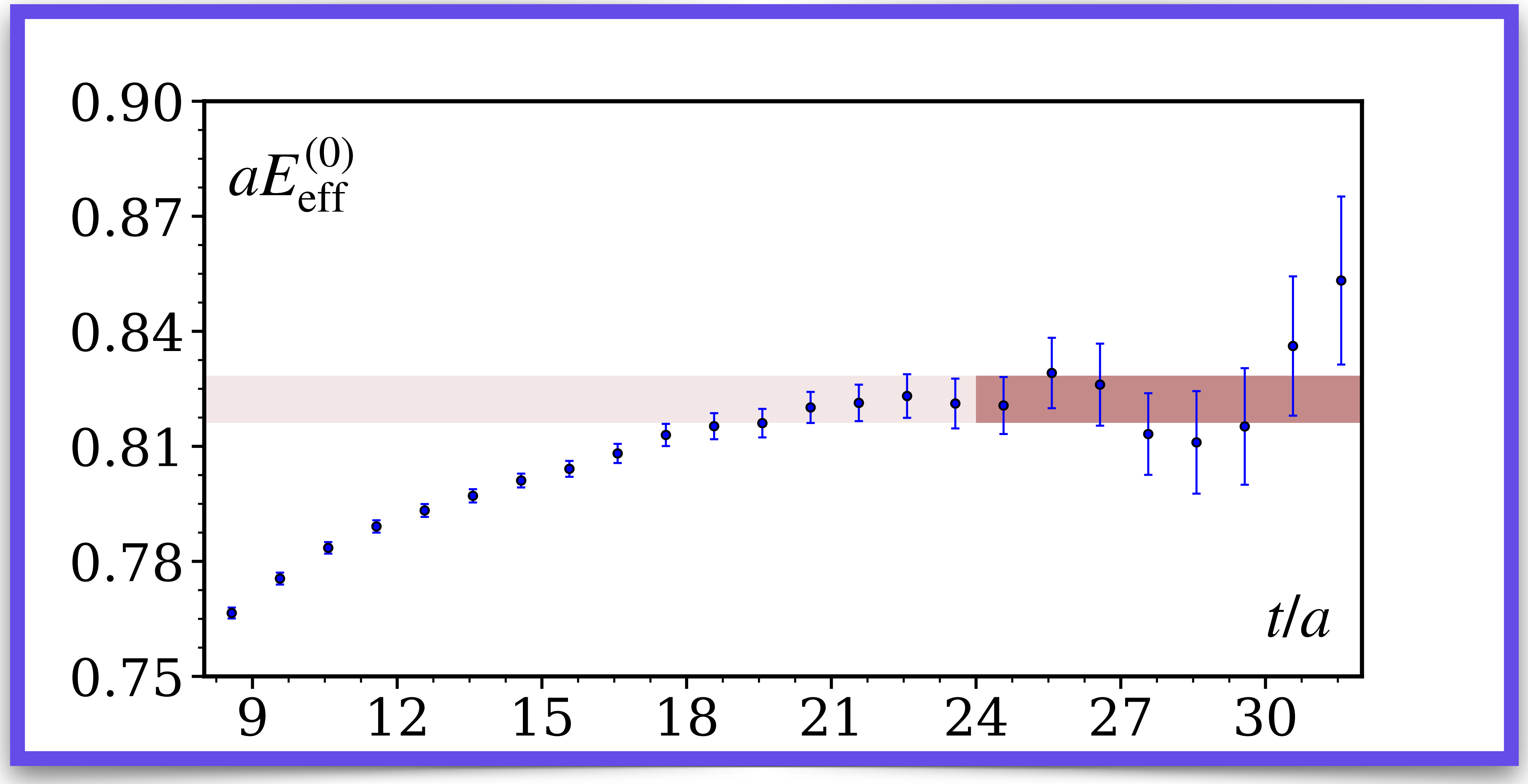

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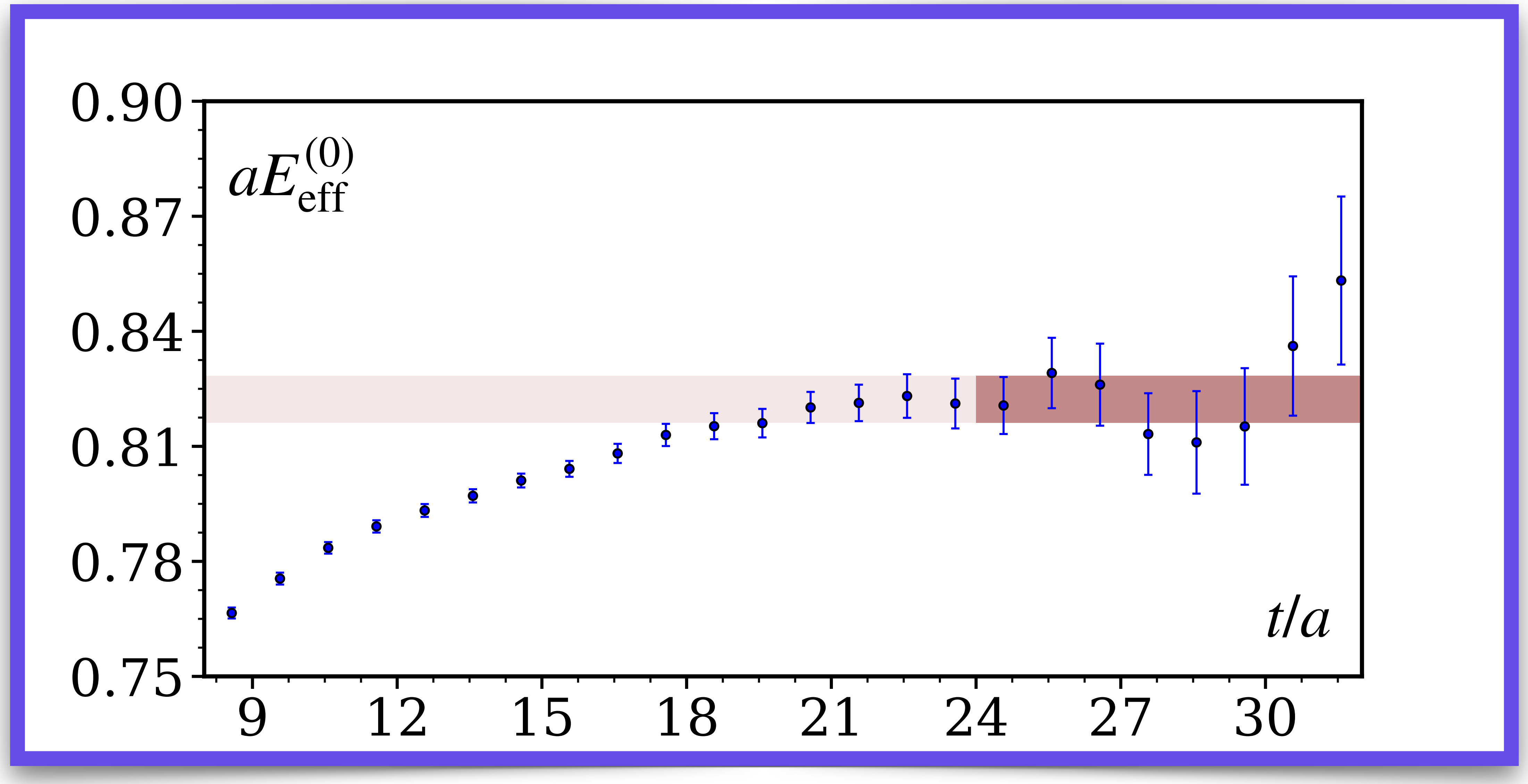
- Repeated for B and B^* mesons.



Non-Hermitian GEVP

- Modification of GEVP to account non-hermiticity.

- Solving asymmetric correlation matrix with left and right Eigenvector with same Eigenvalue.

$$\mathscr{C}(t_d)v_r^{(n)}(t_d) = \tilde{\lambda}^{(n)}(t_d)\mathscr{C}(t_0)v_r^{(n)}(t_d)$$

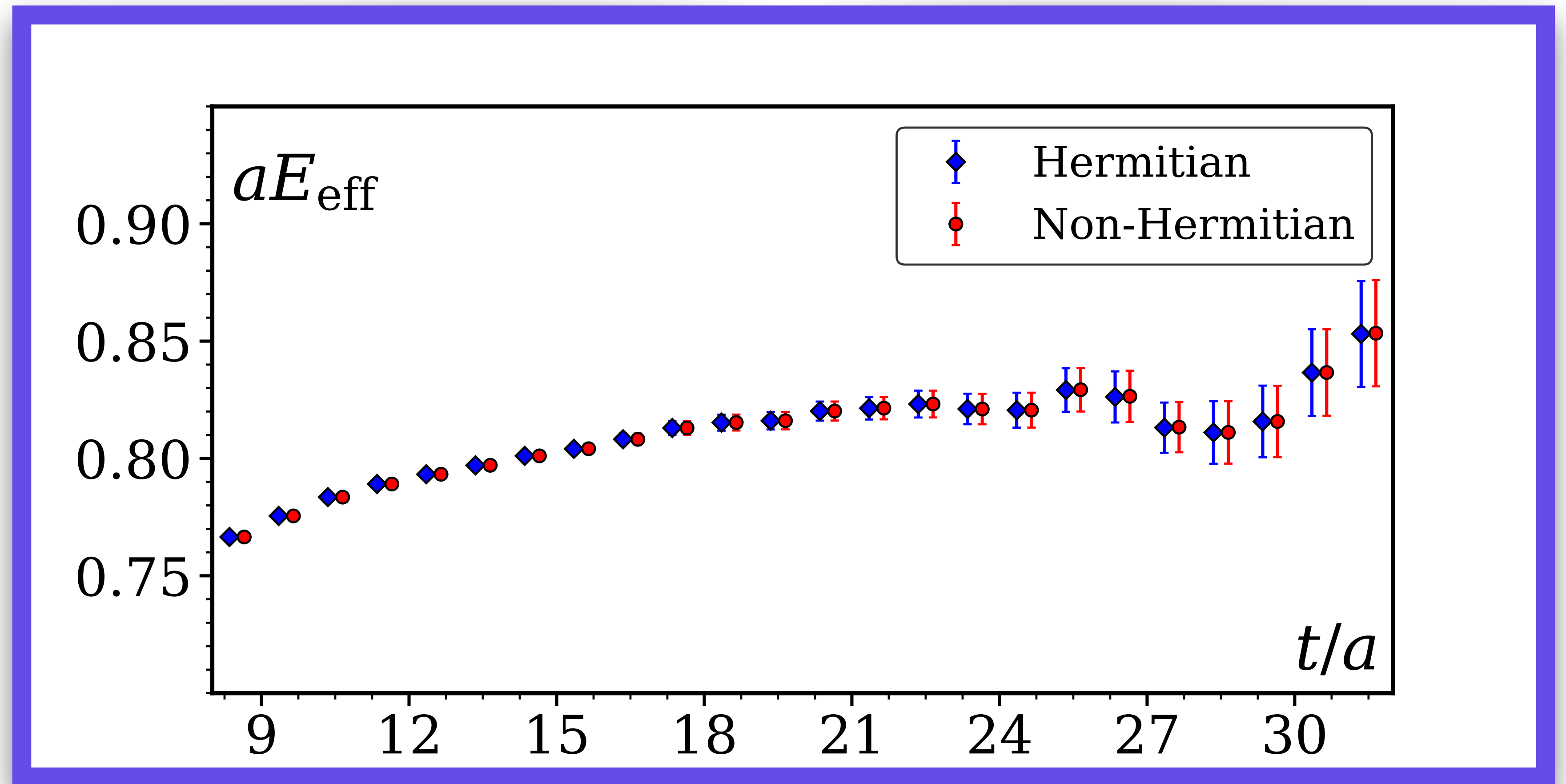
$$v_l^{(n)\dagger}(t_d)\mathscr{C}(t_d) = \tilde{\lambda}^{(n)}(t_d)v_l^{(n)\dagger}(t_d)\mathscr{C}(t_0)$$

t_d -> diagonalization time slice

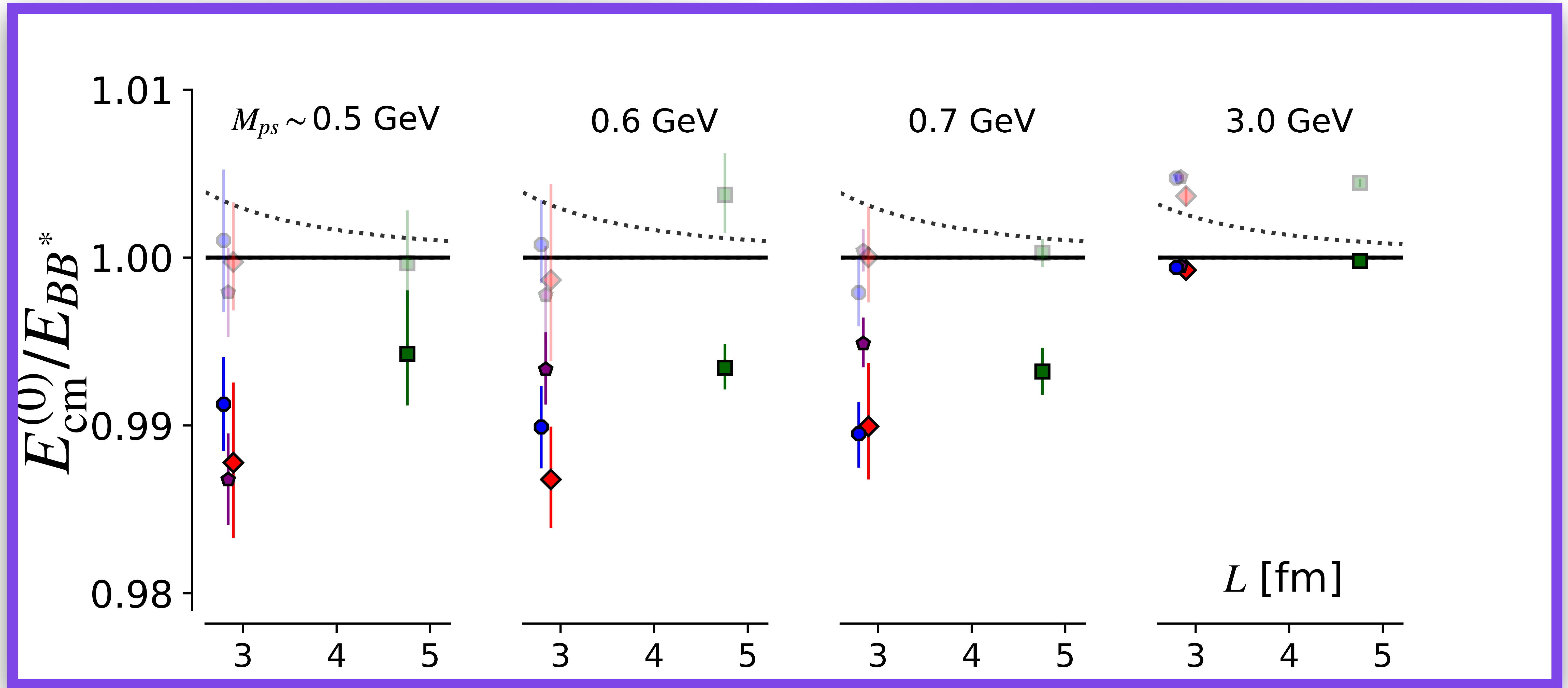
- Solve it for other time slices with $v_l(t_d)$ and $v_r(t_d)$ using

$$\tilde{\lambda}^{(n)}(t) = v_l^{(n)\dagger}(t_d)\mathscr{C}(t)v_r^{(n)}(t_d)$$

- Found $\dfrac{Im(\tilde{\lambda}^{(n)}(t))}{|(\tilde{\lambda}^{(n)}(t))|} < 0.01$ for all correlators used.



Finite Volume Spectrum cont.



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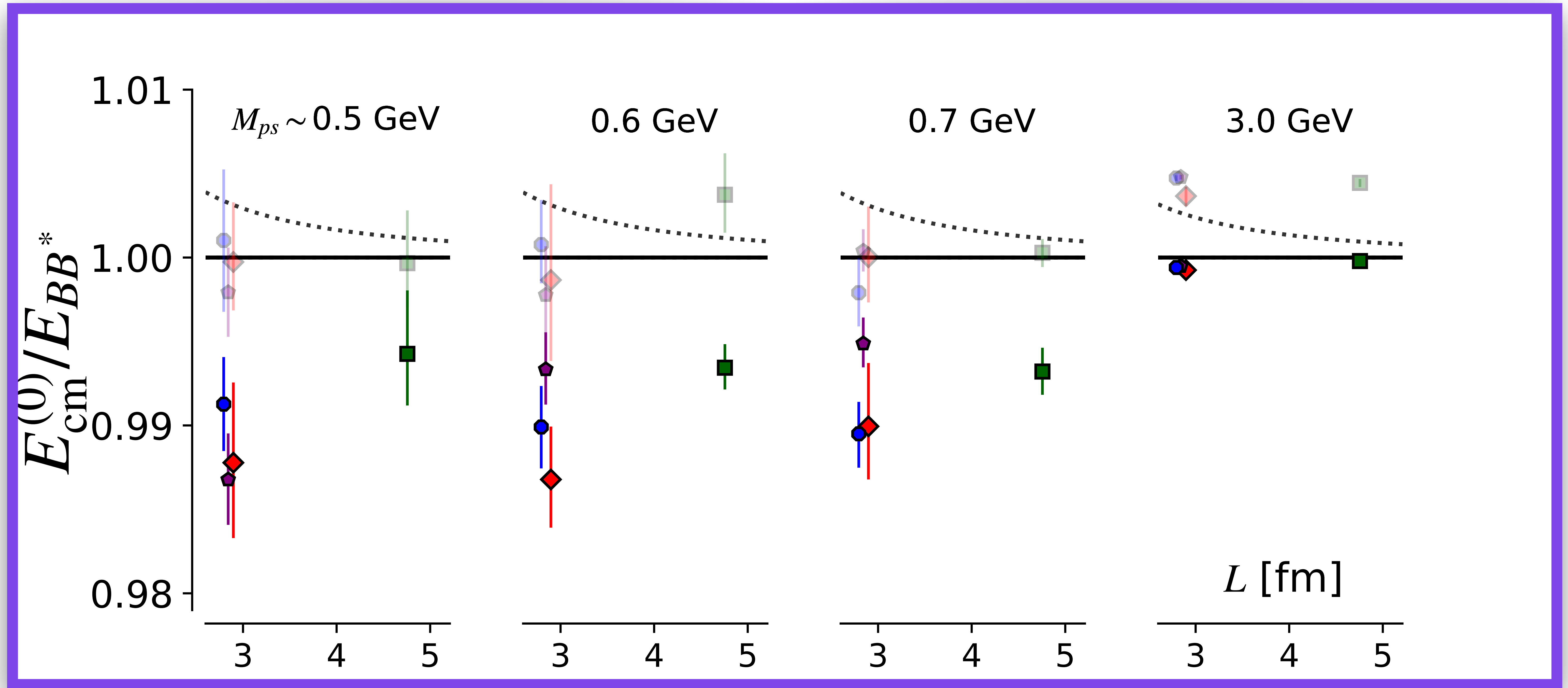
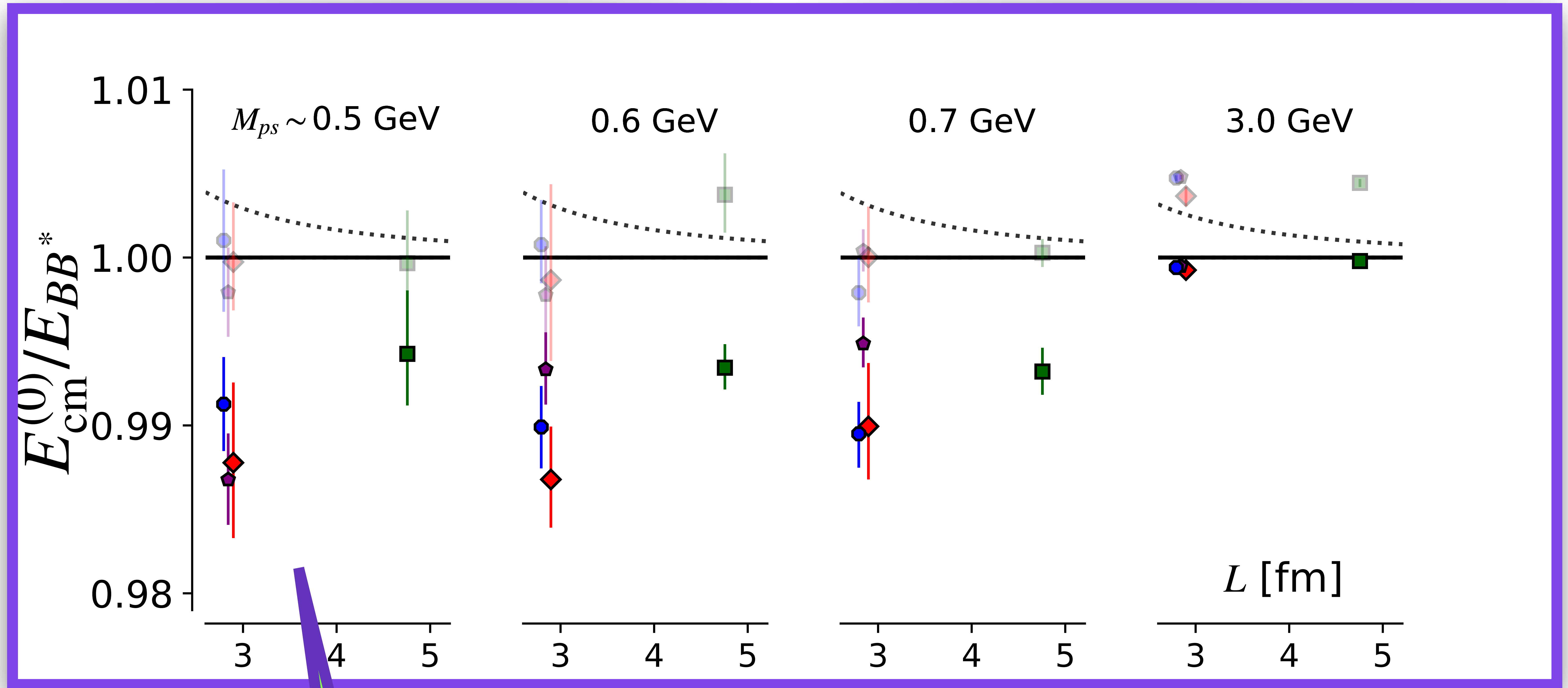


- **Spectrum extraction repeated for every M_{ps} and every ensemble.**

Finite Volume Spectrum cont.



NRQCD Offset corrected

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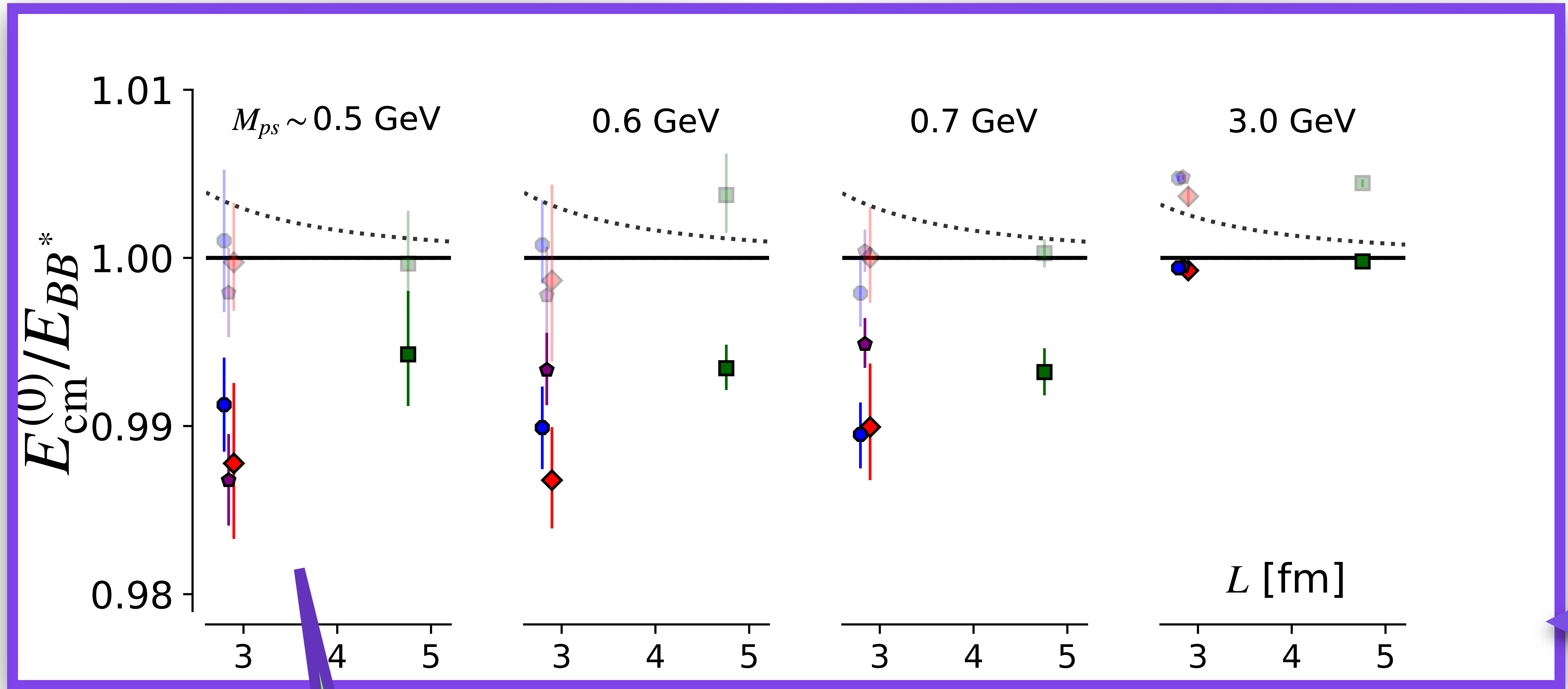
Finite Volume Spectrum cont.

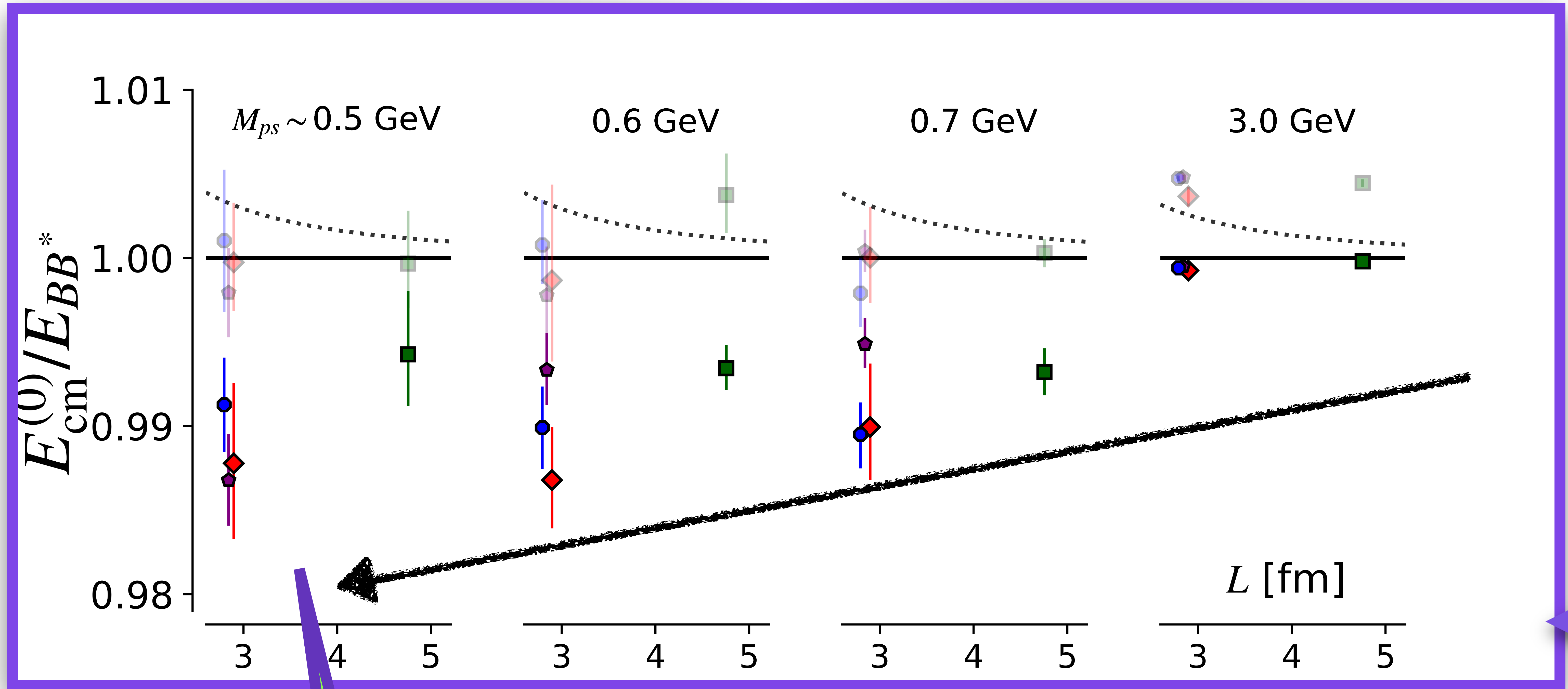


- Spectrum extraction repeated for every M_{ps} and every ensemble.

Spectrum were calculated in units of nearest Two body decay threshold $BB*$.

NRQCD Offset corrected

Finite Volume Spectrum cont.



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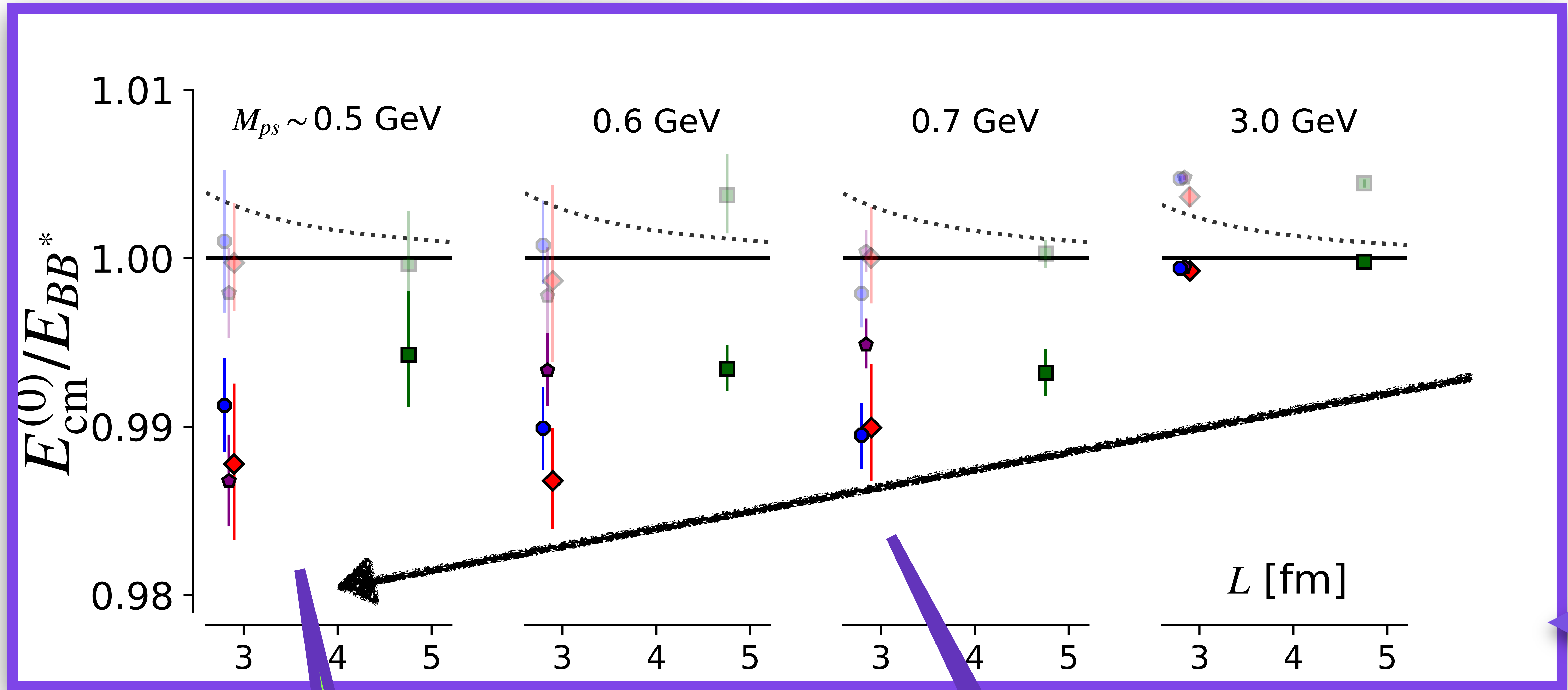

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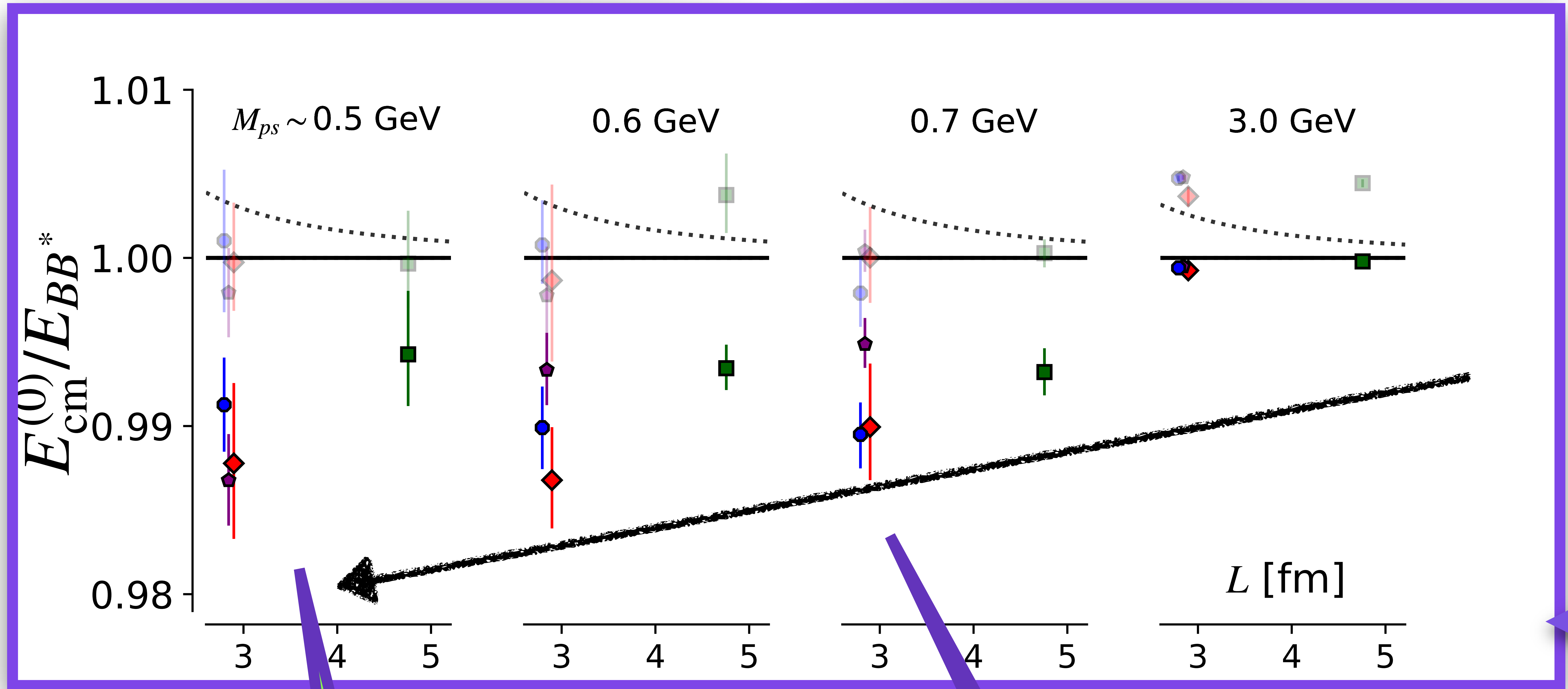

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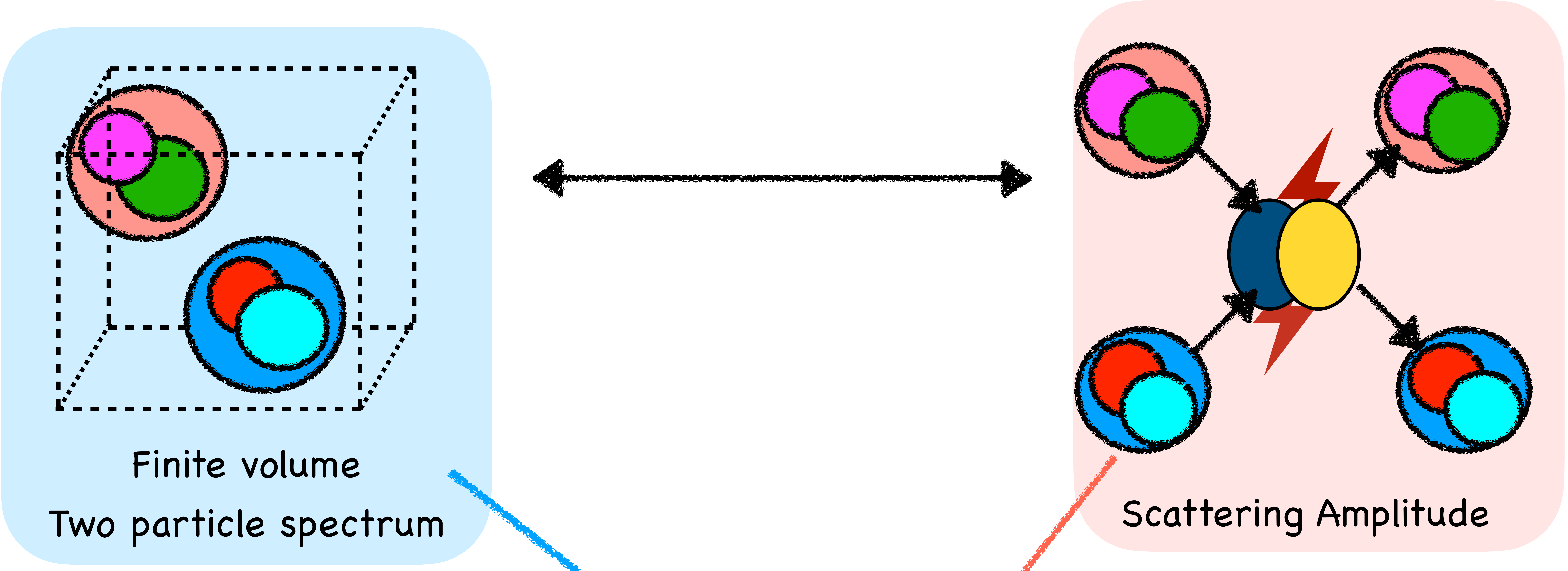
We need continuum extrapolation to have results in physical limit

Amplitude Analysis

Lüscher based quantization condition(1991)



$$det\left[1 + i\mathscr{G}(E)\ \mathscr{M}(E)\right] = 0$$

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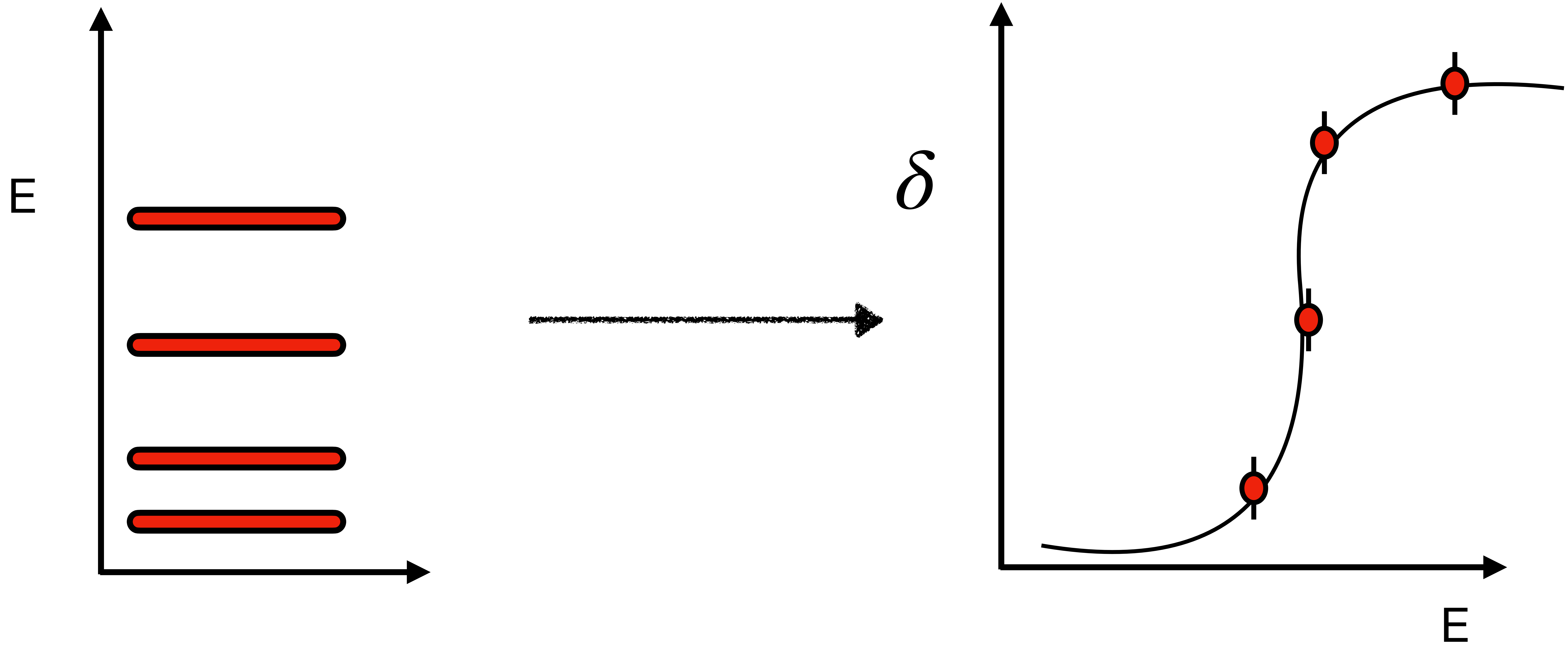
- Search for poles of amplitude near threshold representing bound states.

Amplitude Analysis



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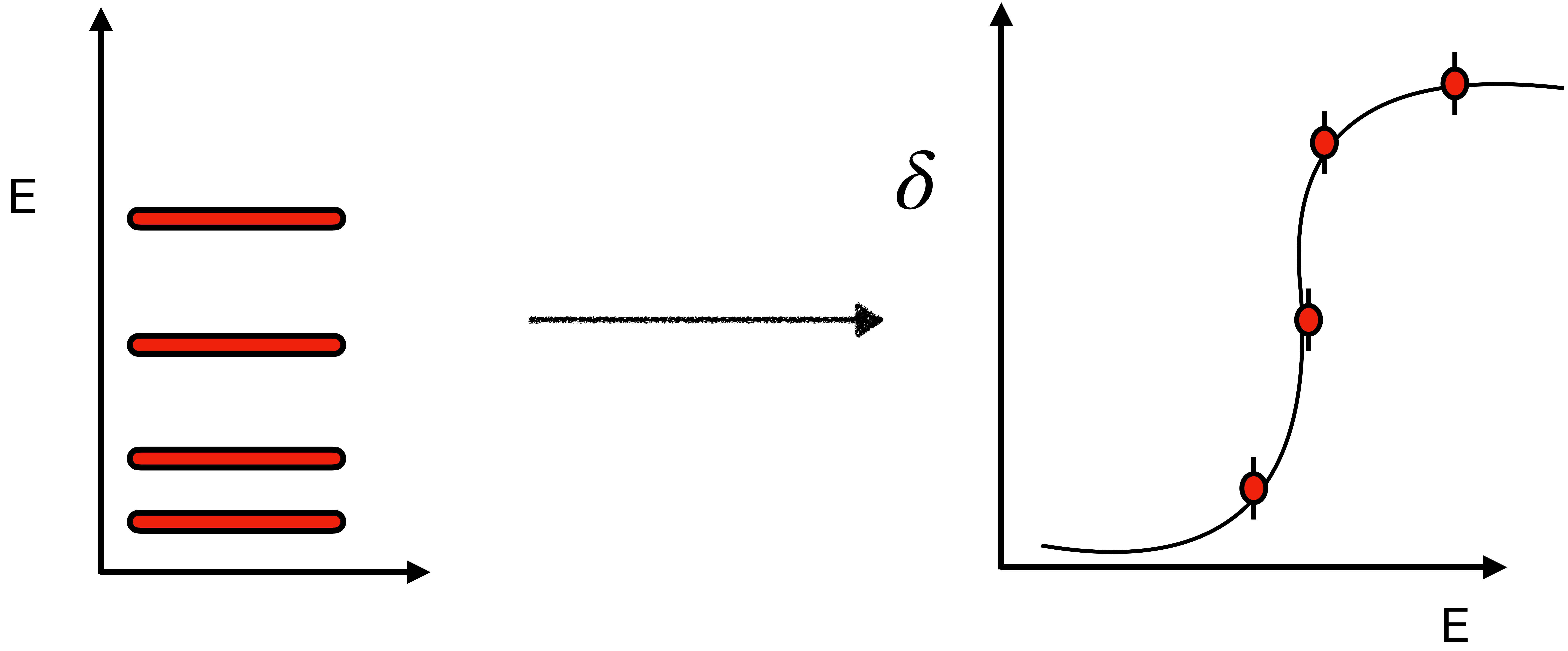

- Energy is discrete.

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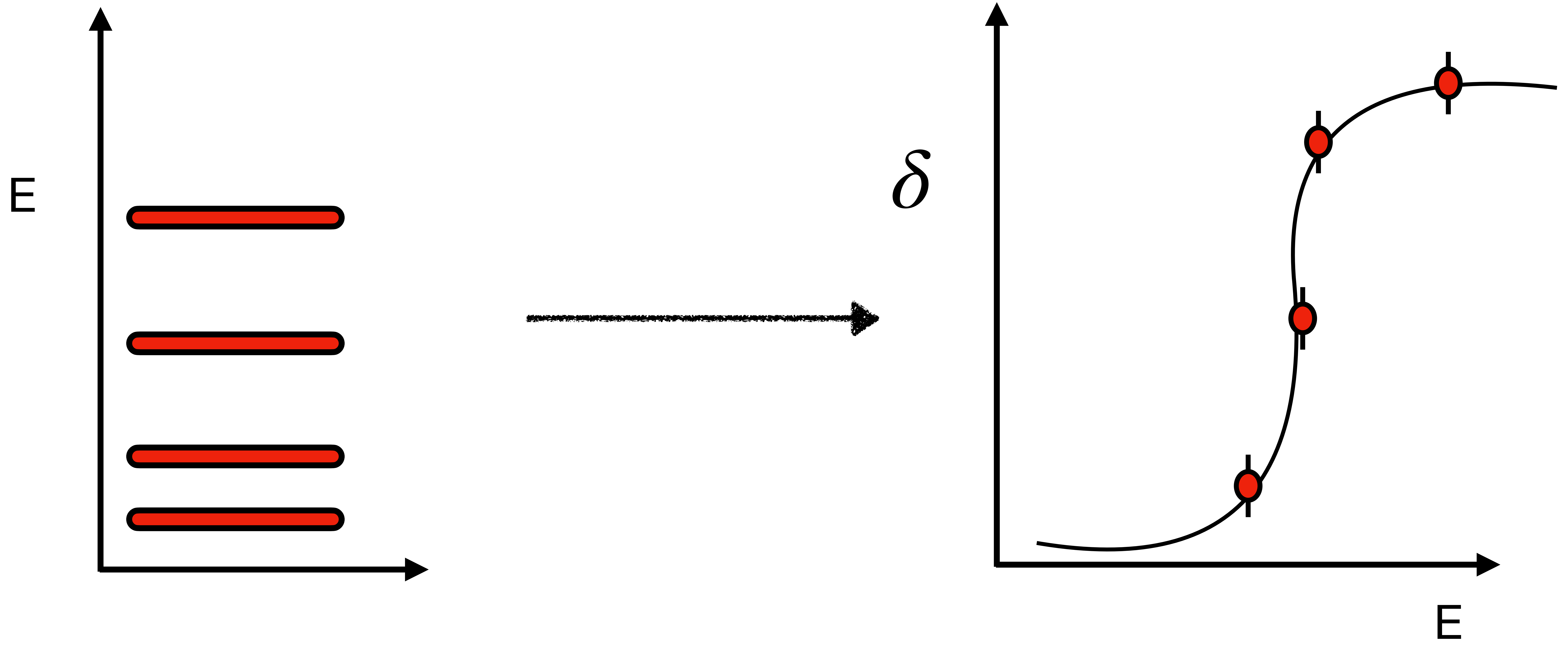

- Energy is discrete.
- Parametrisation of phase shift is required.

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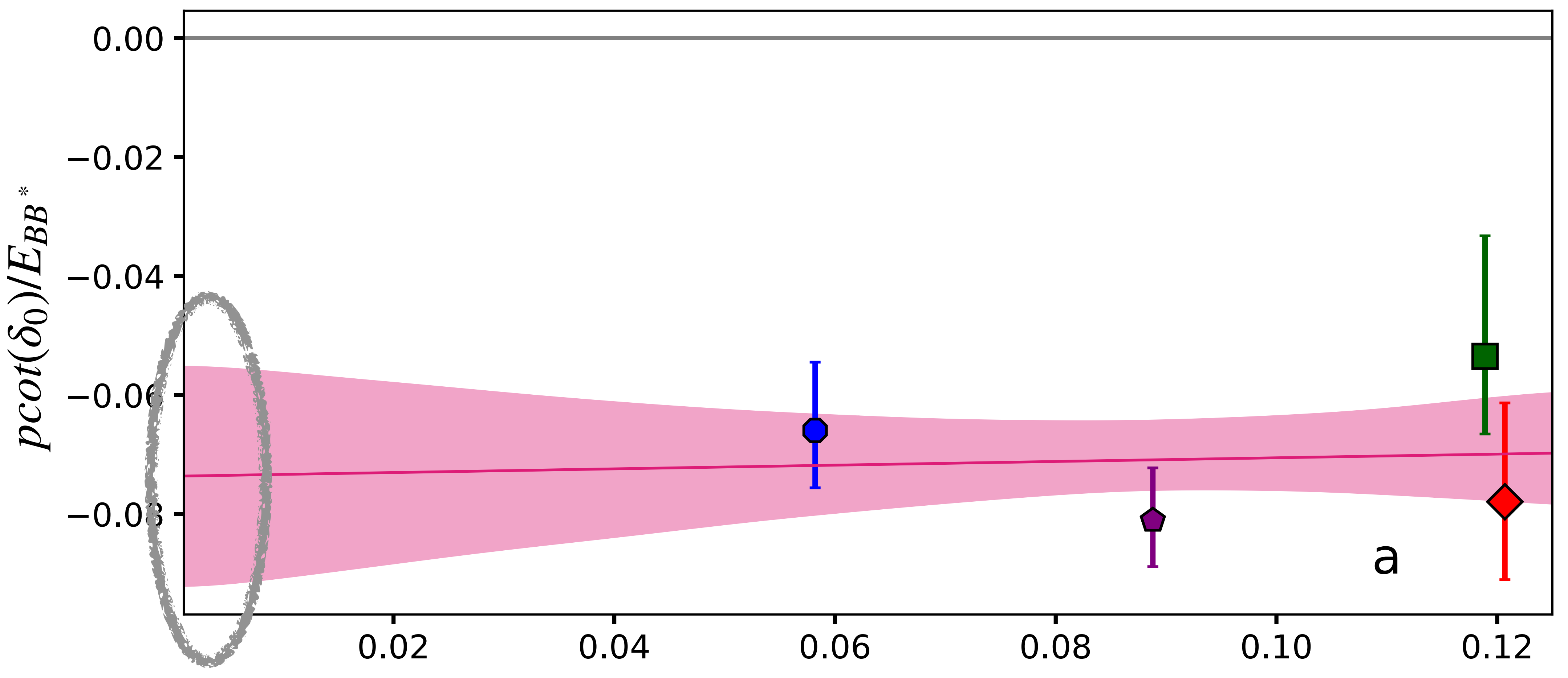
$$\chi^2 = \left[E(L) - E^{sol}(L, A^{[0]}, A^{[1]})\right] C^{-1} \left[E(L) - E^{sol}(L, A^{[0]}, A^{[1]})\right]$$

Continuum Extrapolation

$M_{ps} = 0.5$ GeV



- Scattering Amplitude is given as

$$T \propto (pcot\ \delta - ip)^{-1}$$

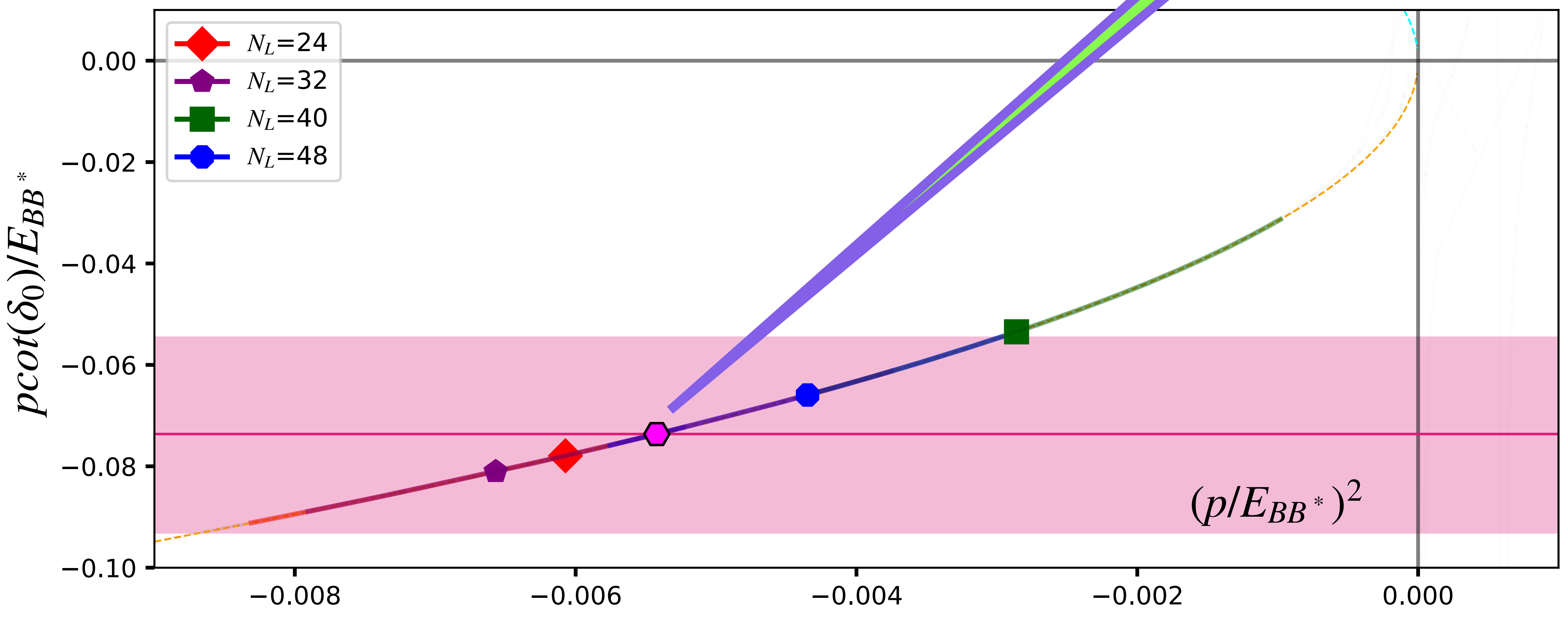
- Phase shift parametrised as

$$pcot\ \delta_0 = -\frac{1}{a_0} + A^{[1]} \cdot a$$

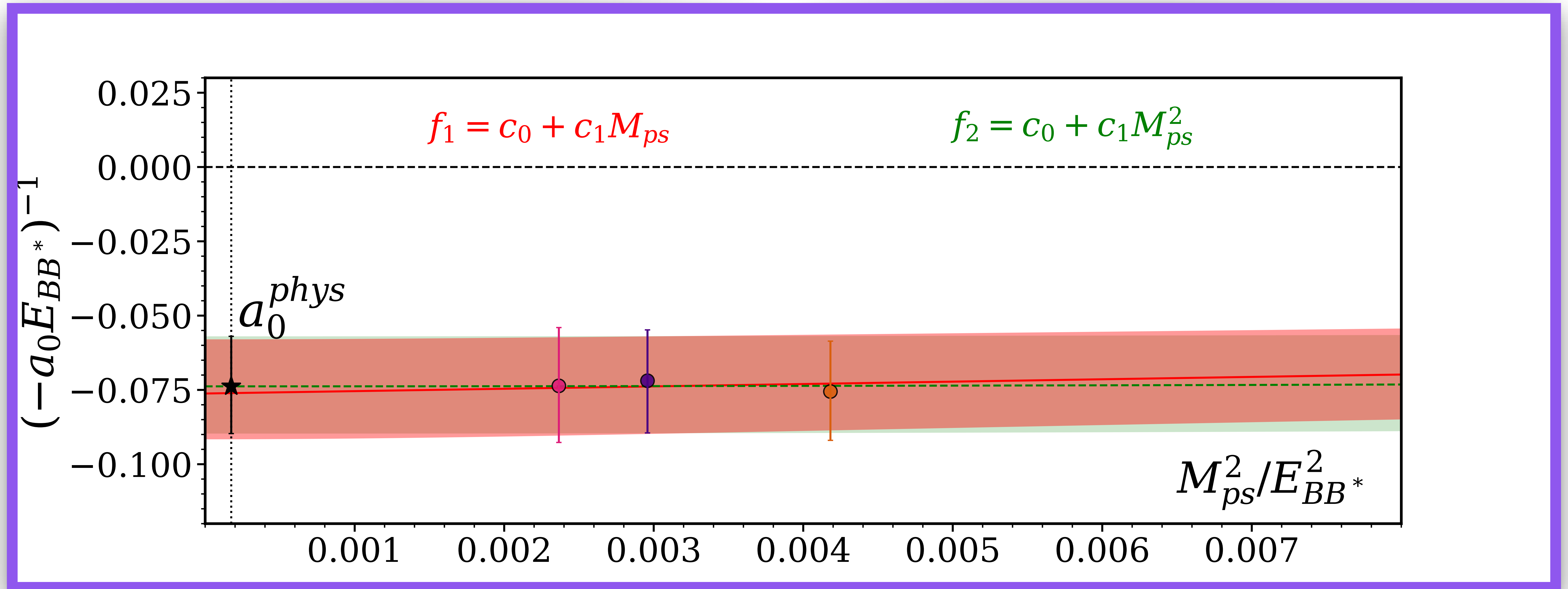
Real Bound State

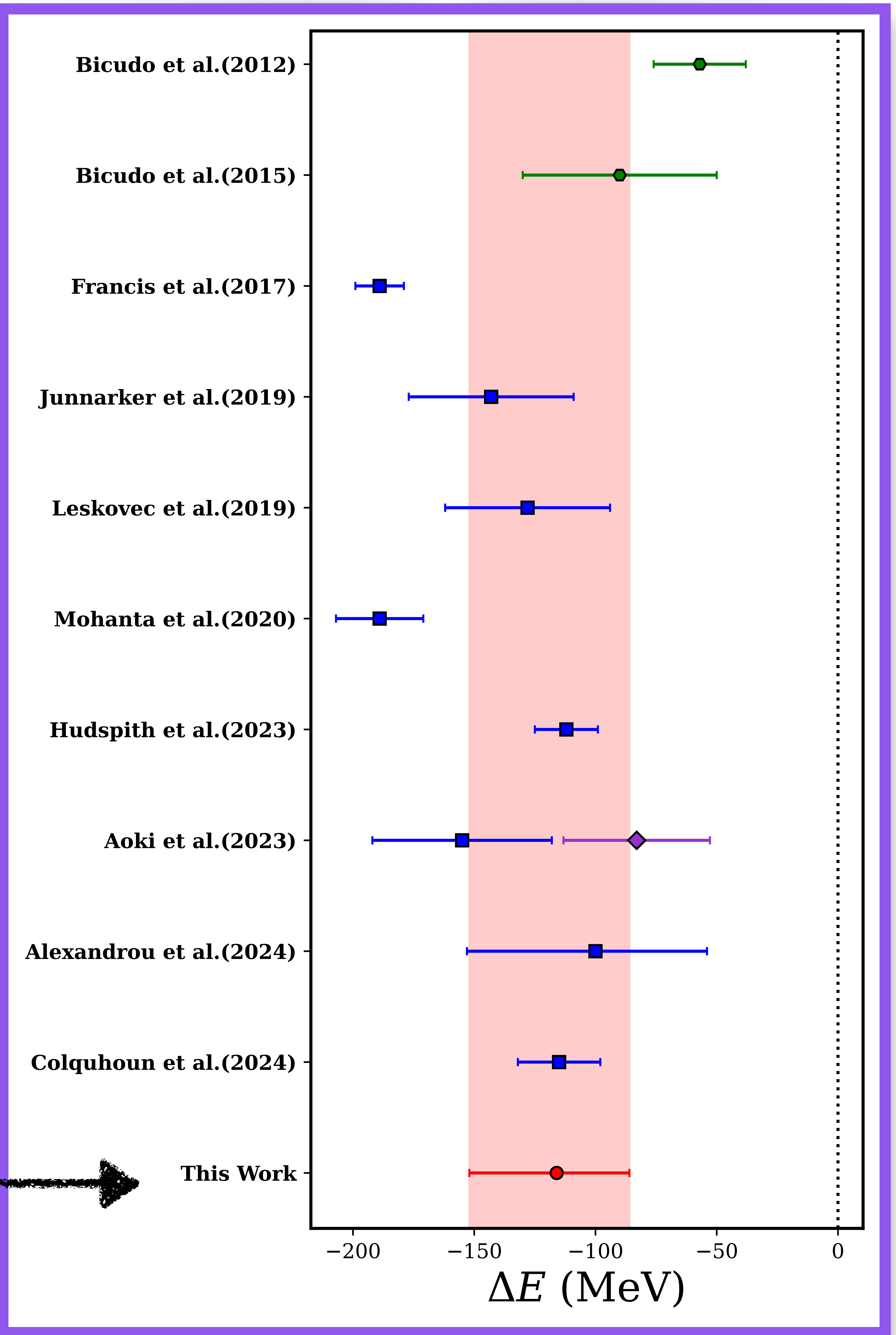
- Same repeated for other M_{ps}(0.6, 0.7, 3.0 GeV).

- Consistent Negative values for other M_{ps} as well as real bound state.



Chiral Extrapolation





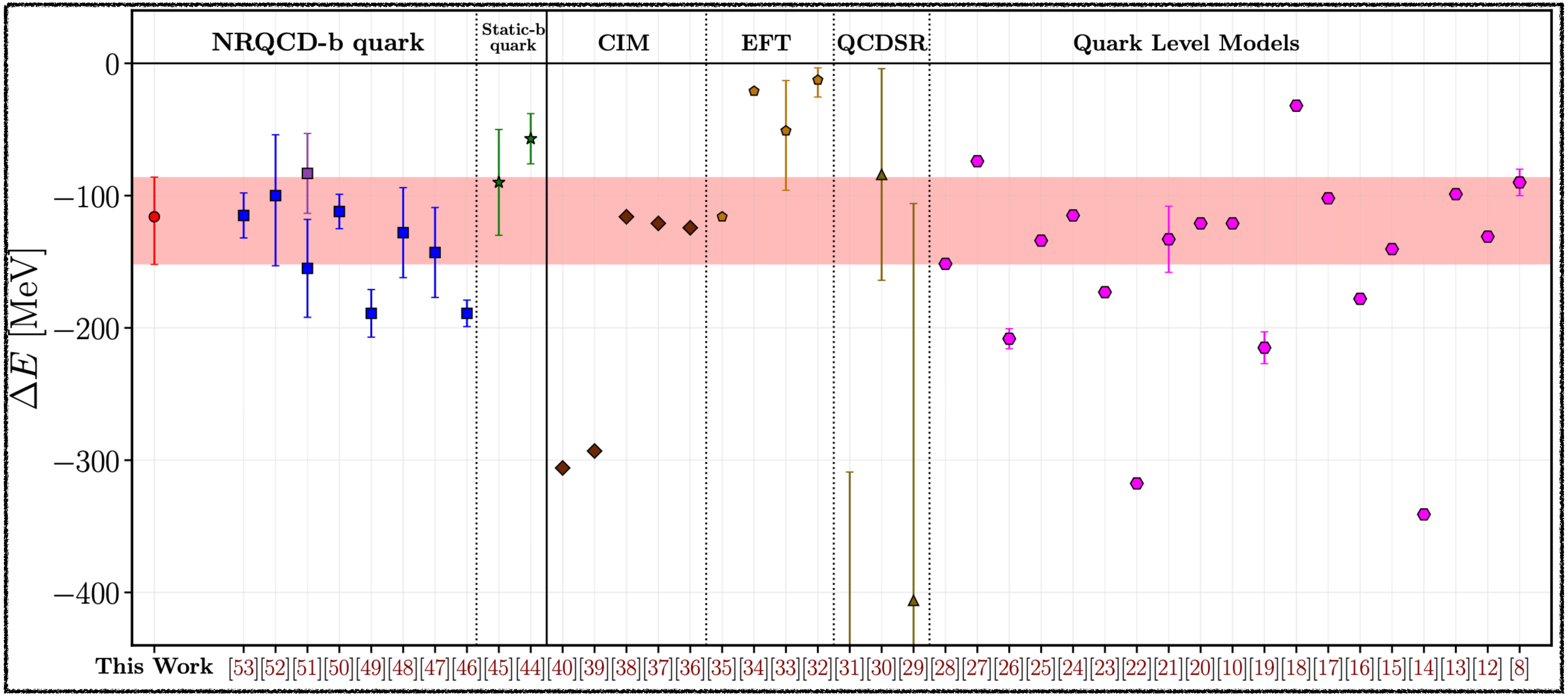
Result:

Scattering length at physical limit

$$a_0^{phy} = 0.25(^4_3)\ fm$$

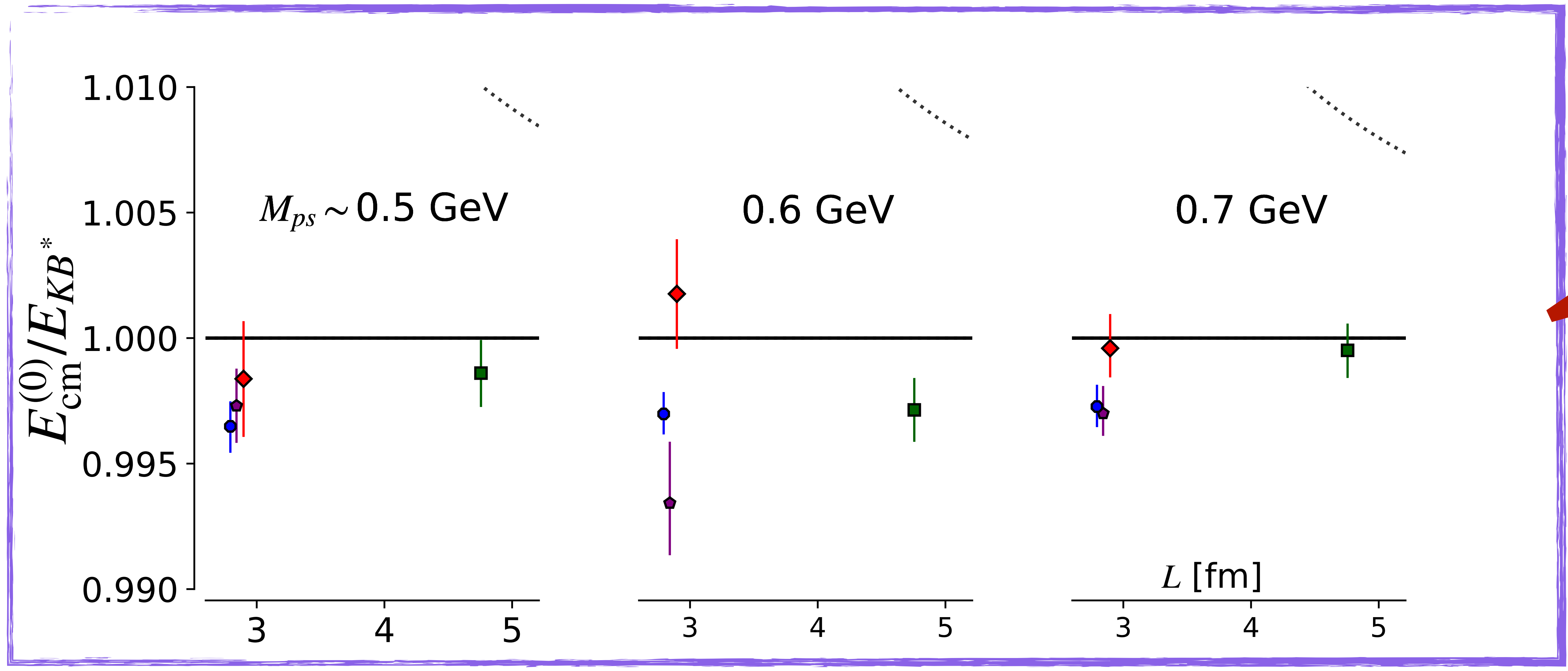
Corresponds to binding energy $\Delta E = -116(^{+30}_{-36})$ **MeV.**

Summary T_{bb}

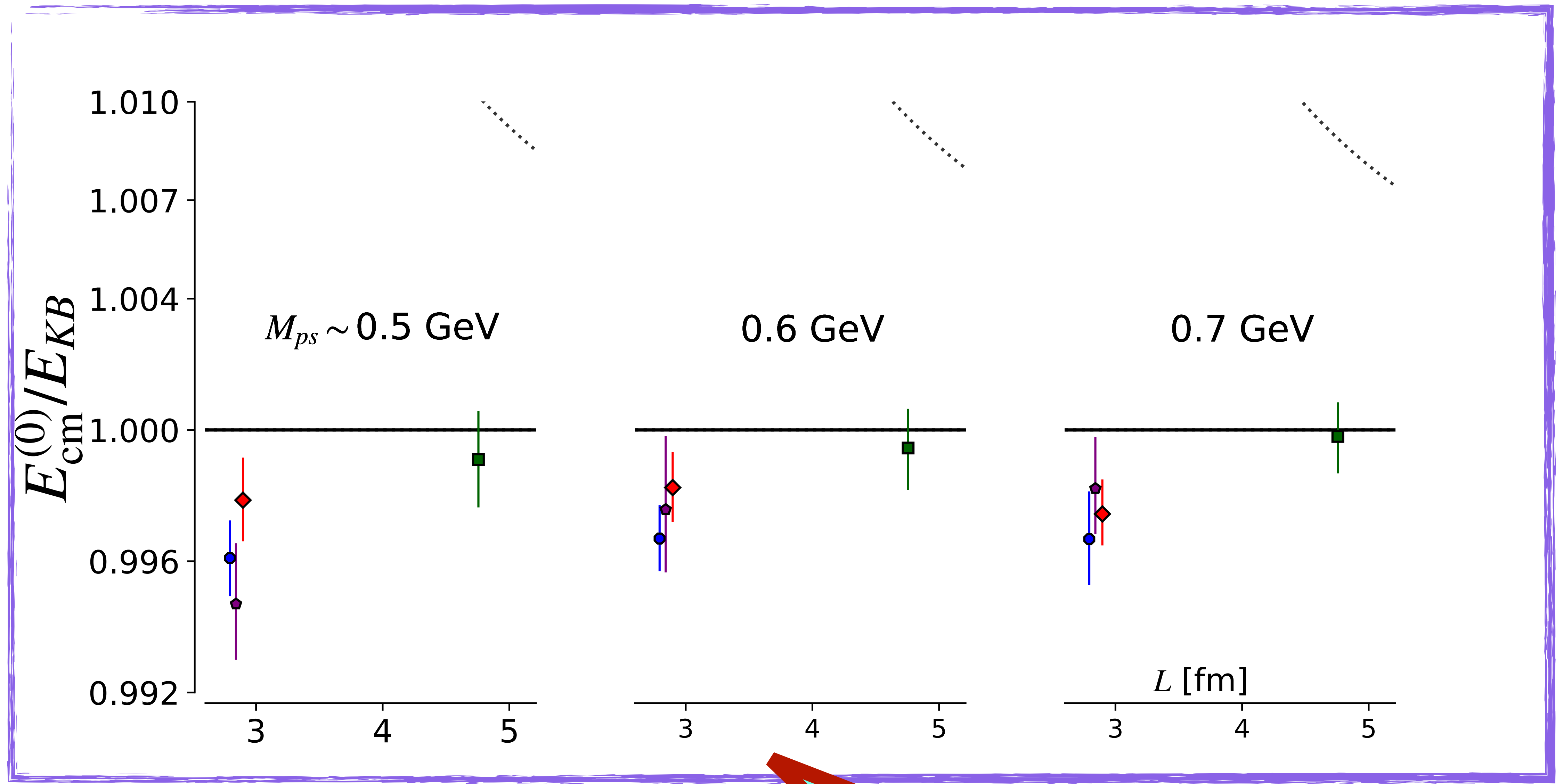


arXiv:2503.09760 BST, Mathur, Padmanath

Results of T_{bs}

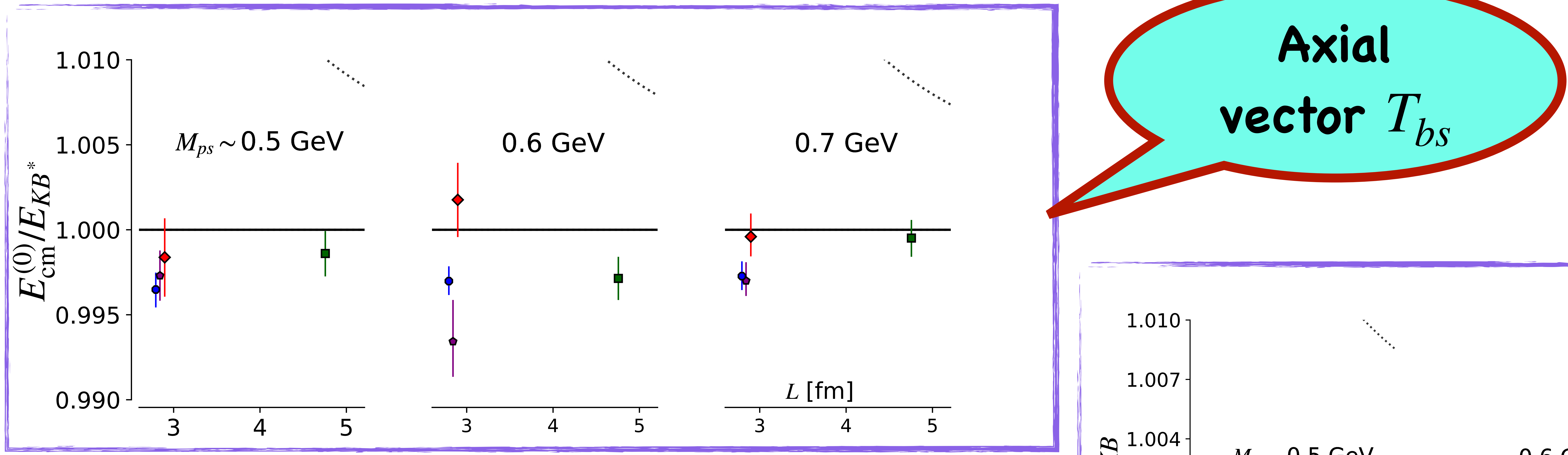


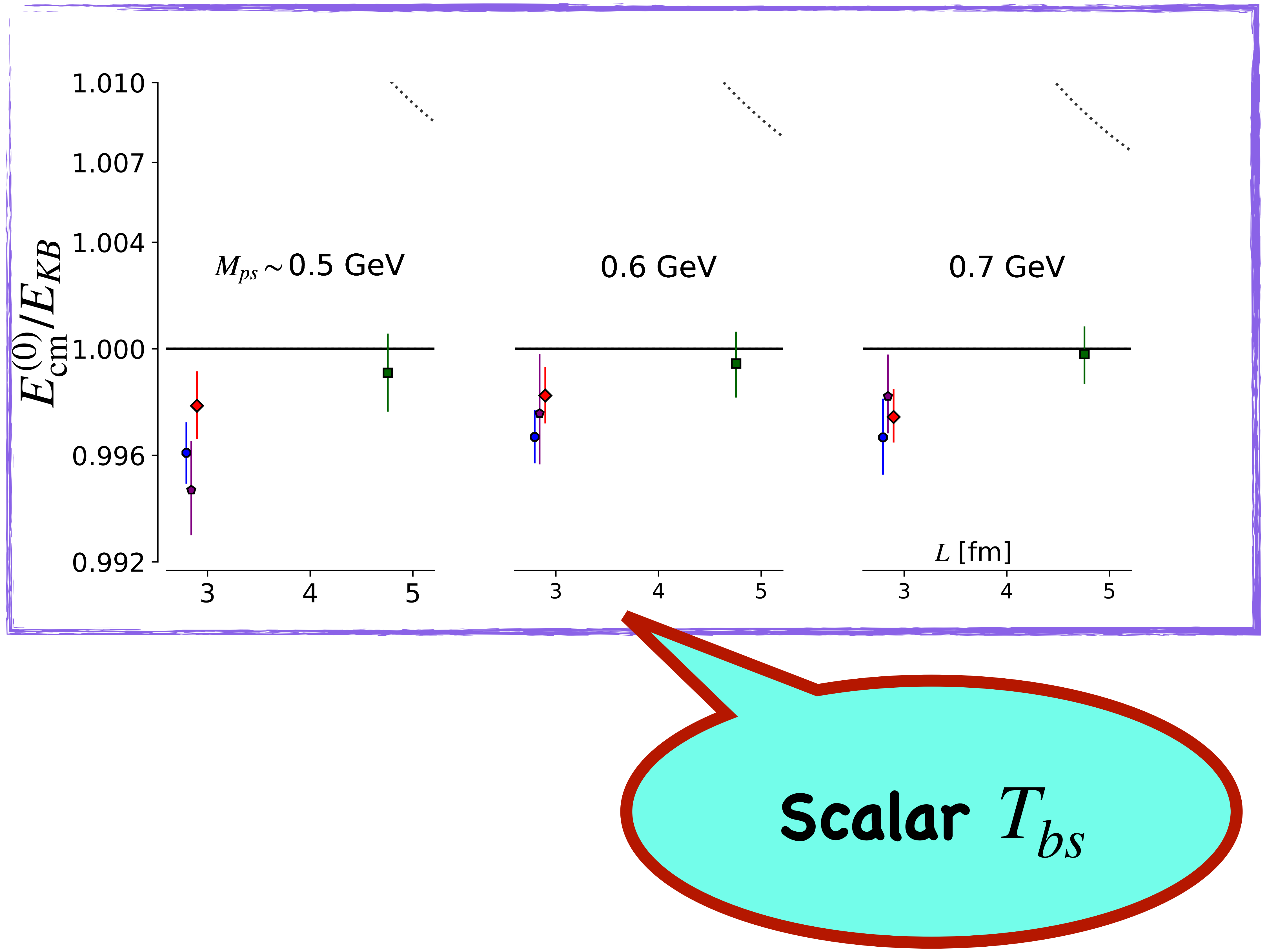
Axial vector T_{bs}



Scalar T_{bs}

Results of T_{bs}





- Results are consistent with threshold in both the cases for larger volume.
- Need low M_{ps} datas for better results.

Results of T_{bs}

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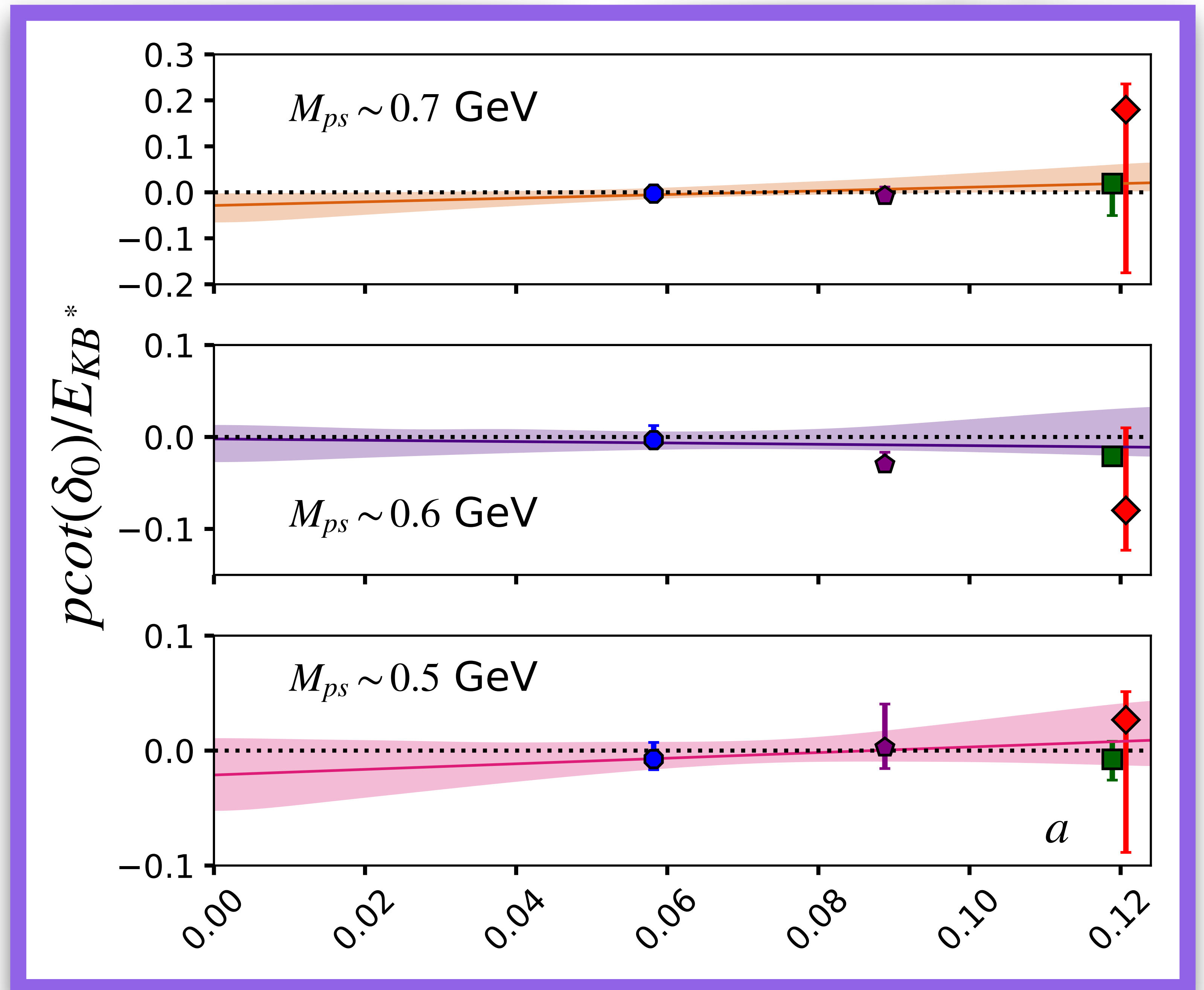

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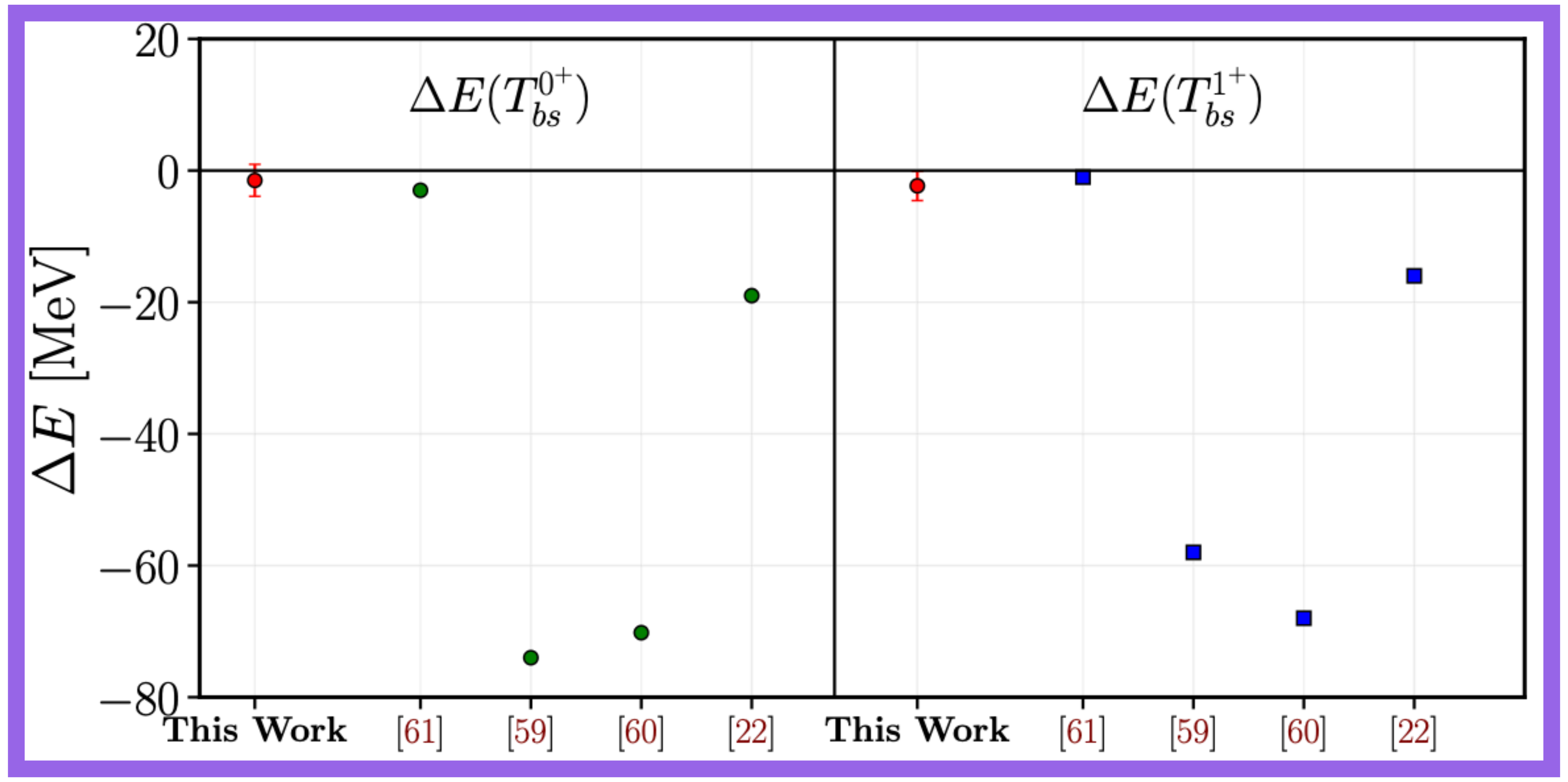

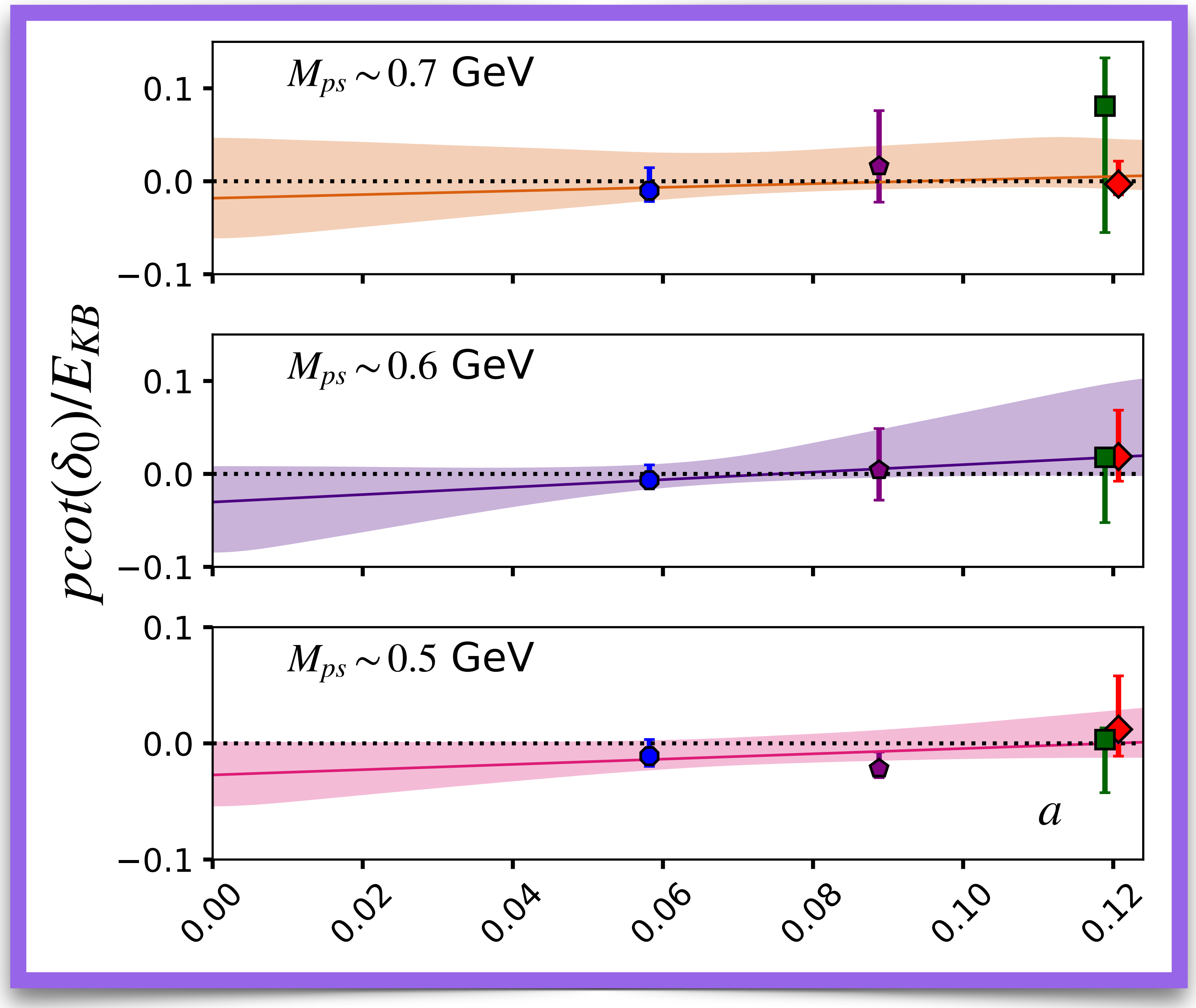

Scalar T_{bs}

Summary and Outlook

- We worked with isoscalar axial vector T_{bb} and both scalar and axial-vector T_{bs}
- Various work widely predicted deep binding in isoscalar axial-vector T_{bb}.
- Rigorous spectrum analysis were done for T_{bb} and T_{bs} tetraquark.
- We worked with multiple lattice spacing, two volumes to control systematics.
- Finite volume spectrum indicates negative energy shift with respect to BB^* threshold.
- Found a possible deeply bound state for T_{bb} not such exciting results in T_{bs}.

Slides will be available at my website https://www.imsc.res.in/~bhabanist/

THANK YOU