# Heavy Tetraquark Spectroscopy in lattice QCD


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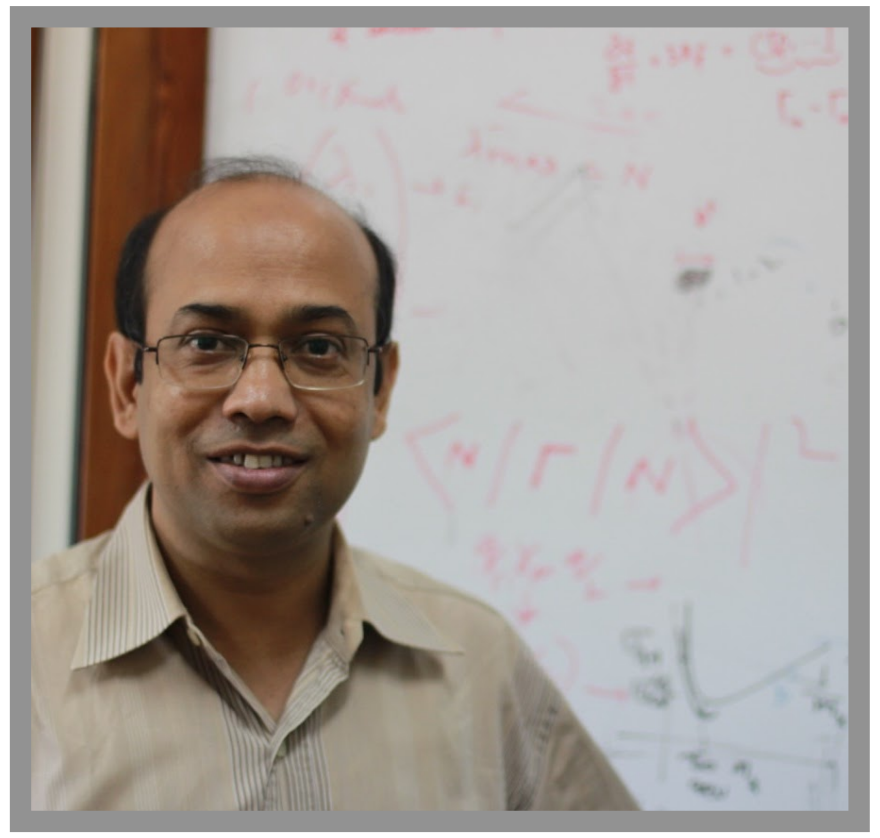
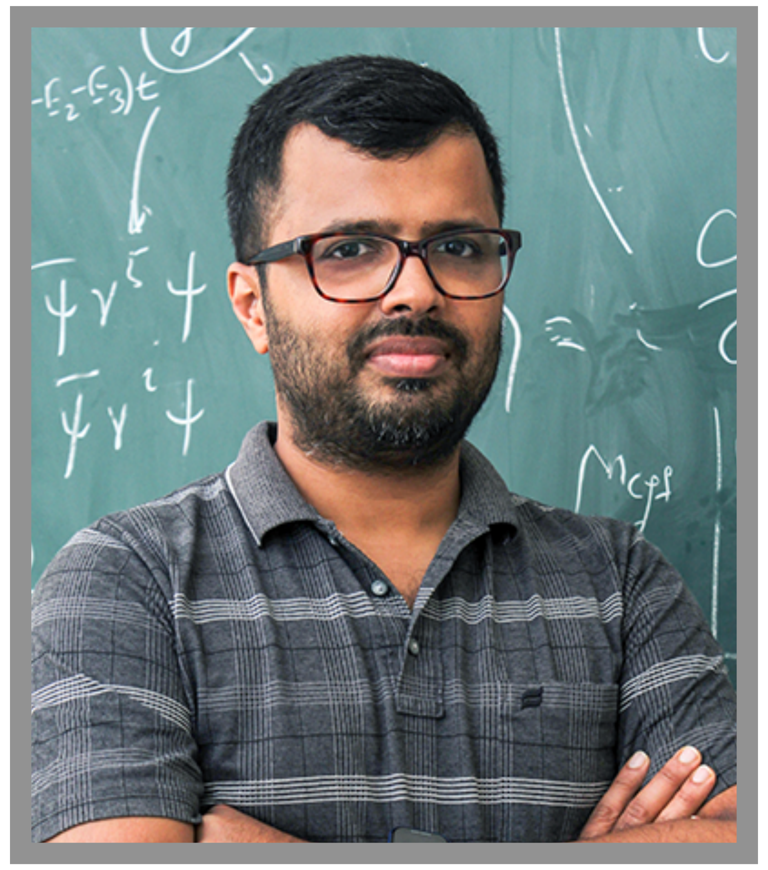


Nilmani Mathur(TIFR)

**arXiv:2503.09760 [hep-lat]**



M. Padmanath(IMSc)

# Introduction and Motivation



- **Color Confinement**:- An infinite amount of energy is required to break the bonds holding quarks within hadrons.
- Quarks cannot be directly observed; only hadronic-level studies are possible.
- The coupling strength is very high at low energy scales, necessitating non-perturbative methods.
- **Lattice QCD**: A powerful first-principles approach for studying QCD in the non-perturbative regime.

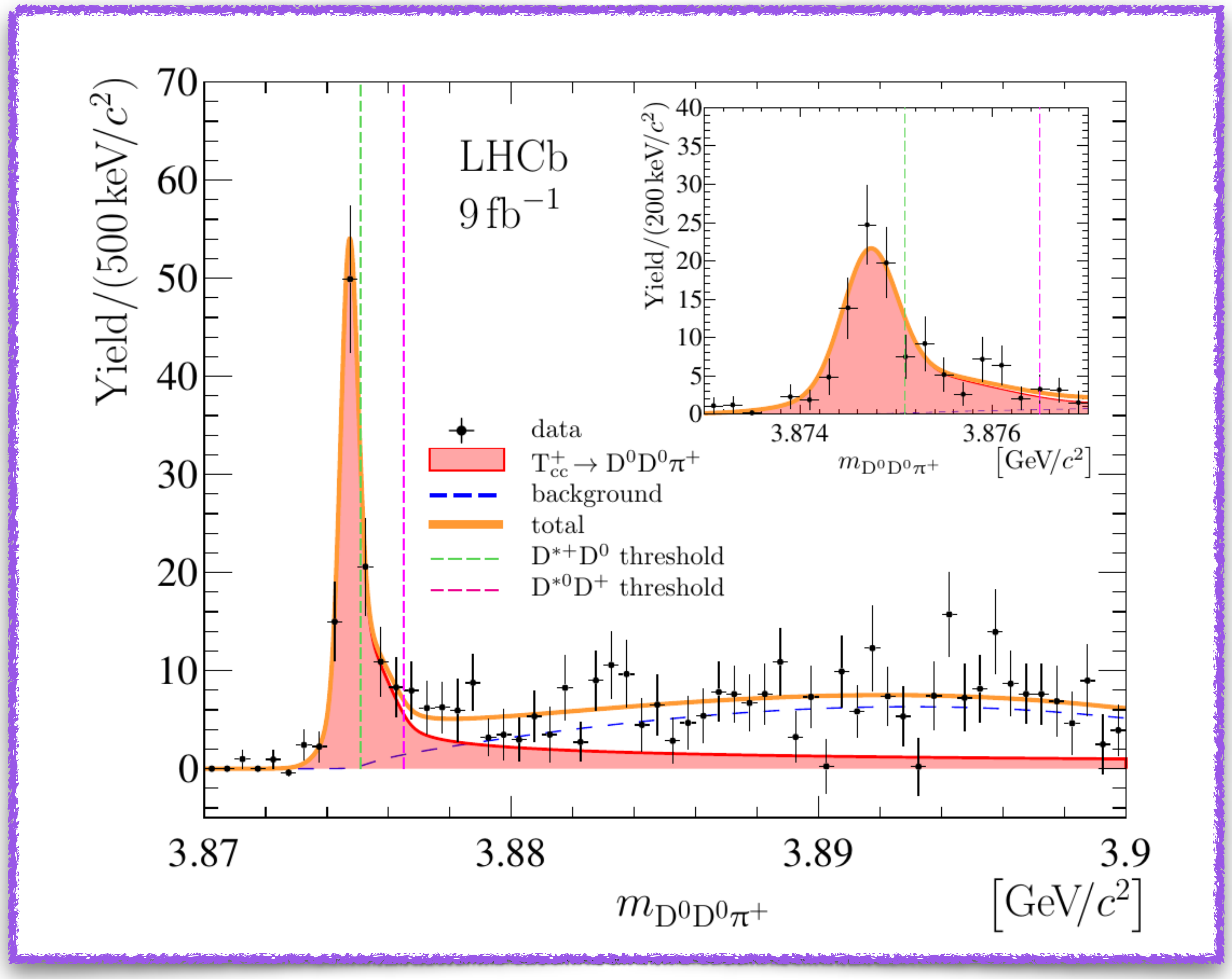
u d s $<< \Lambda_{QCD} <<$ c b t

Light Quarks — Heavy Quarks

What do I mean by heavy?

# $T_{cc}(cc\bar{u}\bar{d})$ discovery at LHC

- In 2021, LHCb made headlines by discovering the longest-lived exotic state ever, observed close to X(3872).
- It was observed in the channel $I=0$, $J^P = 1^+$ below $D^0D^{*+}$ threshold(in $D^0D^0\pi^+$).
- Many more exotic tetraquark discovered recently e.g. $T_{cs}$, $T_{c\bar{s}}$, $Z_c$ and so on. Scope for $T_{bc}$, $T_{bs}$ in near future.




Nature phys: https://rdcu.be/dNMRV
Arxiv:2109.01038

$$\delta M = M_{T_{cc^+}} - (M_{D^{*+}} + M_{D^0})$$

$$\delta M_{pole} = -360 \pm 40(^{+4}_{-0}) keV/c^2$$

$$\Gamma_{pole} = 48 \pm 2(^{+00}_{-14}) KeV$$

# Long History of $T_{bb}(bb\bar{u}\bar{d})$

- Phenomenological calculation of $T_{bb}$ can be trace back to the early 80's.

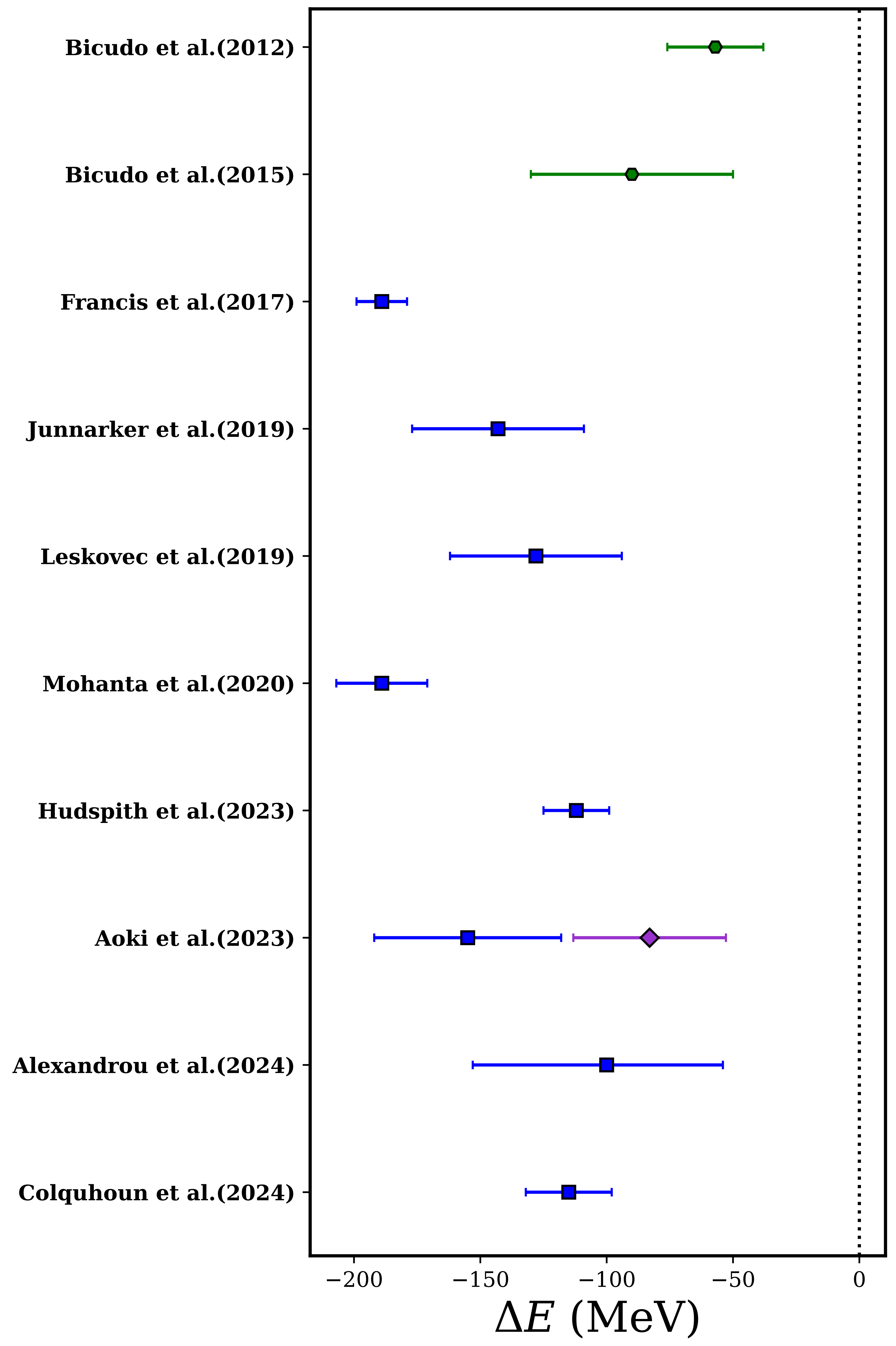
- Prediction of deeply bound state in the heavy quark limit.

Nucl.Phys.B 399 (1993)

- Is bottom Quark Heavy Enough?

- Results from various phenomenological studies suggest possibility of deeply bound state.

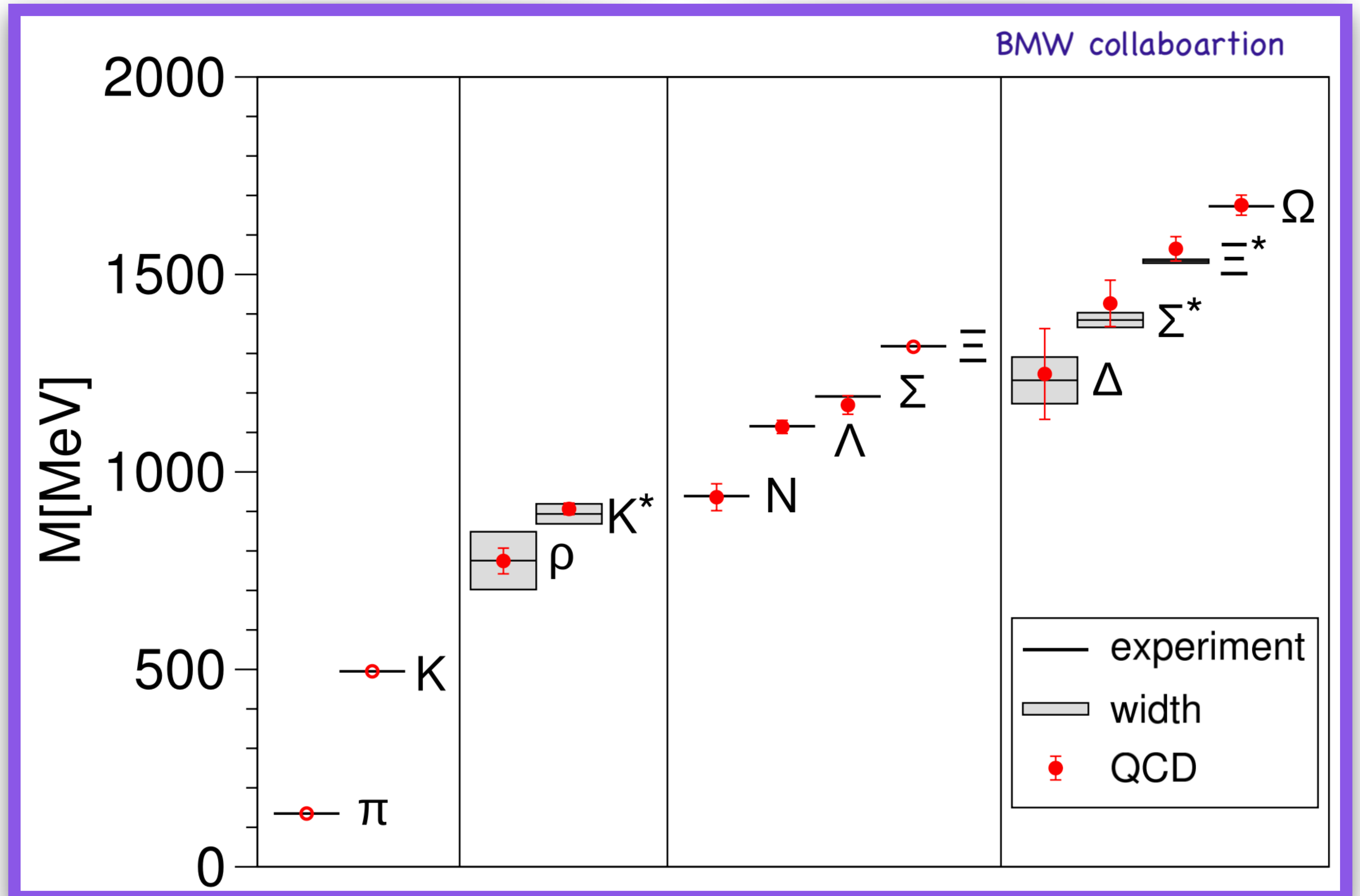
- Previous lattice calculations on $bb\bar{u}\bar{d}$ $I = 0$ shows deep bound state upto systematics.



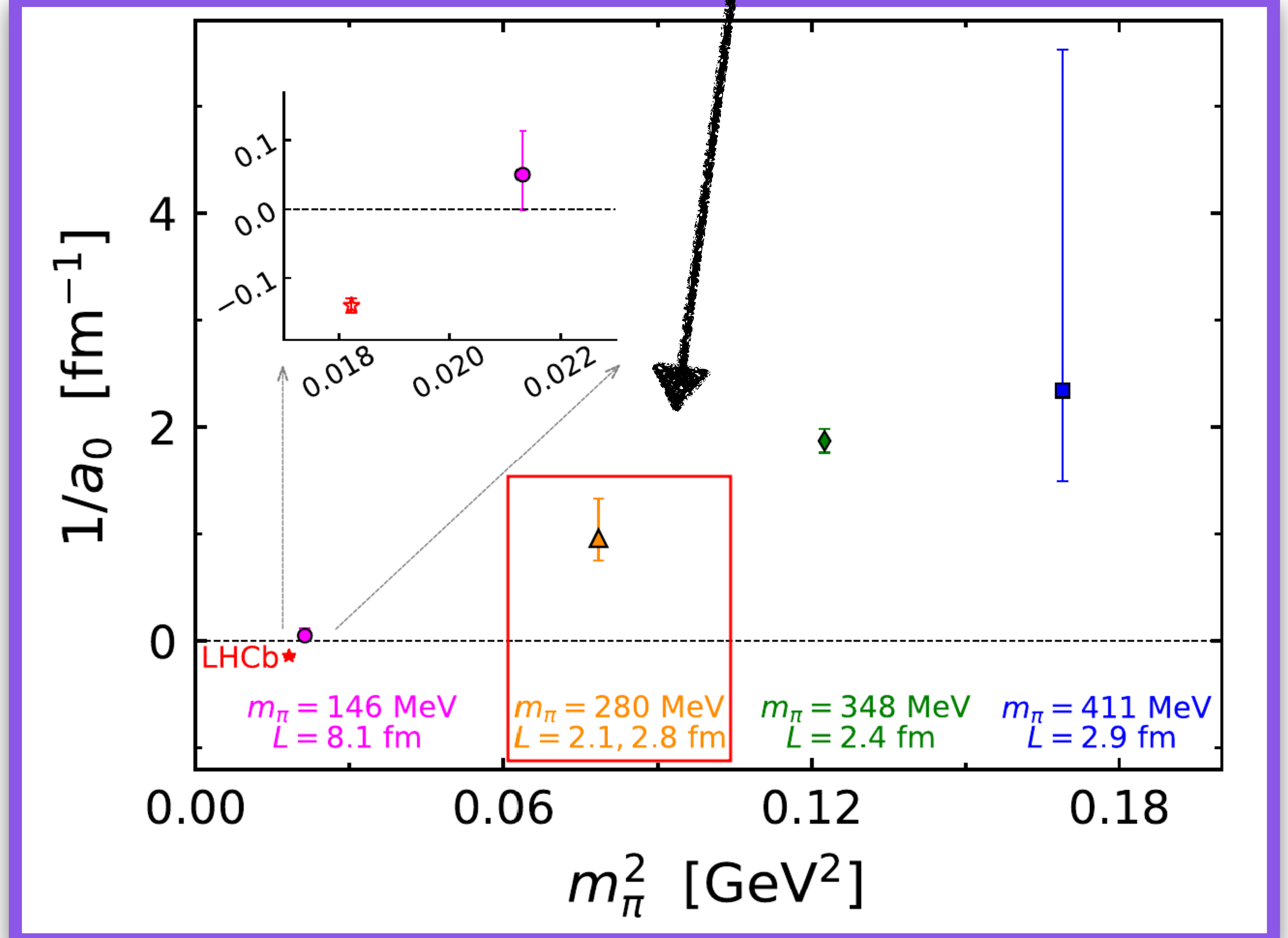
# Lattice Validations

Phys. Rev. Lett. 129, 032002 Padmanath, Prelovsek






Science 322 (2008) 1224-1227




Yan Lyu et al. Phys Rev Lett.131.161901

# Lattice Validations



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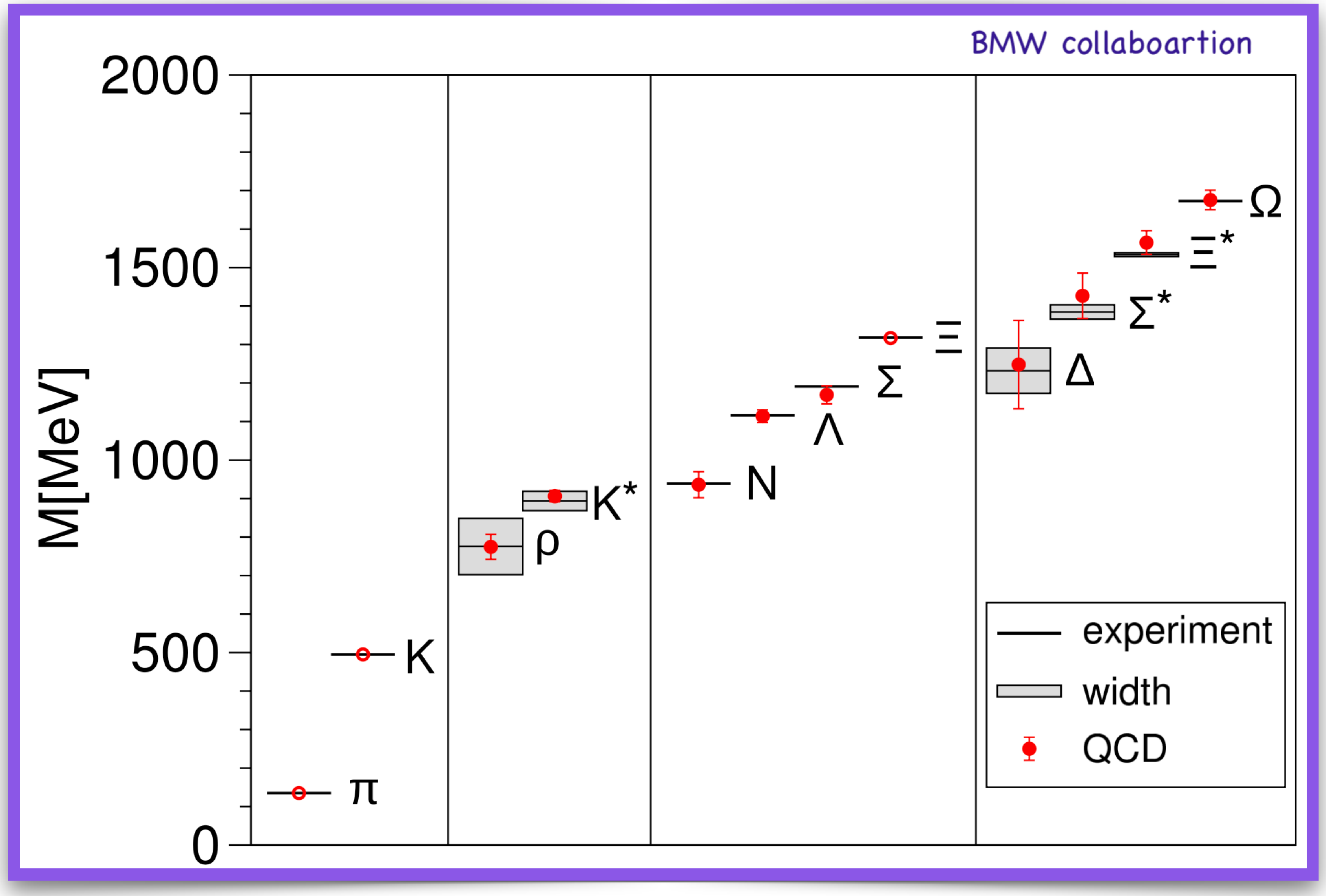


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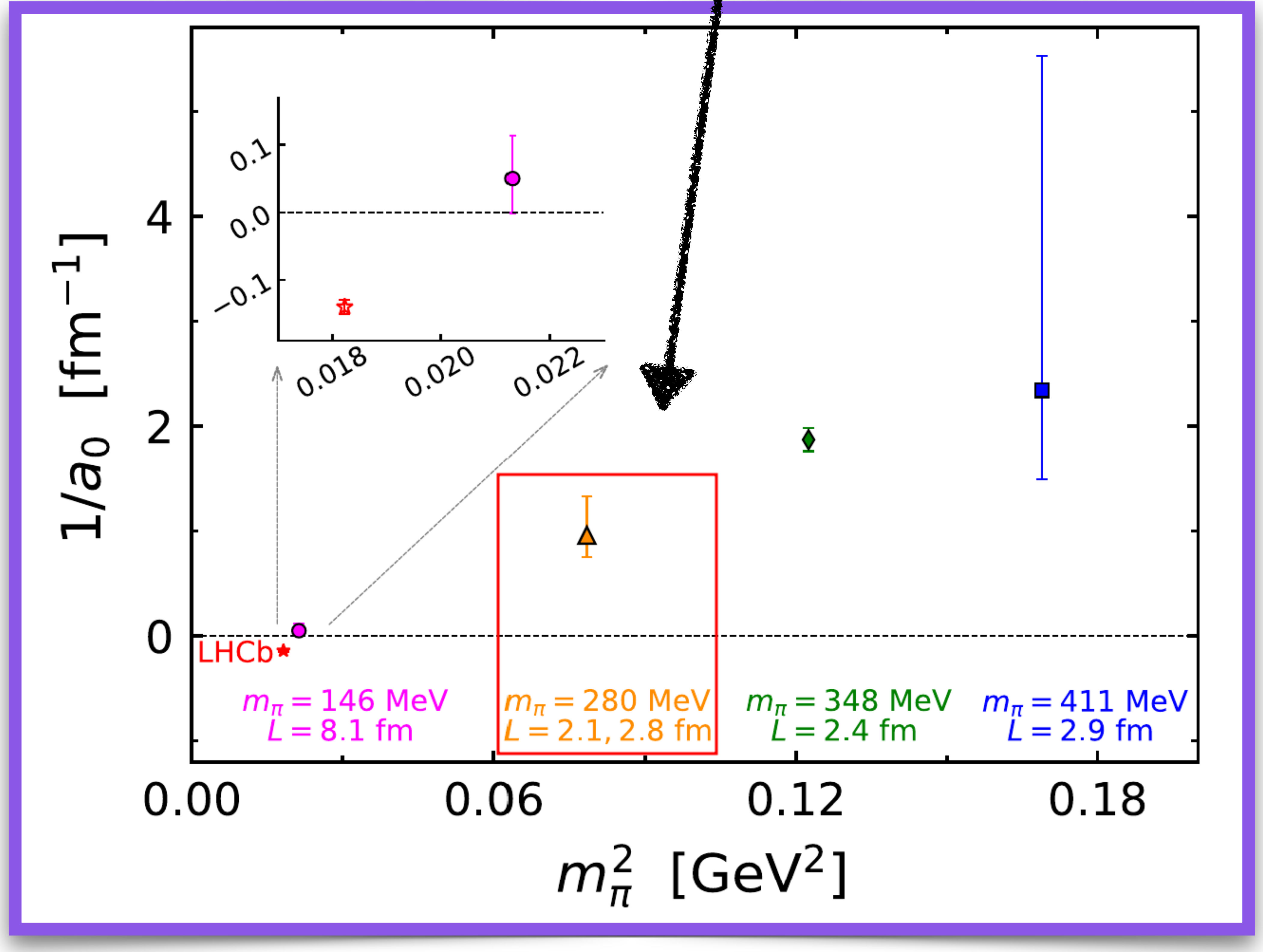


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- **Early Lattice calculations accurately predicts masses of known hadrons.**

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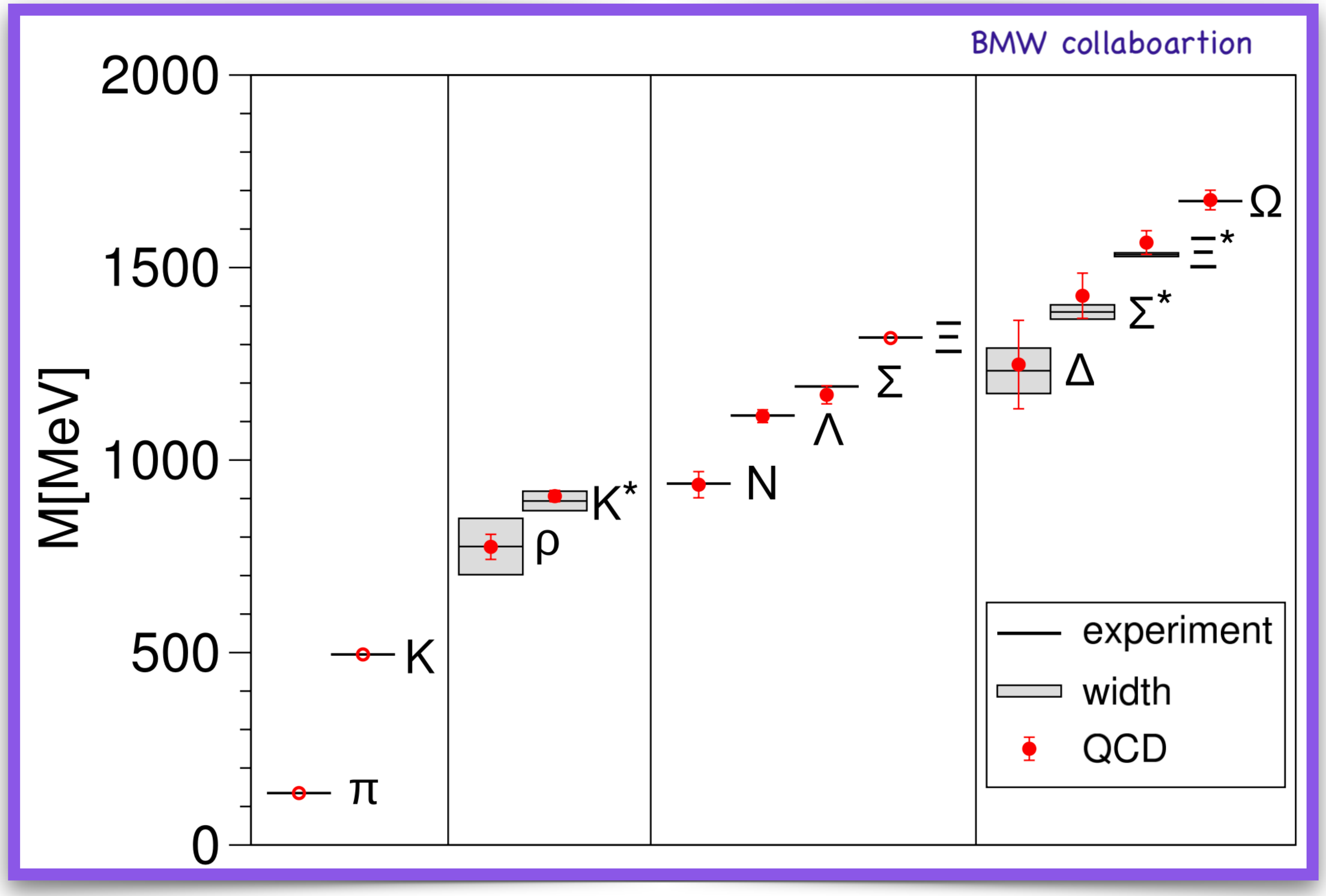


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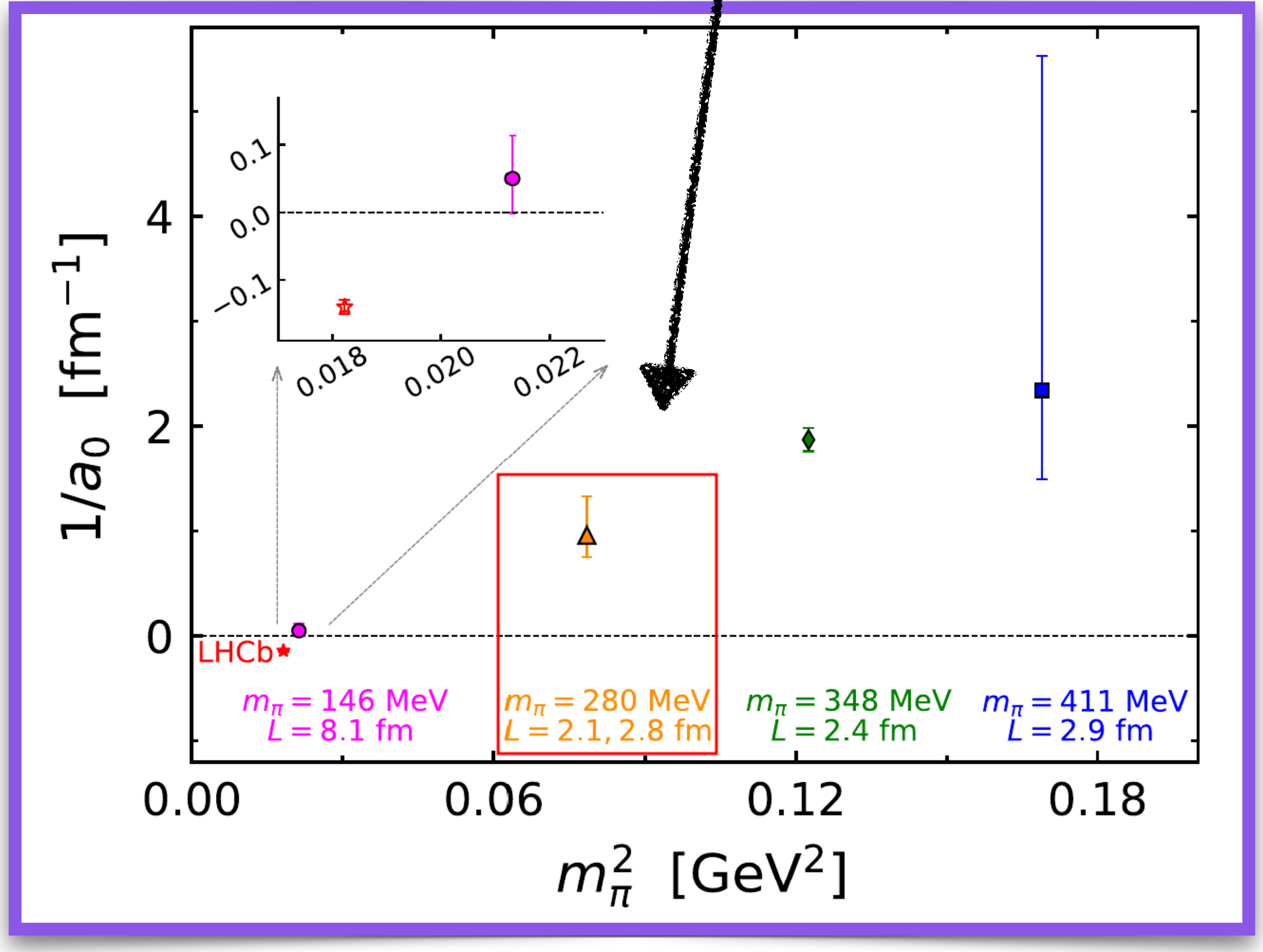


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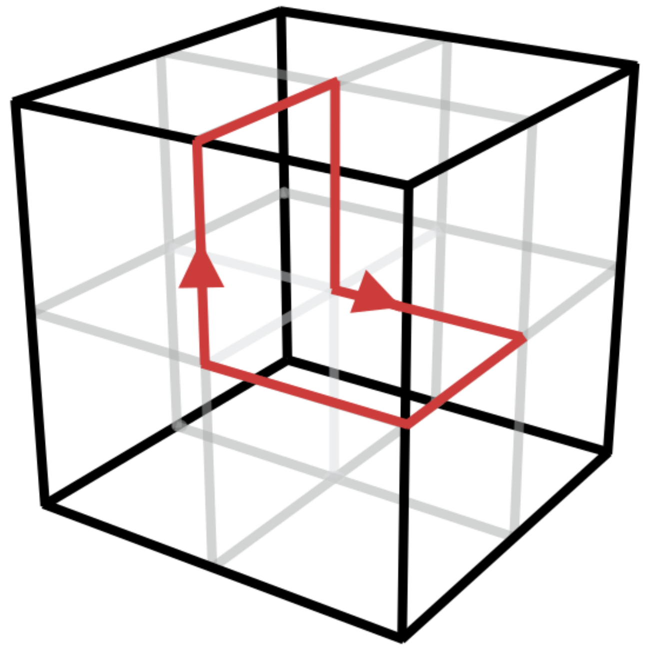
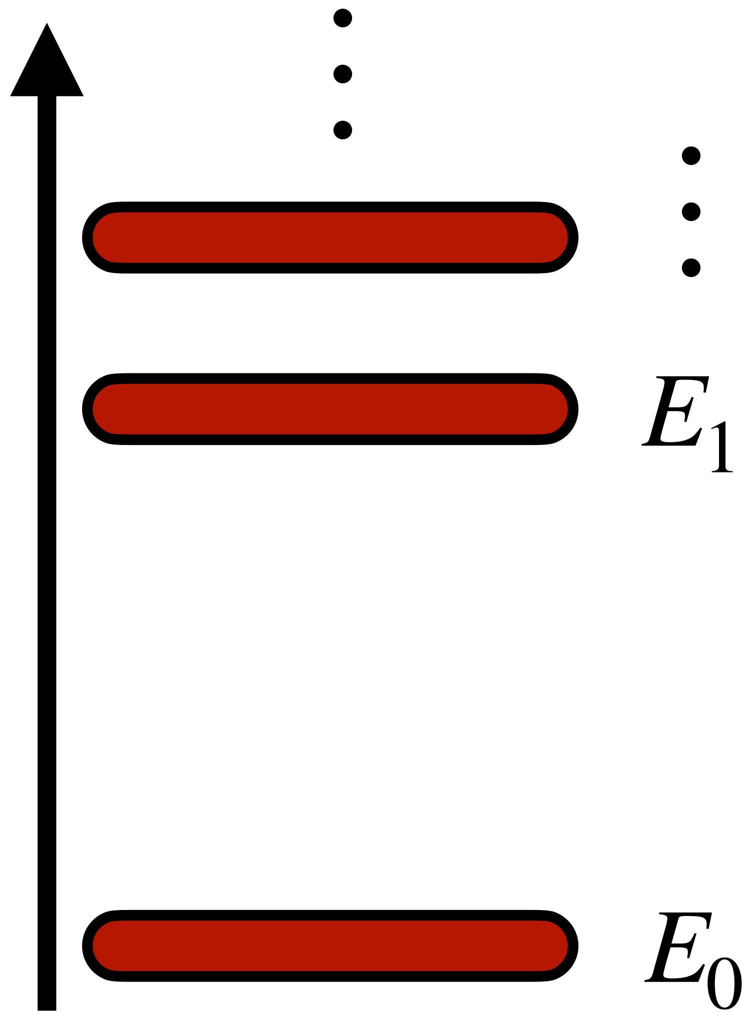
- **Early Lattice calculations accurately predicts masses of known hadrons.**
- $T_{cc}$ **lattice results matches with that of the experimental results as pion mass decreases.**

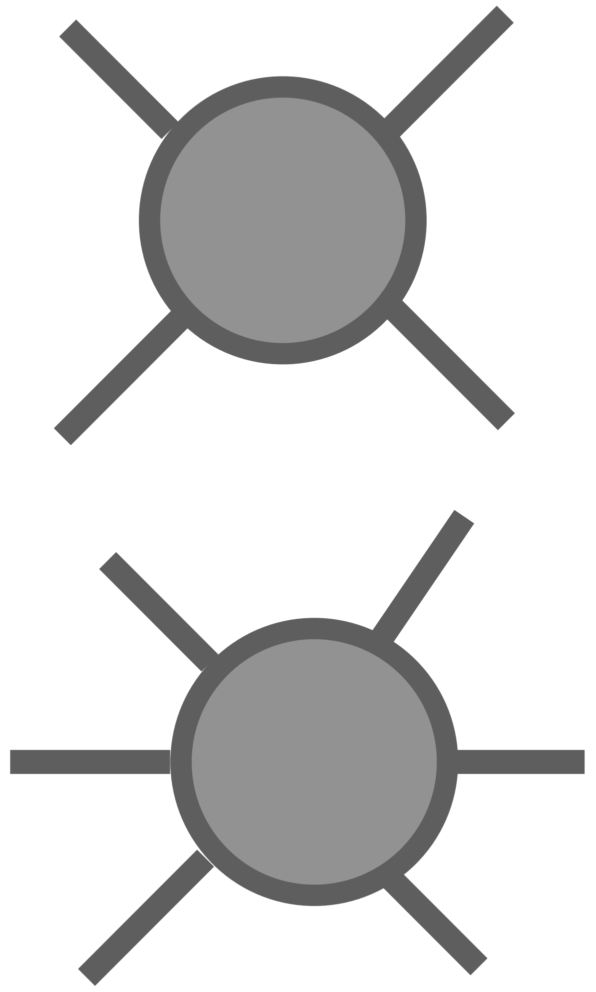
# Hadron Spectroscopy in a Nutshell



Ensemble Generation




Finite Volume Spectra



Scattering Amplitude

$a \to 0$

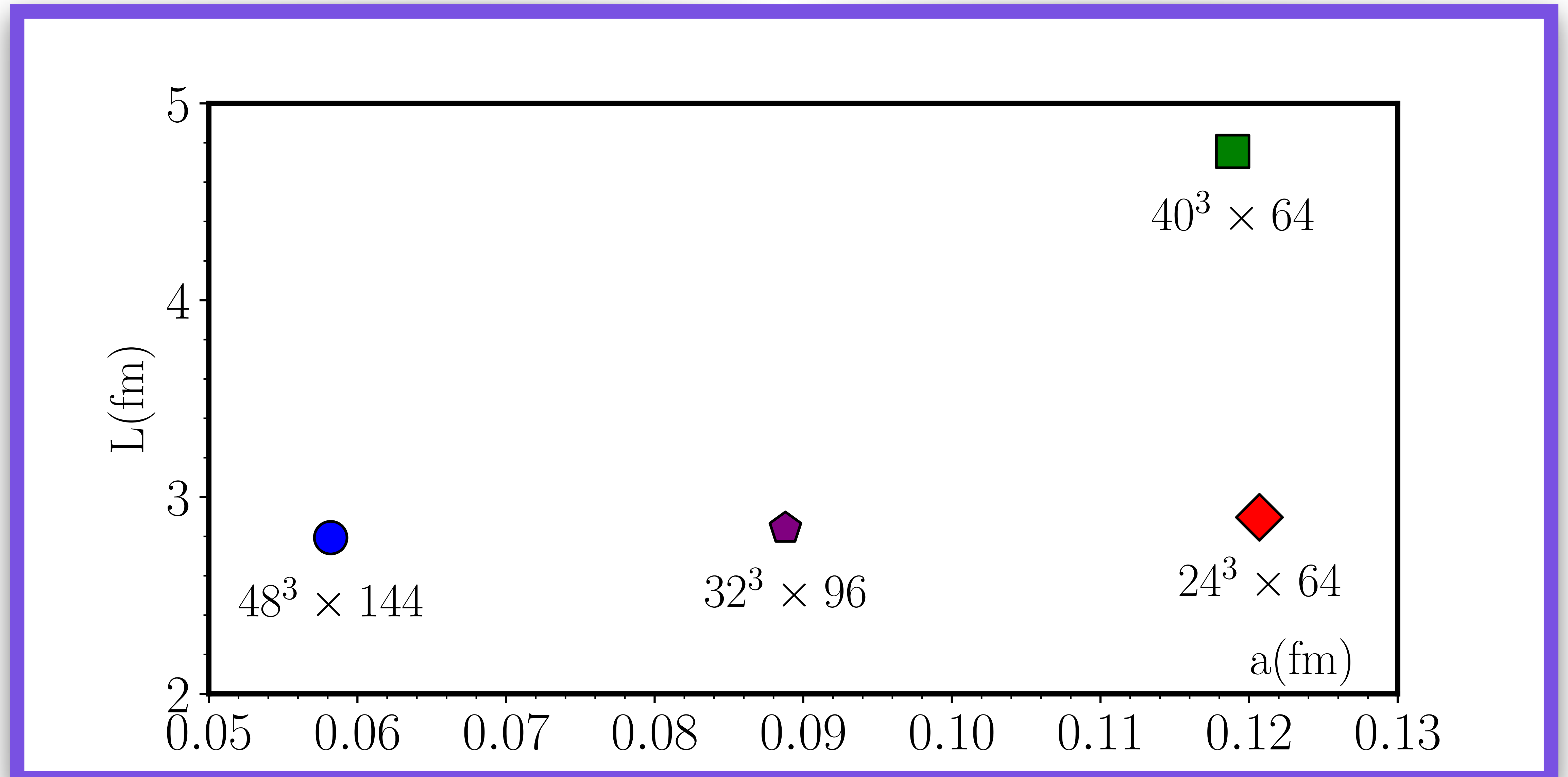
$L \to \infty$

$M_{ps} \to M_{\pi}^{phy}$

Continuum Extrapolation

# Lattice Setup

- We are interested in doubly bottom tetra quarks $bb\bar{u}\bar{d}$ with $I(J^P) = 0(1^+)$ and $bs\bar{u}\bar{d}$ with $I(J^P) = 0(1^+)$ and $0(0^+)$.
- Worked with 4 MILC ensembles with $N_f = 2 + 1 + 1$ using HISQ action.
- Ensembles were generated at unphysical light quarks and physical charm and strange quarks.
- Light quark propagators were constructed using Overlap action. For heavy(bottom) quark, we used NRQCD action.
- Wall-source smearing setup.
- Used multiple volumes, box-sink correlators to reduce systematic effects.
- First lattice calculation for $bs\bar{u}\bar{d}$ with finite volume analysis.



# Building Hadrons



- Goal is to construct interpolating fields coupled with ground state of desired quantum number.

- Properties:-

  1. Flavor Sturture:- Correct combination of quark fields.

  2. Spin and Parity:- Appropriate Dirac bilinear $\Gamma$ and quadrilinear(Tqs) to match desired spin and parity.

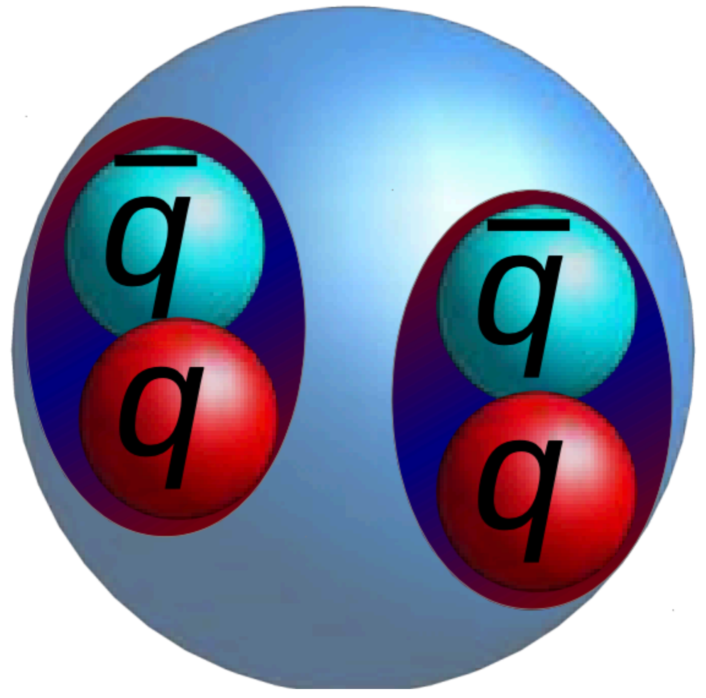
- Lattice breaks O(3) symmetry, operators must transform according to irreps of the cubic group.

- e.g. Pion:- $\Phi_\pi = \bar{d}^\alpha_c(\gamma_5)_{\alpha\beta}u^\beta_c$ , c -> color index

  $\gamma_5$ → Pseudo scalar

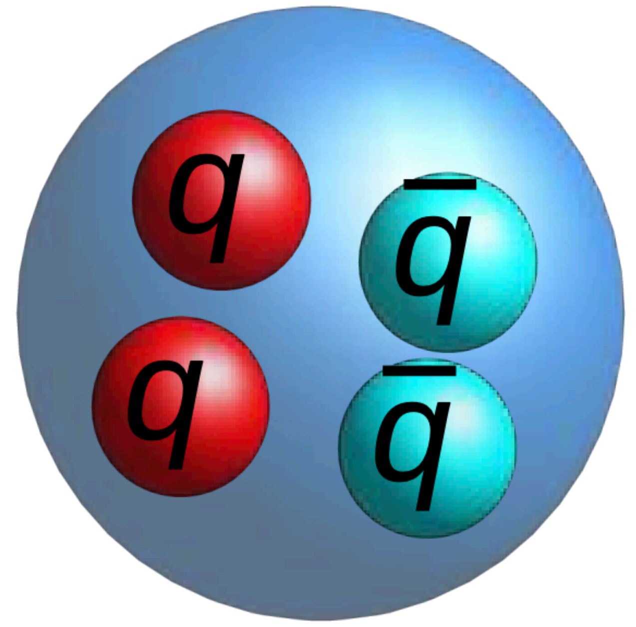
# Extracting Finite Volume Spectrum

- To extract spectrum we need good interpolating operators.
- Here we are using two types of operators.




**Meson-Meson** :

$$\Phi_{\mathscr{M}_{BB^*}}(x) = [\bar{u}(x)\gamma_i b(x)][\bar{d}(x)\gamma_5 b(x)] - [\bar{u}(x)\gamma_5 b(x)][\bar{d}(x)\gamma_i b(x)]$$




**Diquark-antidiquark**:

$$\Phi_{\mathscr{D}}(x) = [((\bar{u}(x)^T C\gamma_5 \bar{d}(x)) - (\bar{d}(x)^T C\gamma_5 \bar{u}(x))) \times (b^T(x) C\gamma_i b(x))]$$

- Finite volume spectrum can be calculated using Euclidean $\mathscr{C}_{ij}(t)$, between $\Phi's$

$$\mathscr{C}_{ij}(t) = \sum_X \left\langle \Phi_i(\mathbf{x}, t)\tilde{\Phi}_j^\dagger(0) \right\rangle$$

# Extracting Finite Volume Spectrum

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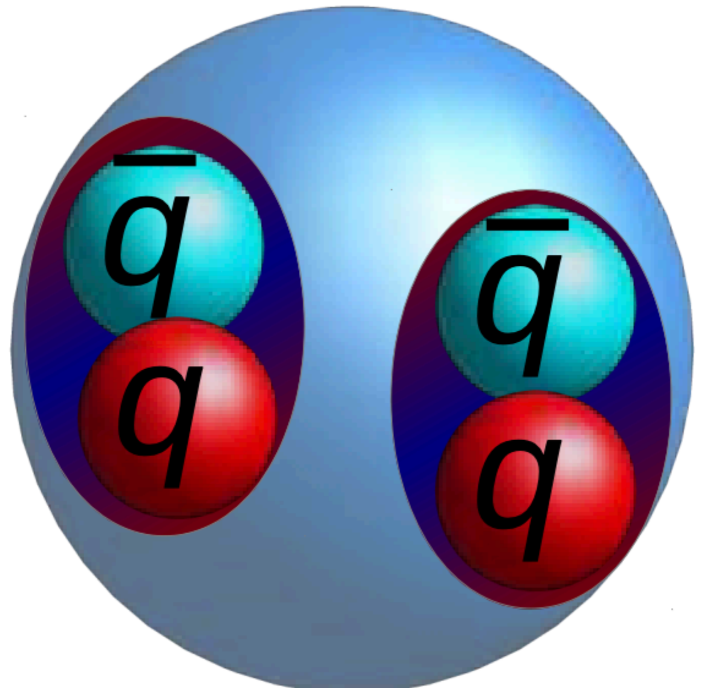

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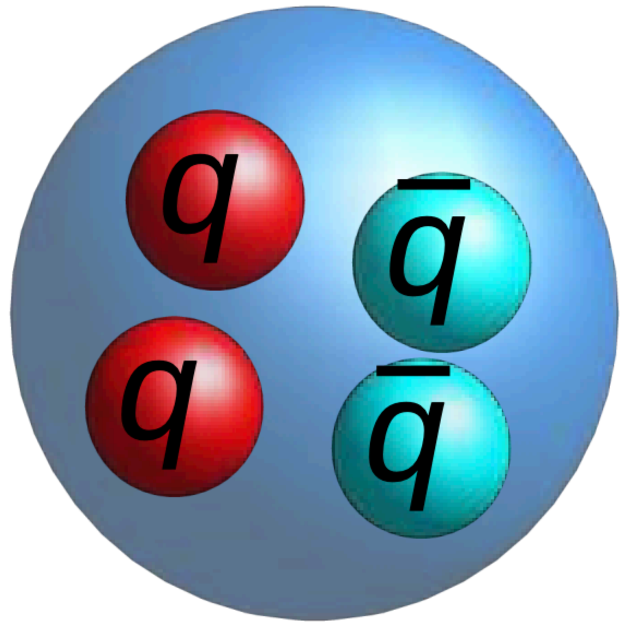

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$$\mathscr{C}_{ij}(t) = \sum_X \left\langle \Phi_i(\mathbf{x}, t)\tilde{\Phi}_j^\dagger(0) \right\rangle \sim e^{-E_n t}$$

# Operators for $T_{bs}$

- For axial-vector $I(J^P) = 0(1^+)$, we use three operators,

$$\Phi_{\mathcal{M}_{KB^*}}(x) = \left[\bar{u}(x)\gamma_i b(x)\right]\left[\bar{d}(x)\gamma_5 s(x)\right] - \left[\bar{u}(x)\gamma_5 s(x)\right]\left[\bar{d}(x)\gamma_i b(x)\right]$$

$$\Phi_{\mathcal{M}_{BK^*}}(x) = \left[\bar{u}(x)\gamma_5 b(x)\right]\left[\bar{d}(x)\gamma_i s(x)\right] - \left[\bar{u}(x)\gamma_i s(x)\right]\left[\bar{d}(x)\gamma_5 b(x)\right]$$

$$\Phi_{\mathcal{D}}(x) = \left[\left(\bar{u}(x)^T C\gamma_5 \bar{d}(x) - \bar{d}(x)^T C\gamma_5 \bar{u}(x)\right) \times \left(b^T(x) C\gamma_i s(x)\right)\right]$$

- For scalar $I(J^P) = 0(0^+)$, we use two operators,

$$\Phi_{\mathcal{M}_{BK}}(x) = \left[\bar{u}(x)\gamma_5 b(x)\right]\left[\bar{d}(x)\gamma_5 s(x)\right] - \left[\bar{u}(x)\gamma_5 b(x)\right]\left[\bar{d}(x)\gamma_5 s(x)\right]$$

$$\Phi_{\mathcal{D}}(x) = \left[\left(\bar{u}(x)^T C\gamma_5 \bar{d}(x) - \bar{d}(x)^T C\gamma_5 \bar{u}(x)\right) \times (b^T(x) C\gamma_5 s(x))\right]$$

# Finite Volume Spectrum cont.



- We use GEVP to extract finite volume spectrum from correlation-matrix variationaly.

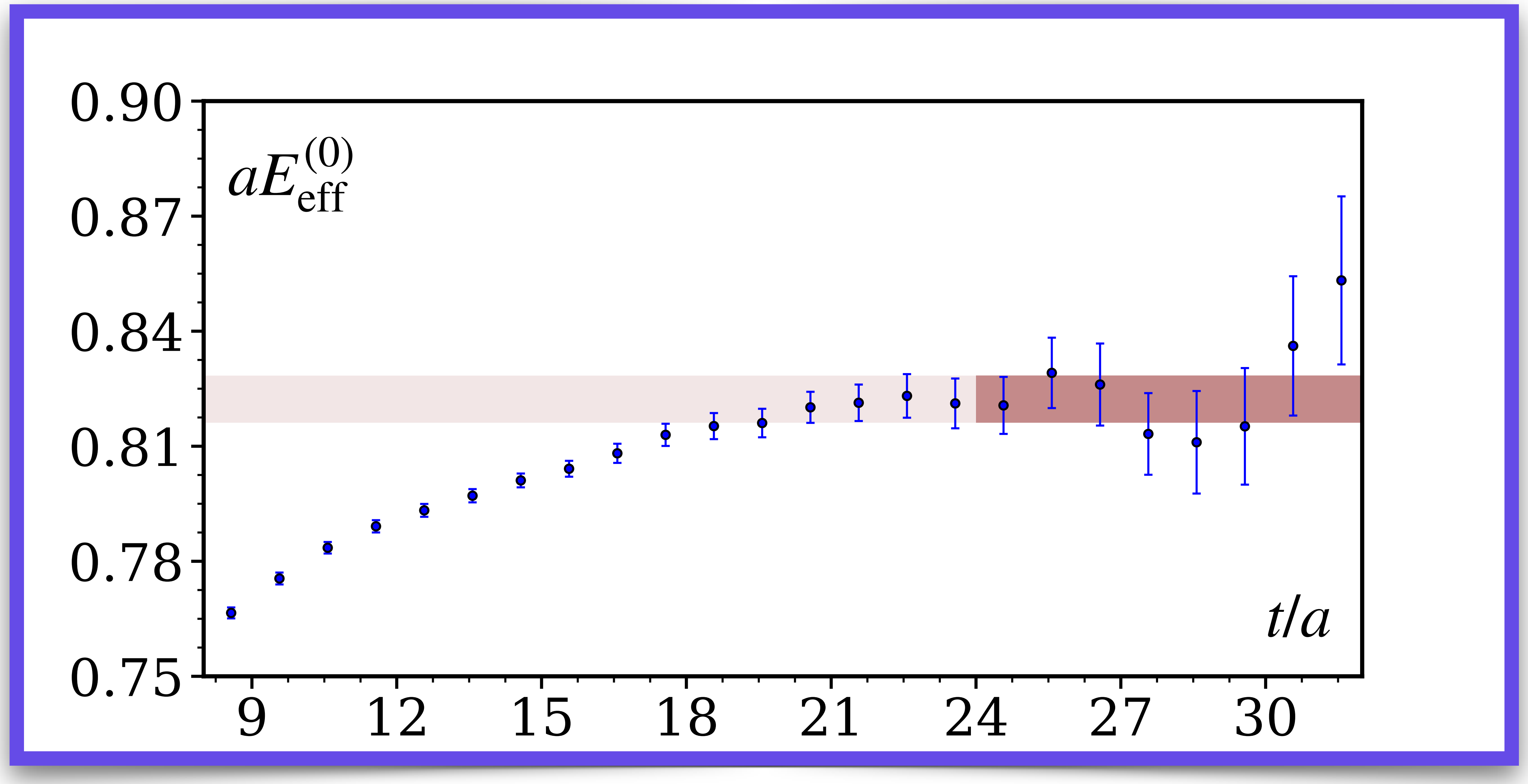
$$\mathscr{C}(t)v^{(n)}(t) = \lambda^{(n)}(t, t_0)\mathscr{C}(t_0)v^{(n)}(t)$$

- Fitting the leading exponential of $\lambda^n(t)$, yields the energy Eigen state $E_n$.

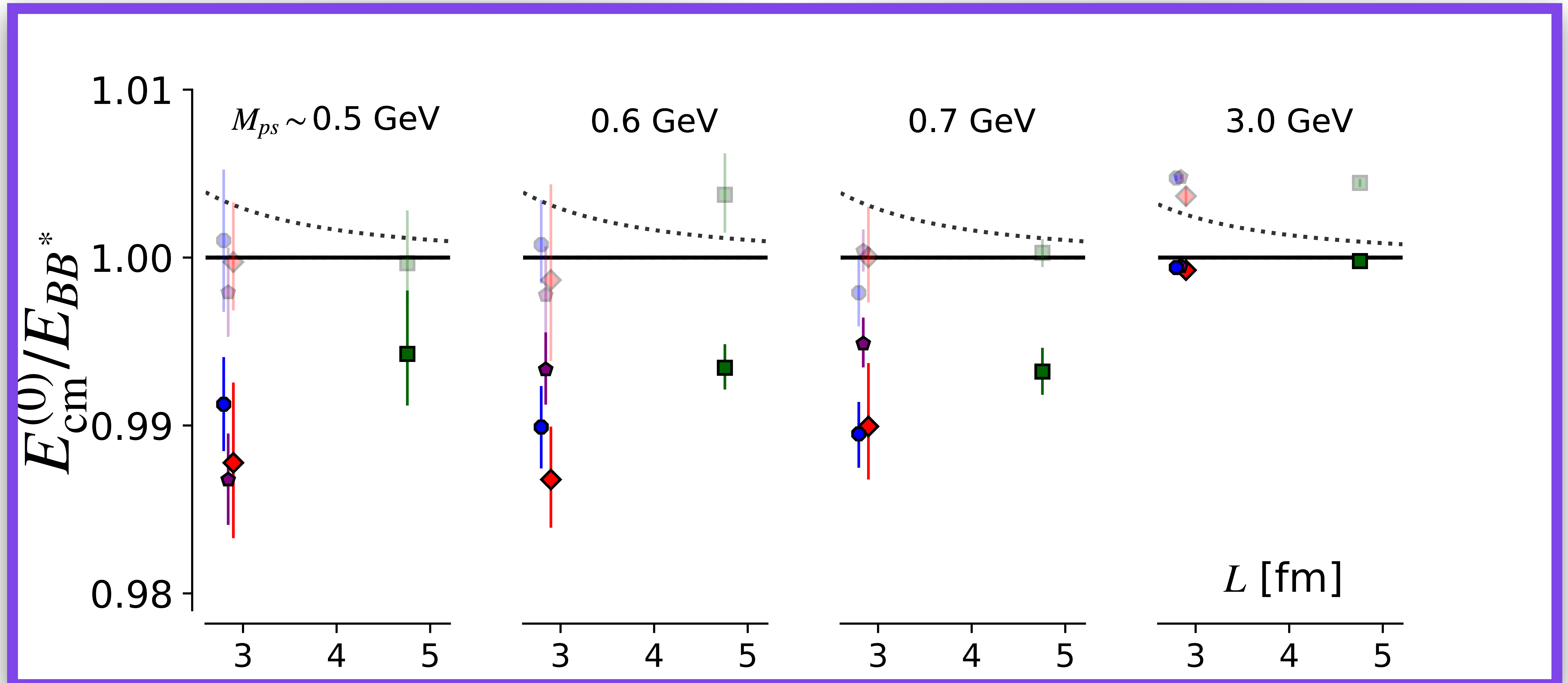
$$\lambda^n(t, t_0) = |A_n|^2 e^{-E_n(t-t_0)}[1 + \mathcal{O}(e^{-\Delta_n(t-t_0)})]$$

- Excited states can be determined with this method.

- Repeated for $B$ and $B^*$ mesons.



# Finite Volume Spectrum cont.



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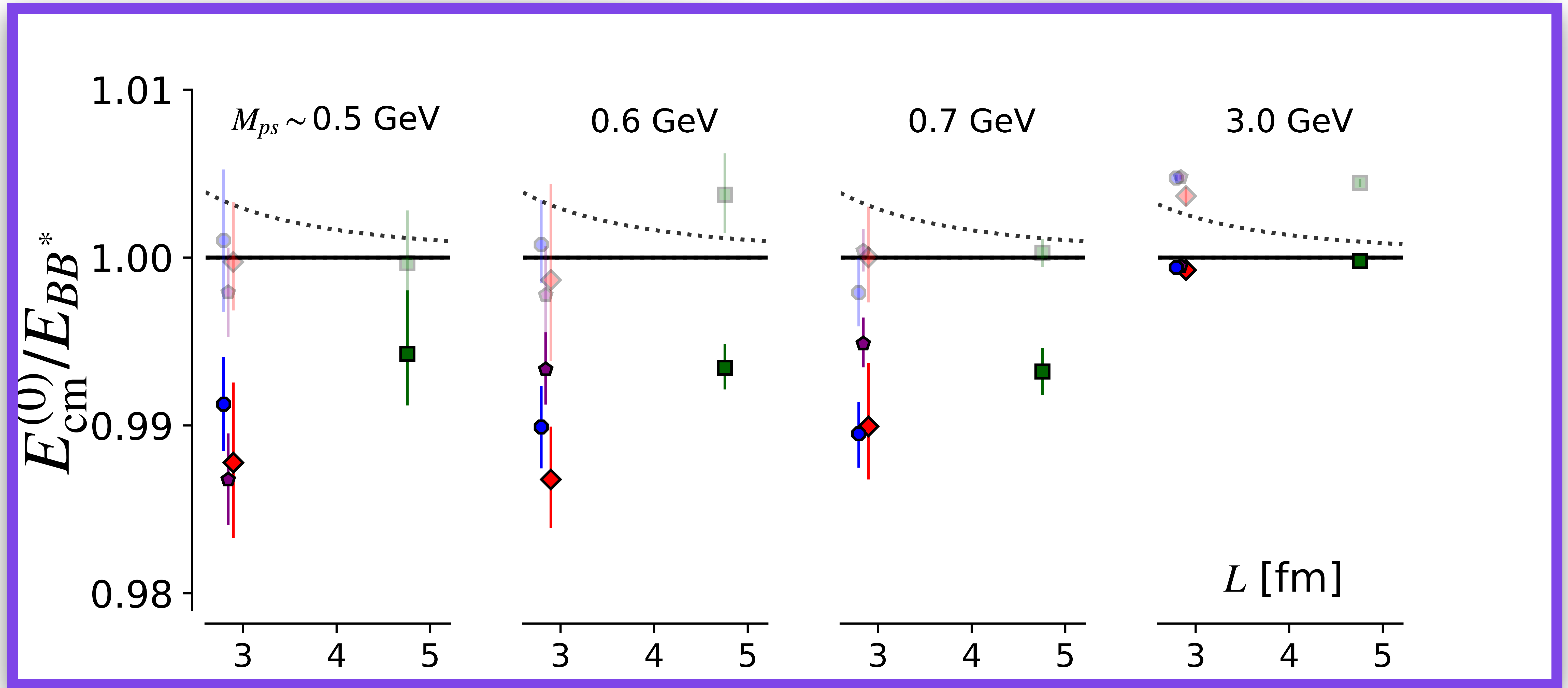
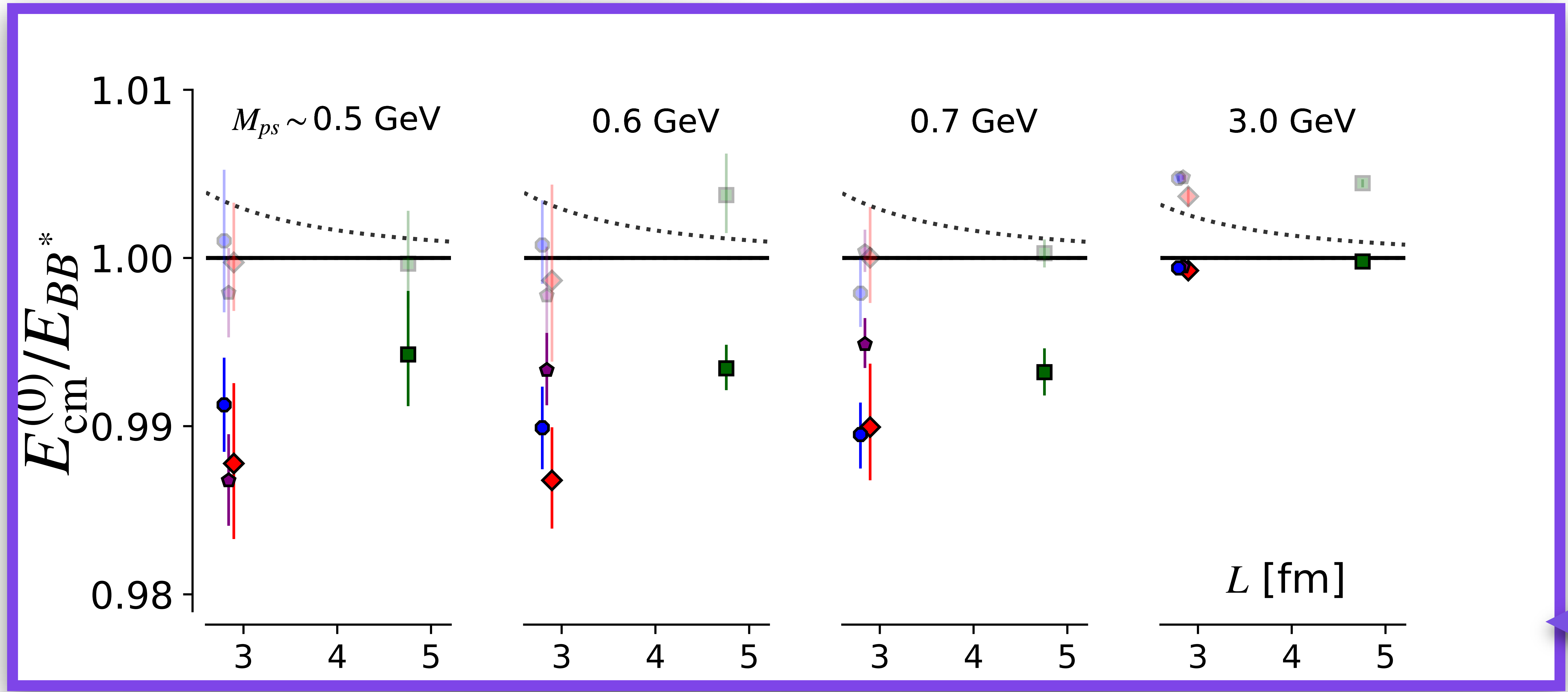



- **Spectrum extraction repeated for every $M_{ps}$ and every ensemble.**

# Finite Volume Spectrum cont.

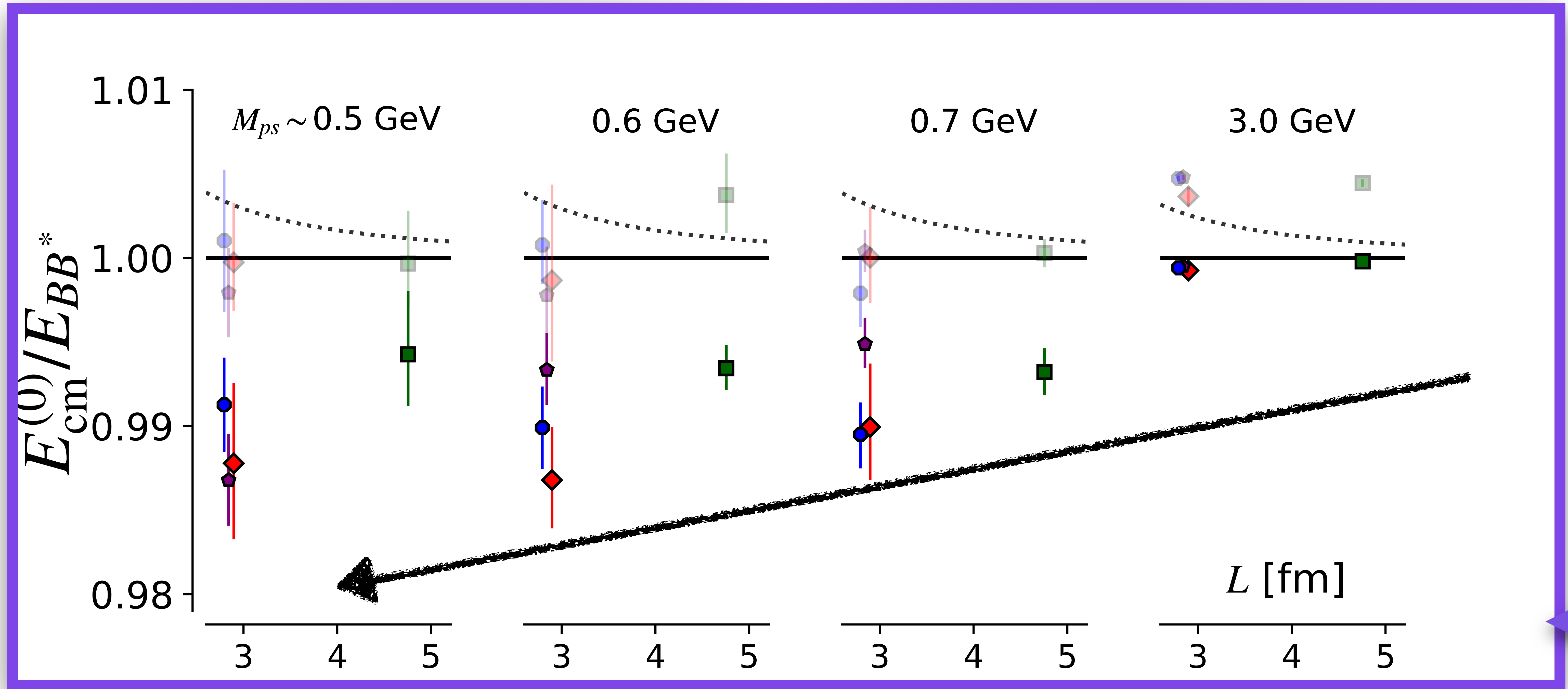





- Spectrum extraction repeated for every $M_{ps}$ and every ensemble.

Spectrum were calculated in units of nearest Two body decay threshold $BB*$.

# Finite Volume Spectrum cont.

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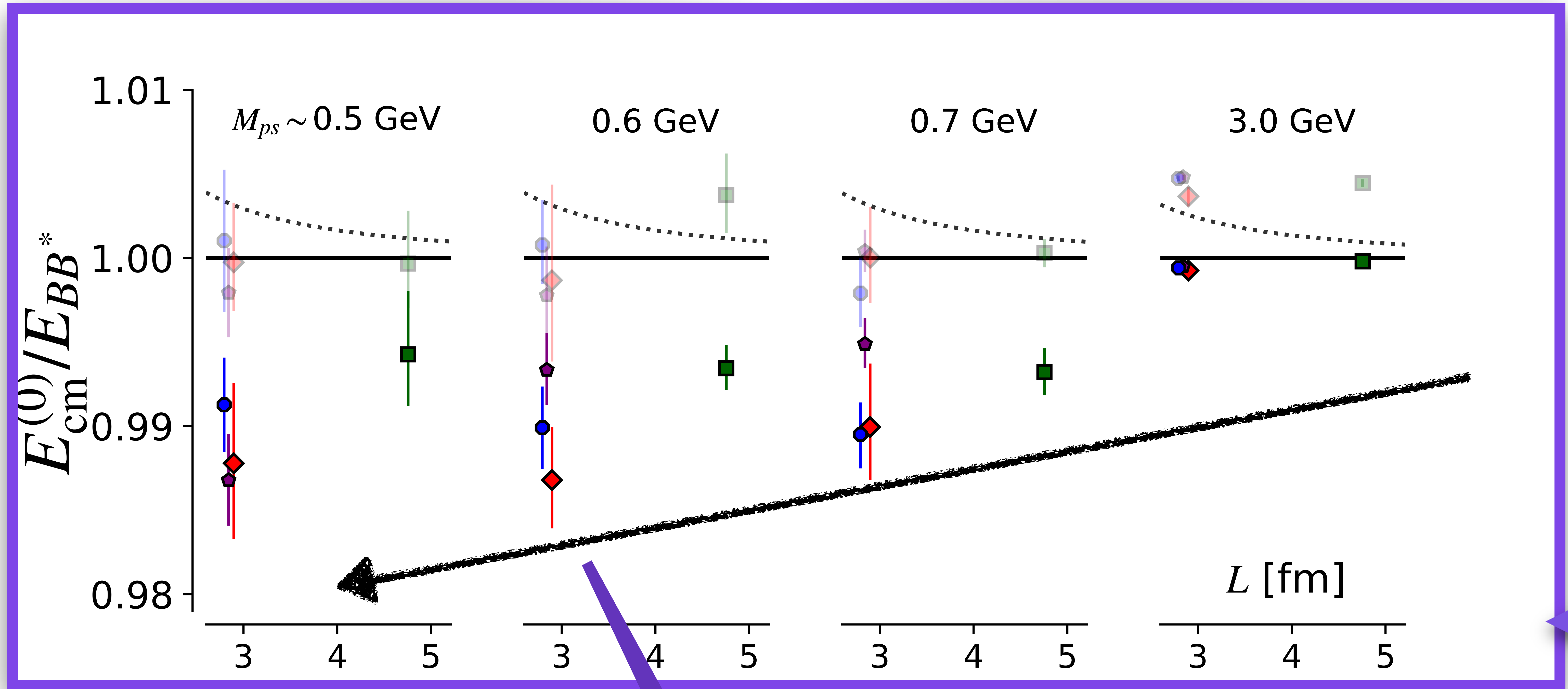
# Finite Volume Spectrum cont.





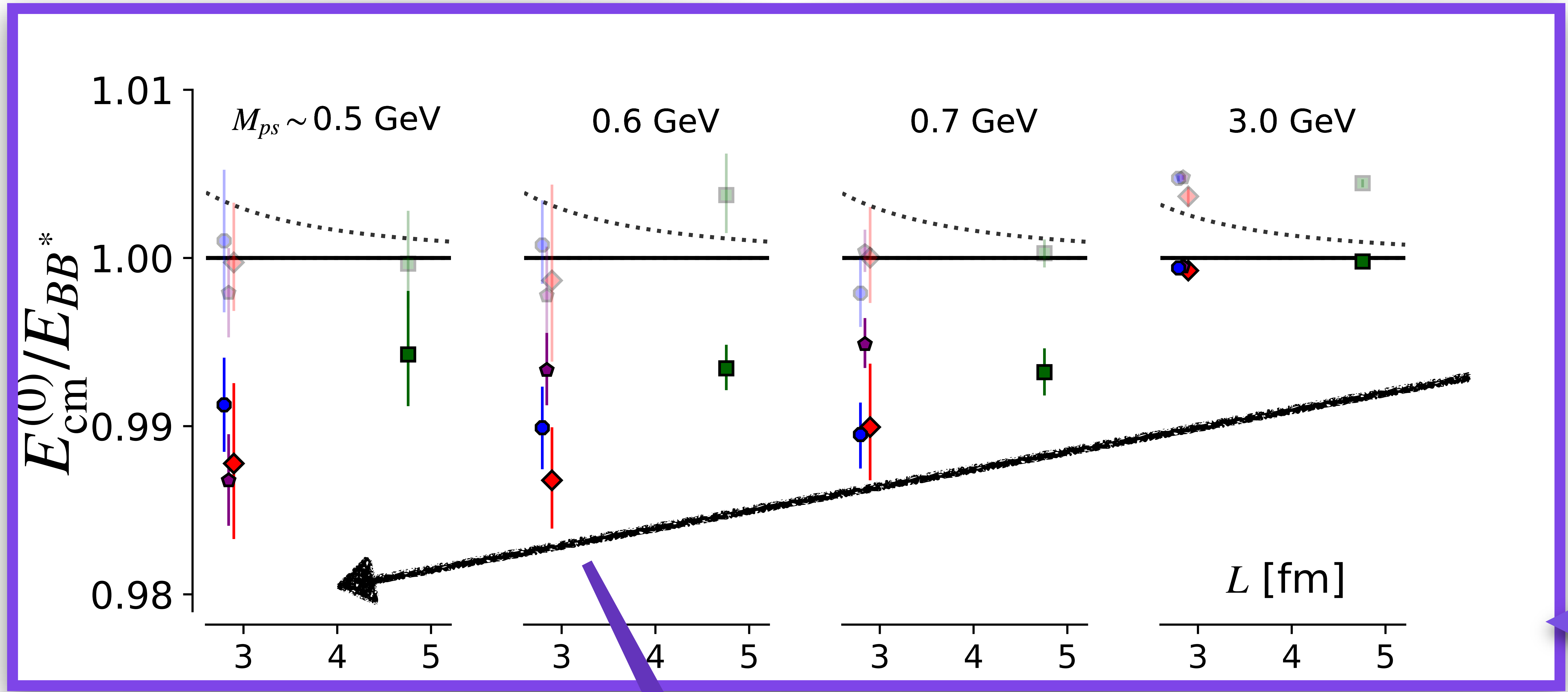

- Spectrum extraction repeated for every $M_{ps}$ and every ensemble.

Spectrum were calculated in units of nearest Two body decay threshold $BB*$.

A decreasing trend can be observed

# Finite Volume Spectrum cont.






- Spectrum extraction repeated for every $M_{ps}$ and every ensemble.

Spectrum were calculated in units of nearest Two body decay threshold $BB*$.

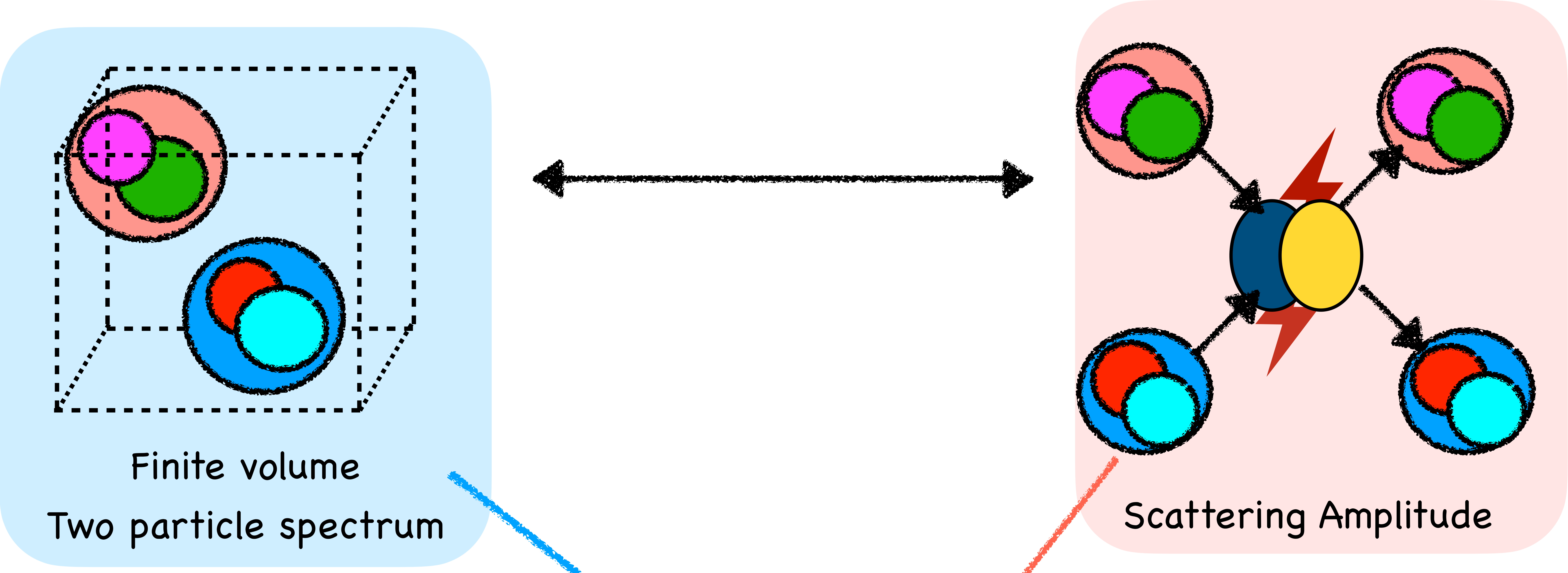
A decreasing trend can be observed

We need continuum extrapolation to have results in physical limit

# Amplitude Analysis

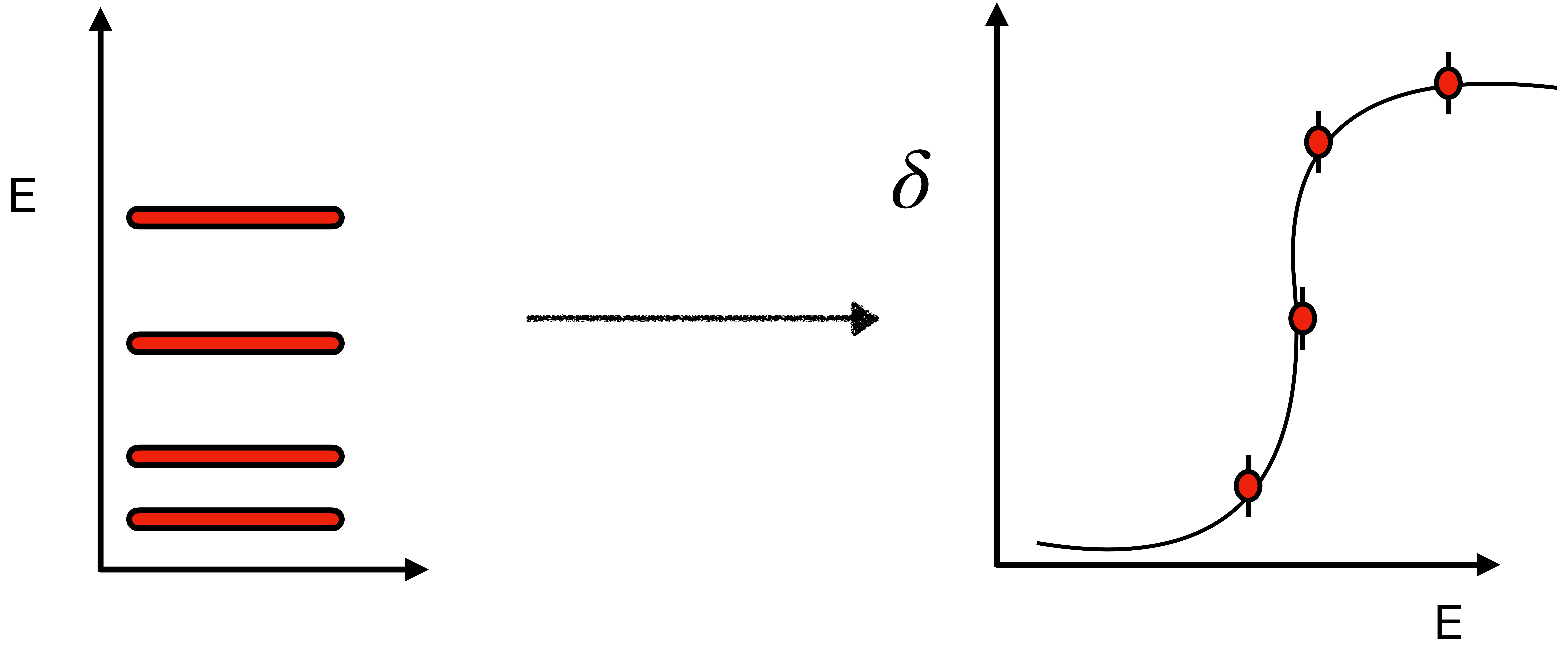


**Lüscher based quantization condition(1991)**




$$det\left[1 + i\mathscr{G}(E)\ \mathscr{M}(E)\right] = 0$$

# Amplitude Analysis



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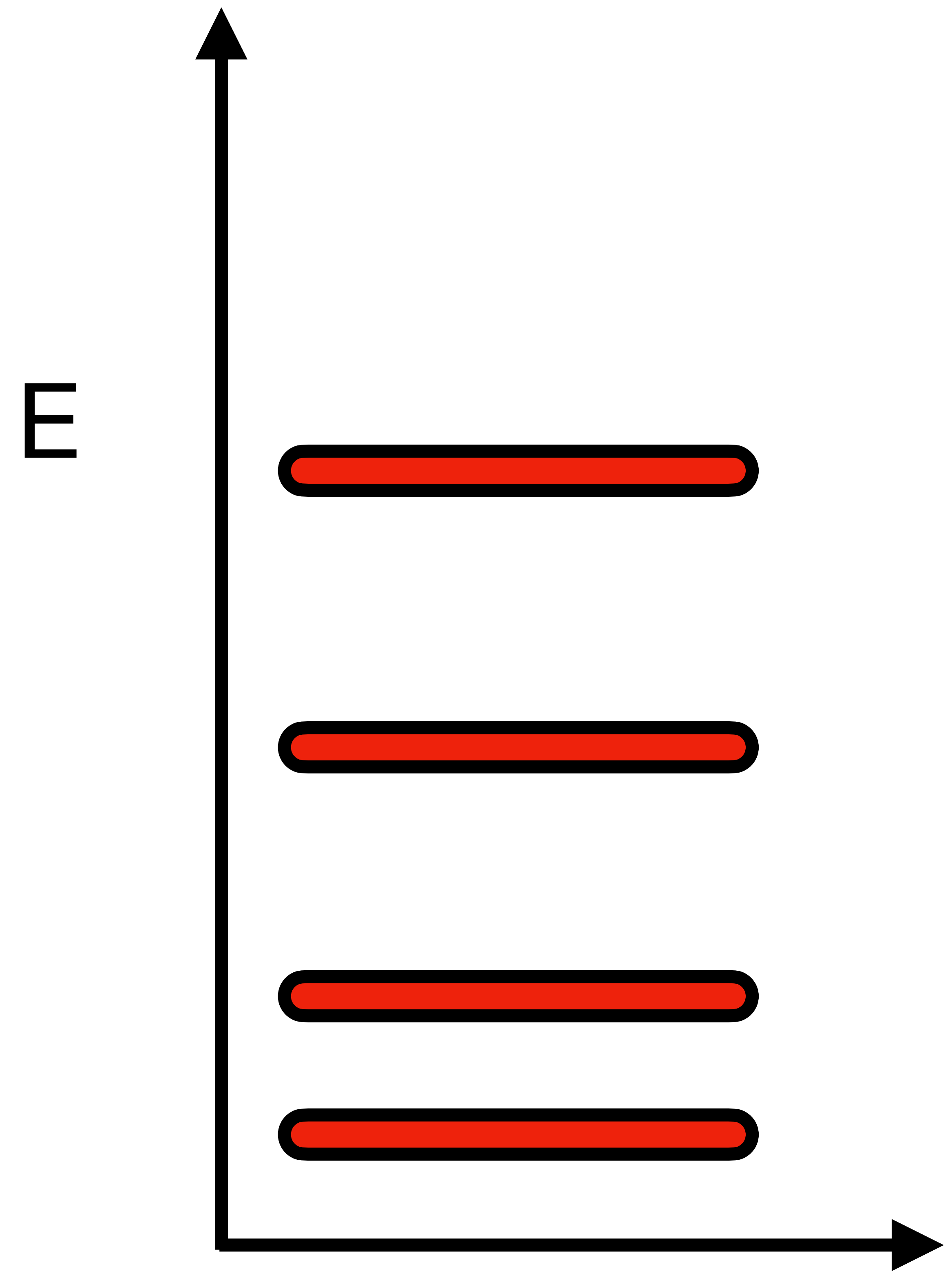
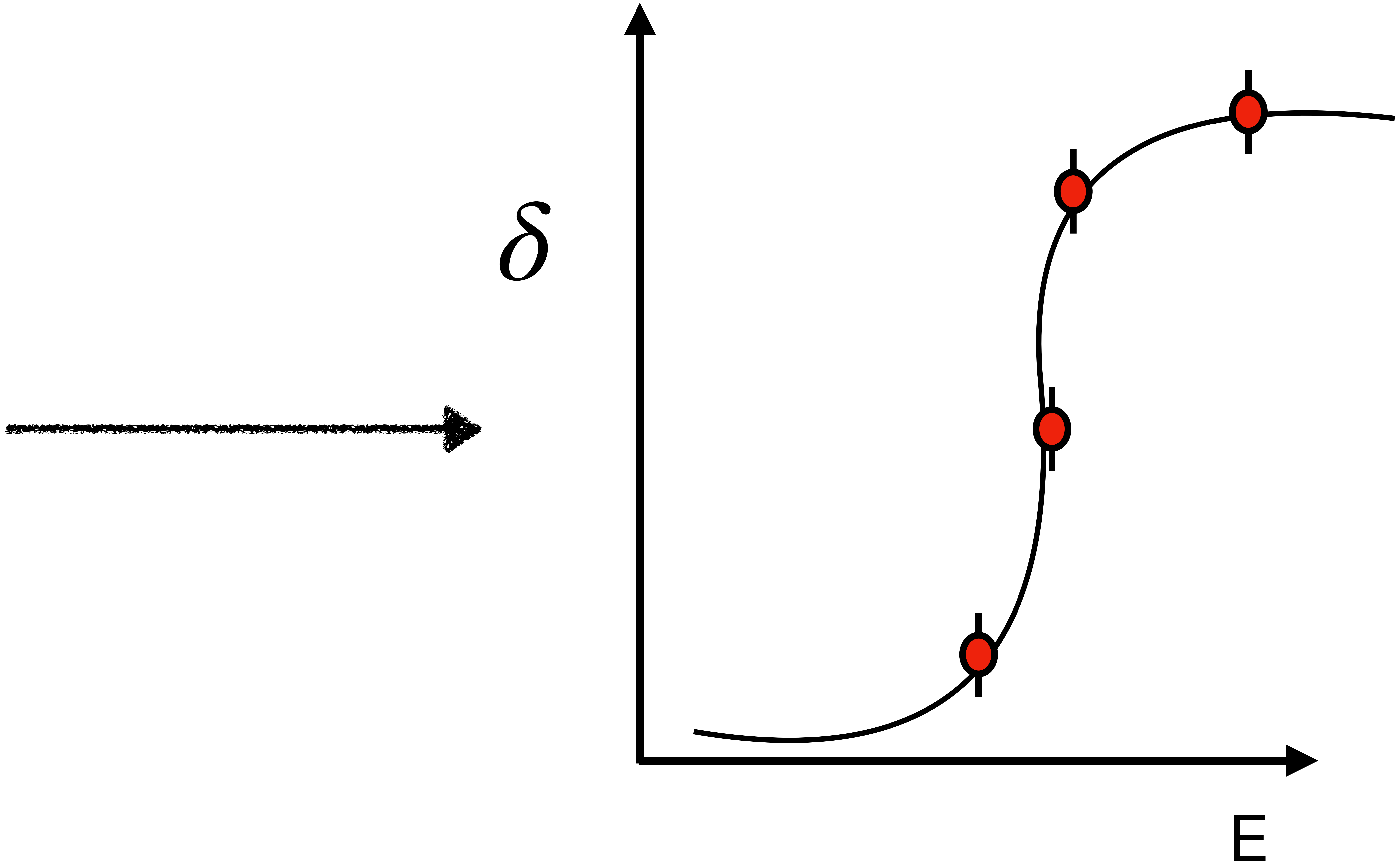
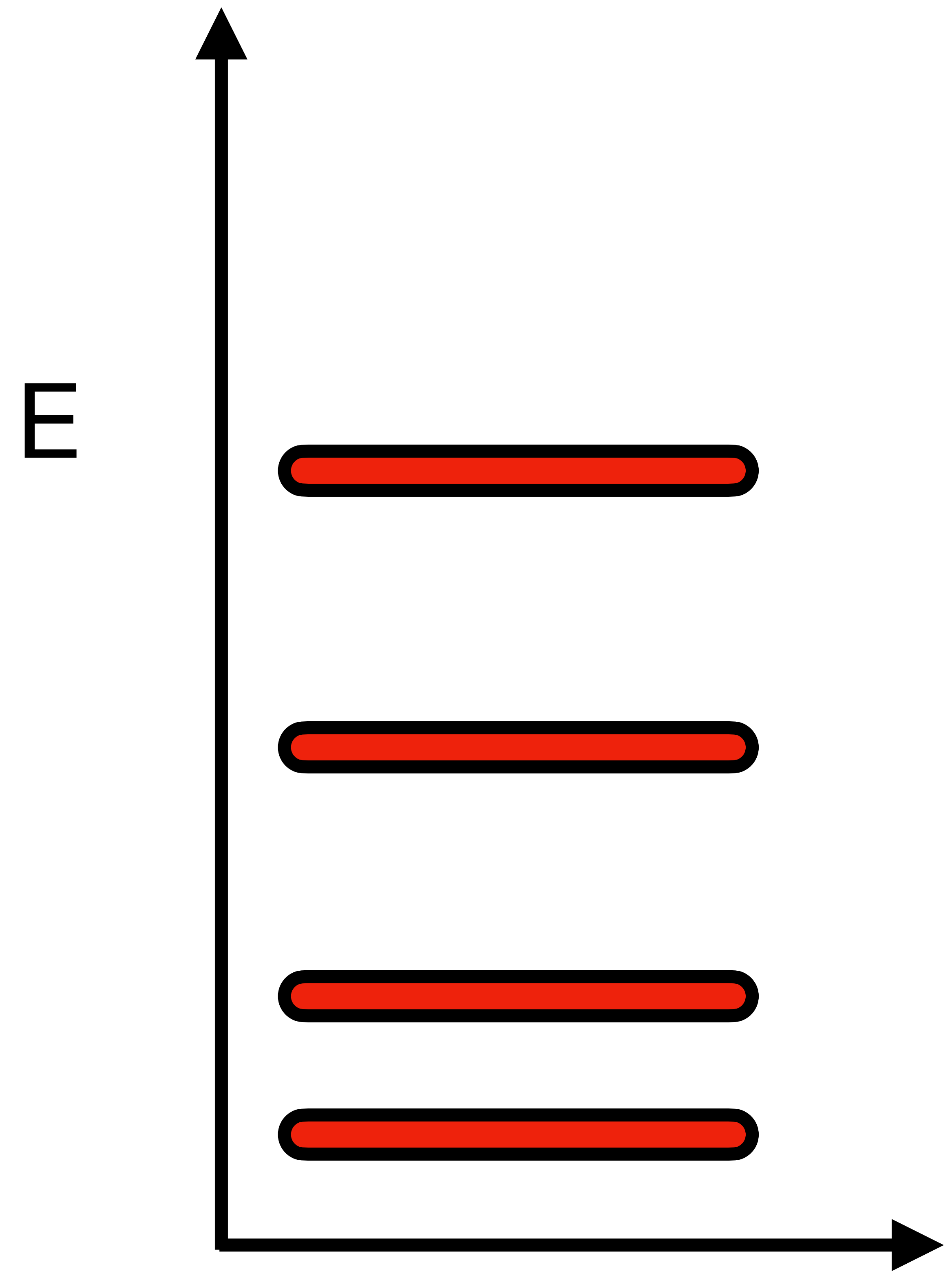


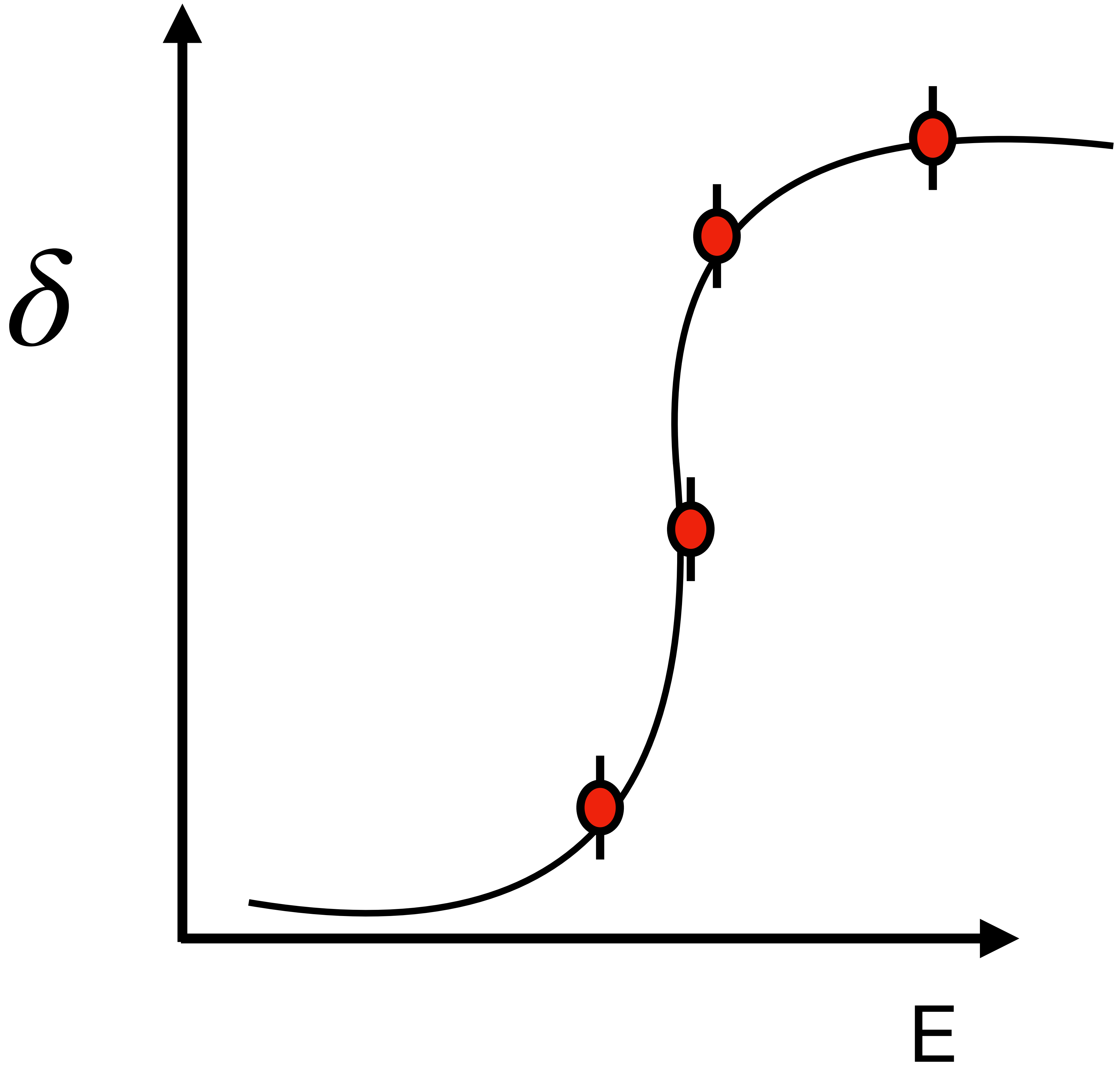




- Energy is discrete.

# Amplitude Analysis



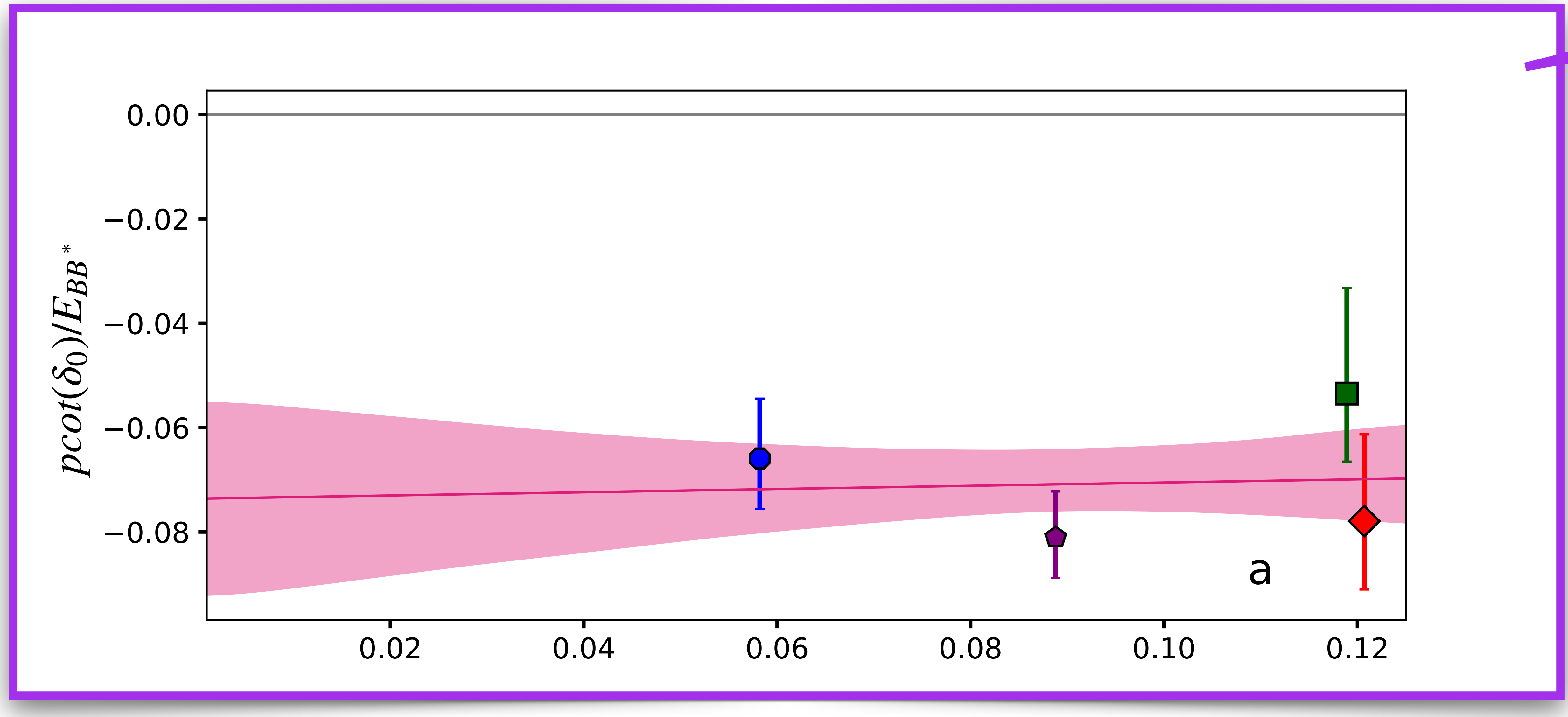



- Energy is discrete.
- Energy dependence is required.

# Continuum Extrapolation

$M_{ps} = 0.5$ GeV



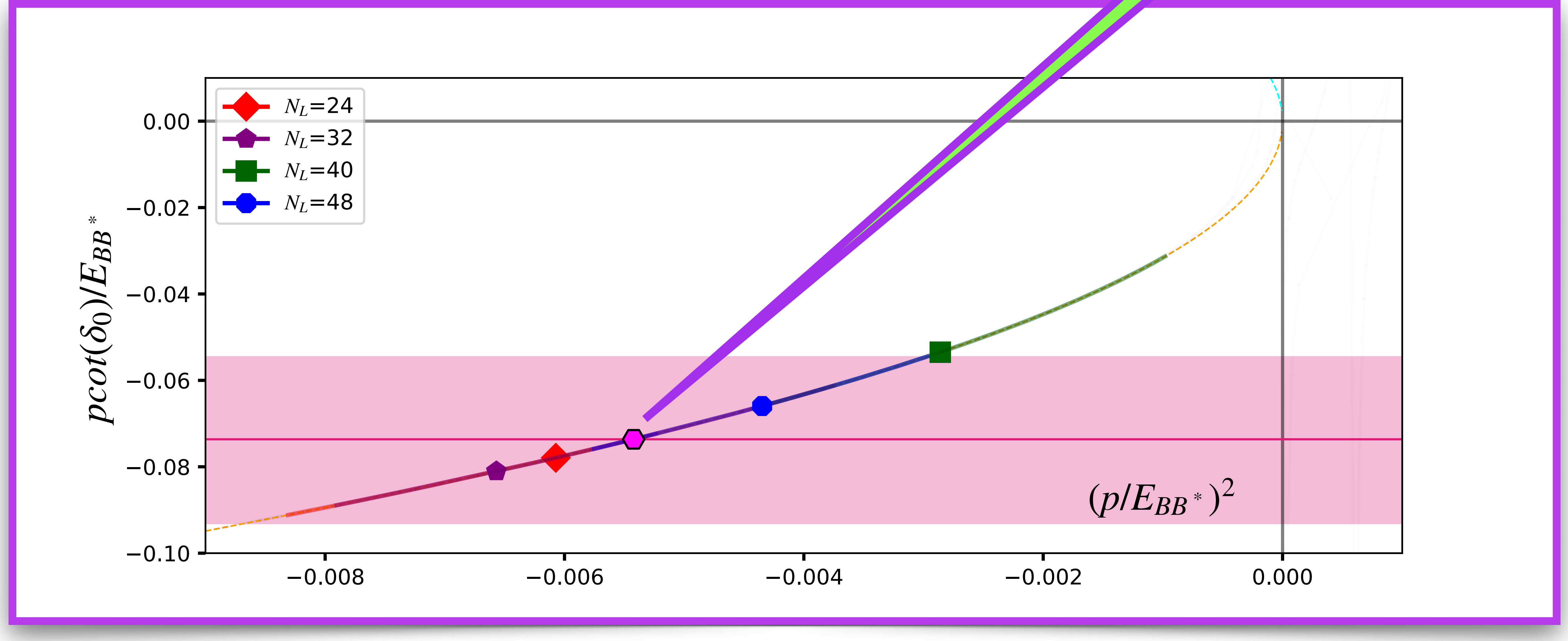

- Scattering Amplitude is given as

$$T \propto (pcot\ \delta - ip)^{-1}$$

- It is parametrised as

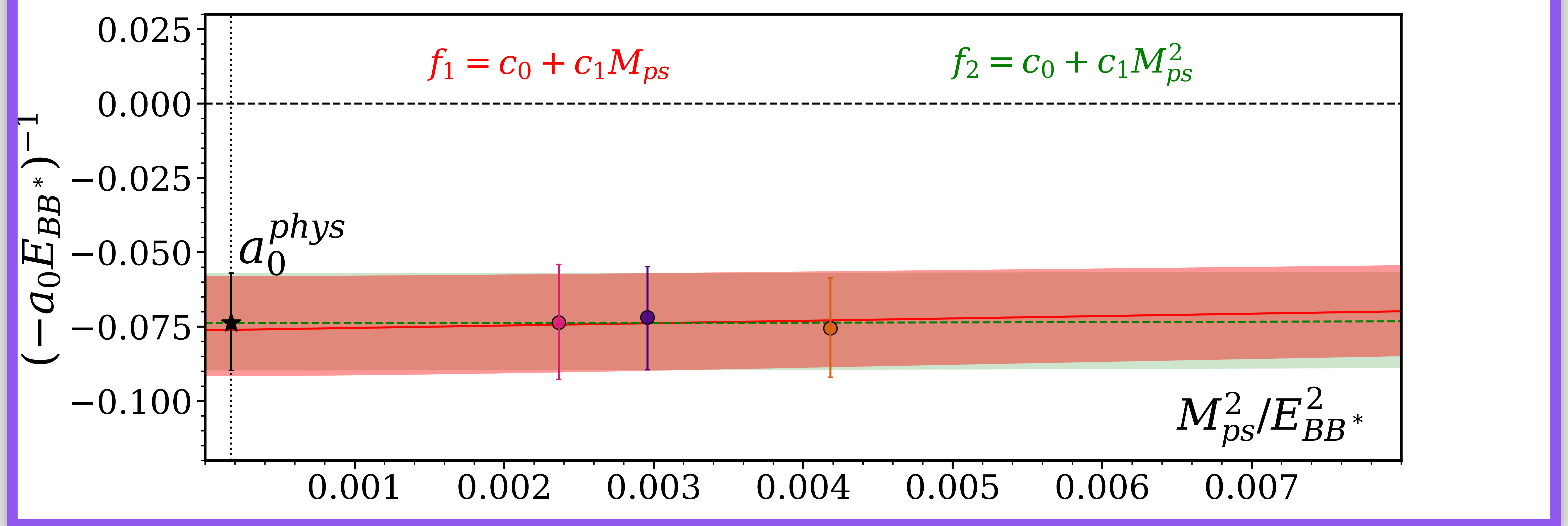
$$pcot\ \delta = -\frac{1}{a_0} + B \cdot a$$

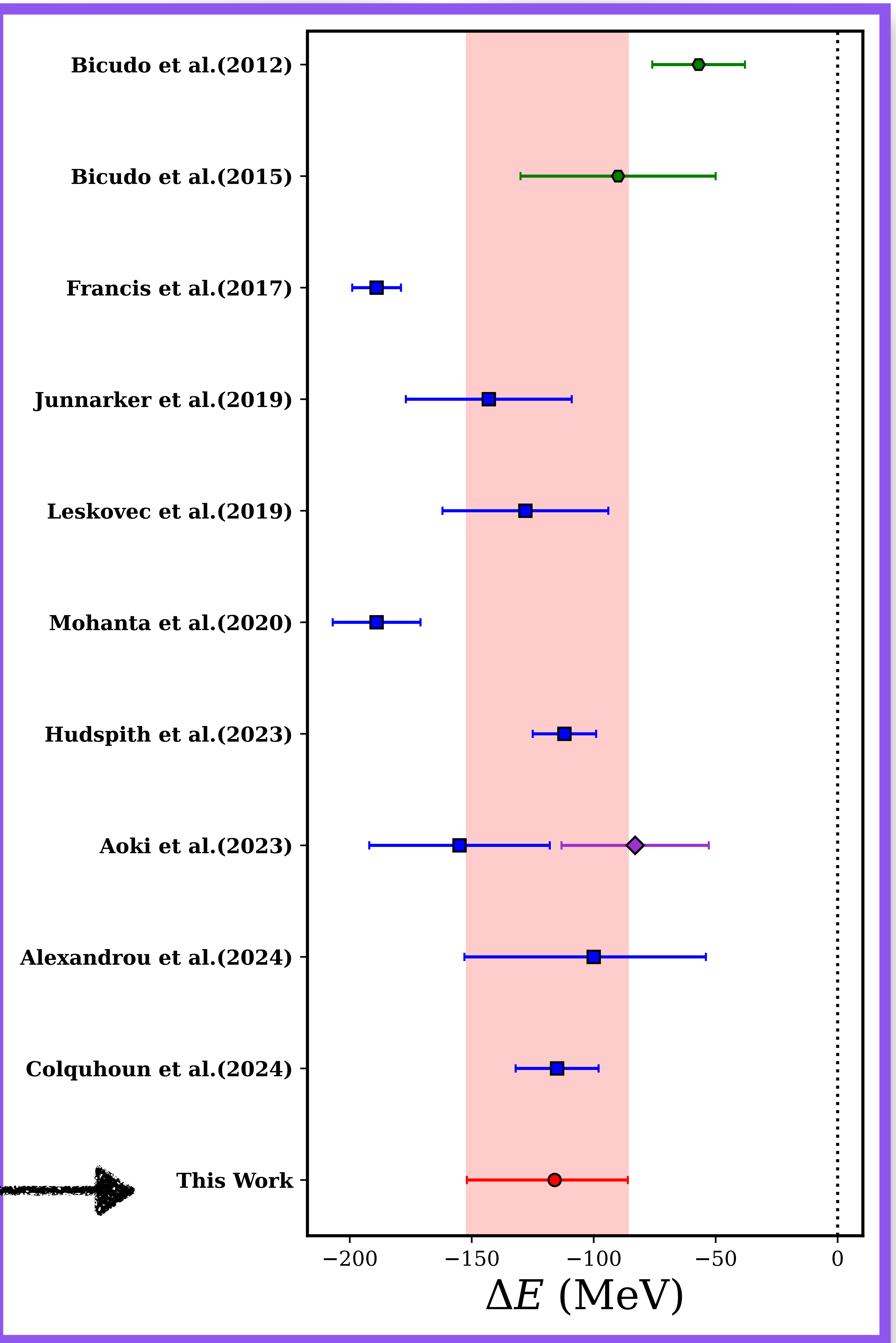
Real Bound State




- Same repeated for other $M_{ps}$.
- Consistent Negative values for other $M_{ps}$ as well as real bound state.

# Chiral Extrapolation



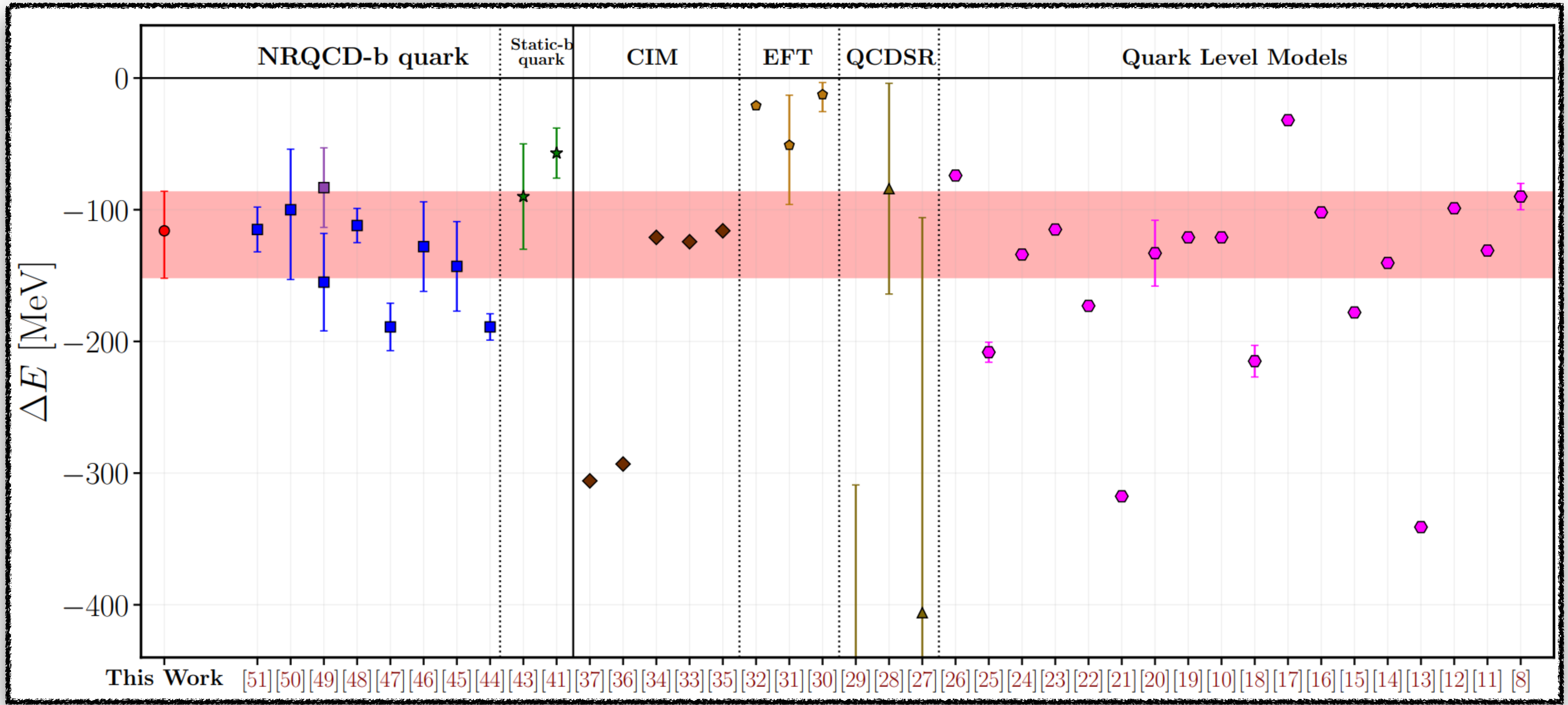

**Result:**

**Scattering length at physical limit**

$$a_0^{phy} = 0.25(^4_3)\ fm$$

**Corresponds to binding energy** $\Delta E = -116(^{+30}_{-36})$ **MeV.**

# Summary $T_{bb}$




arXiv:2503.09760 BST, Mathur, Padmanath

# Results of $T_{bs}$



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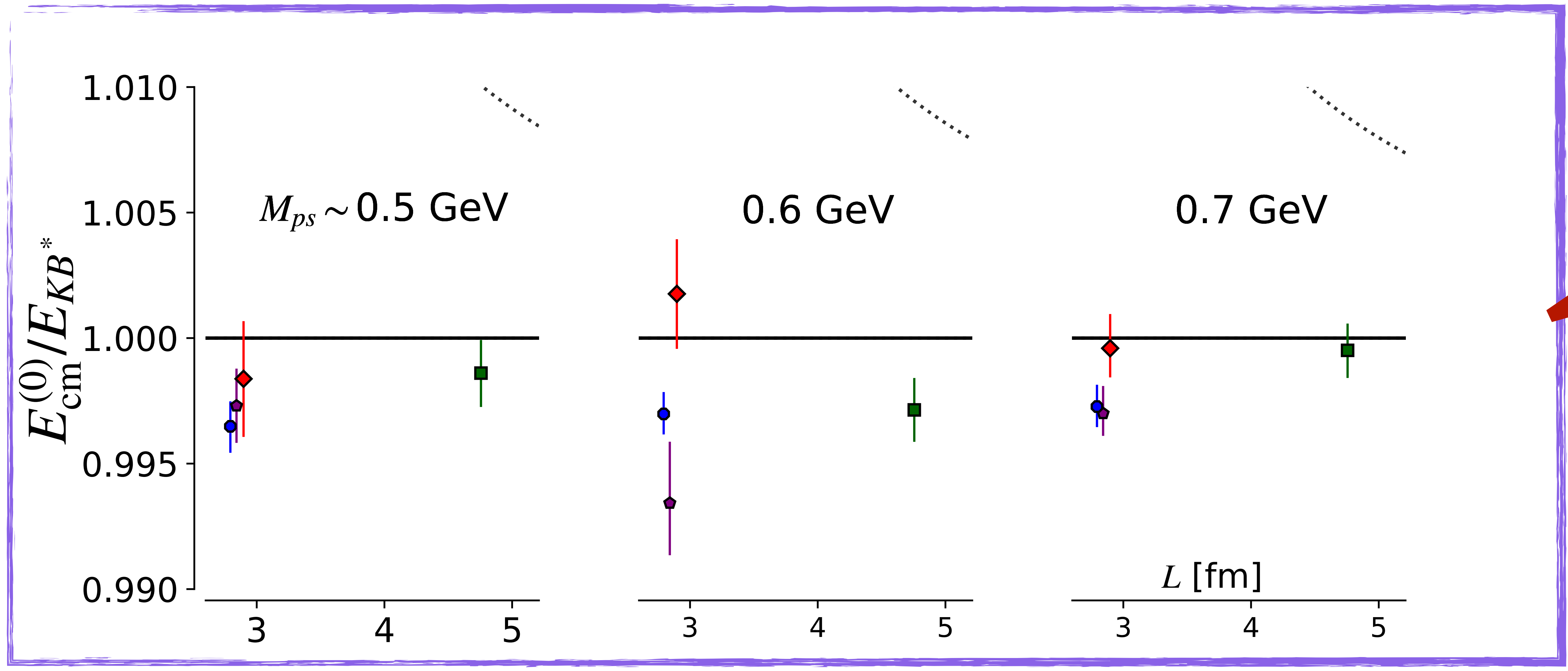
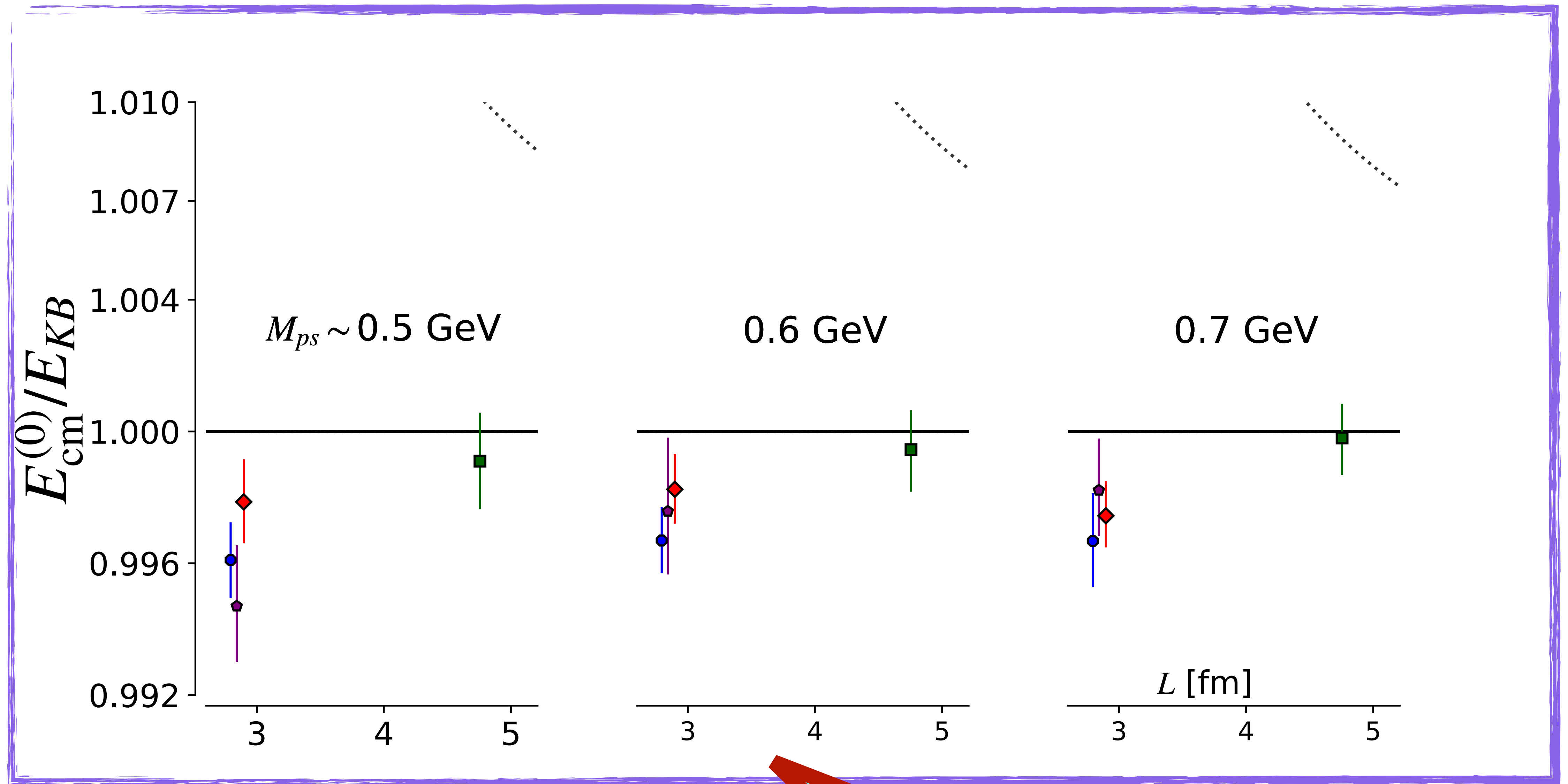



Axial vector $T_{bs}$




Scalar $T_{bs}$

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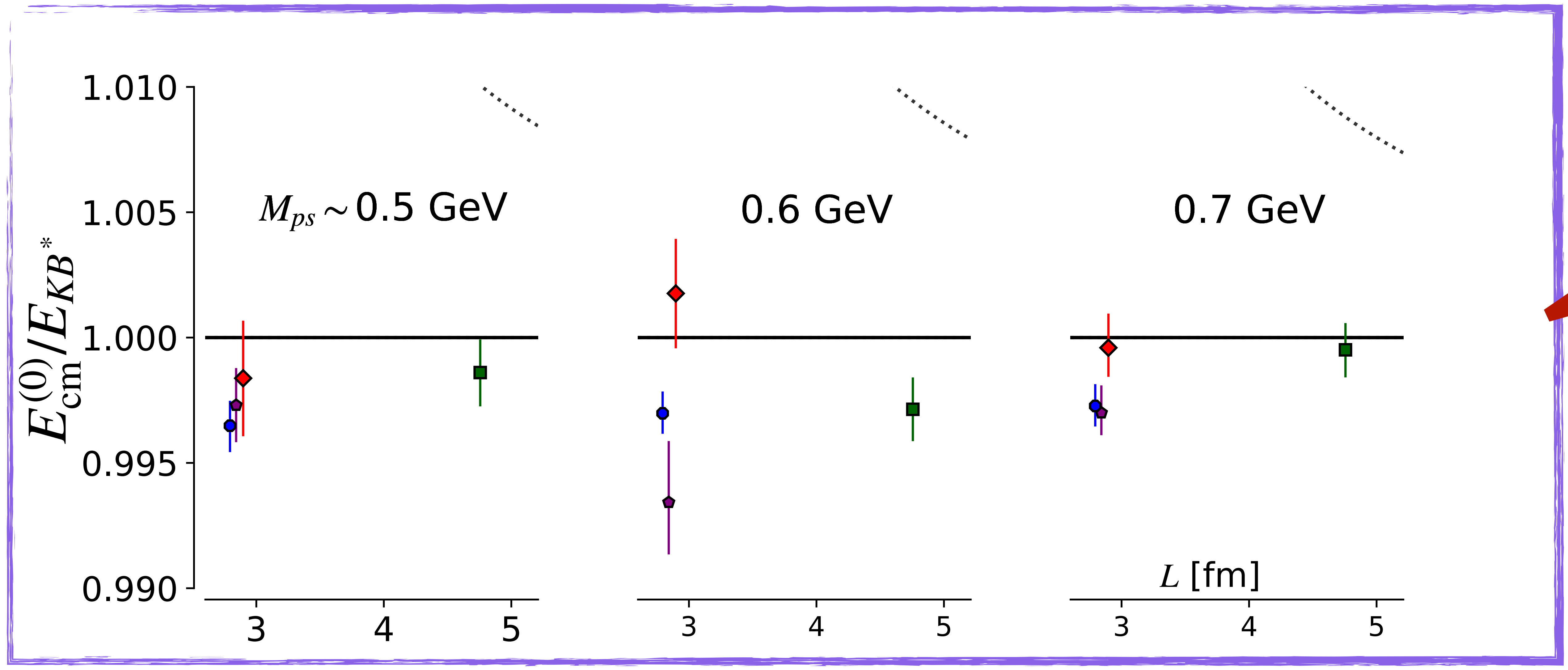
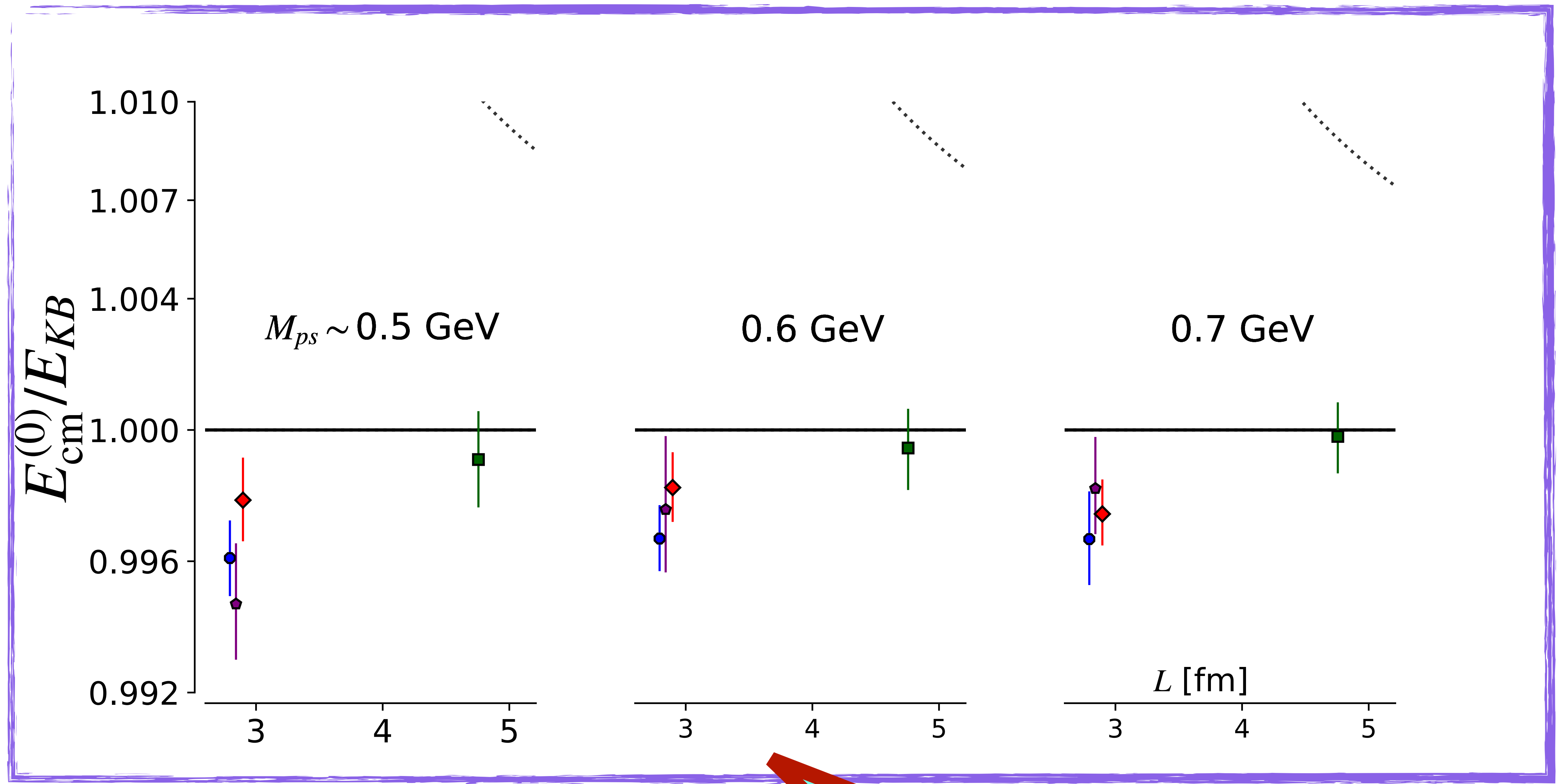
arXiv:2503.09760 BST, Mathur, Padmanath




Axial vector $T_{bs}$

- Results are consistent with threshold in both the cases for larger volume.
- Need low $M_{ps}$ datas for better results.




Scalar $T_{bs}$

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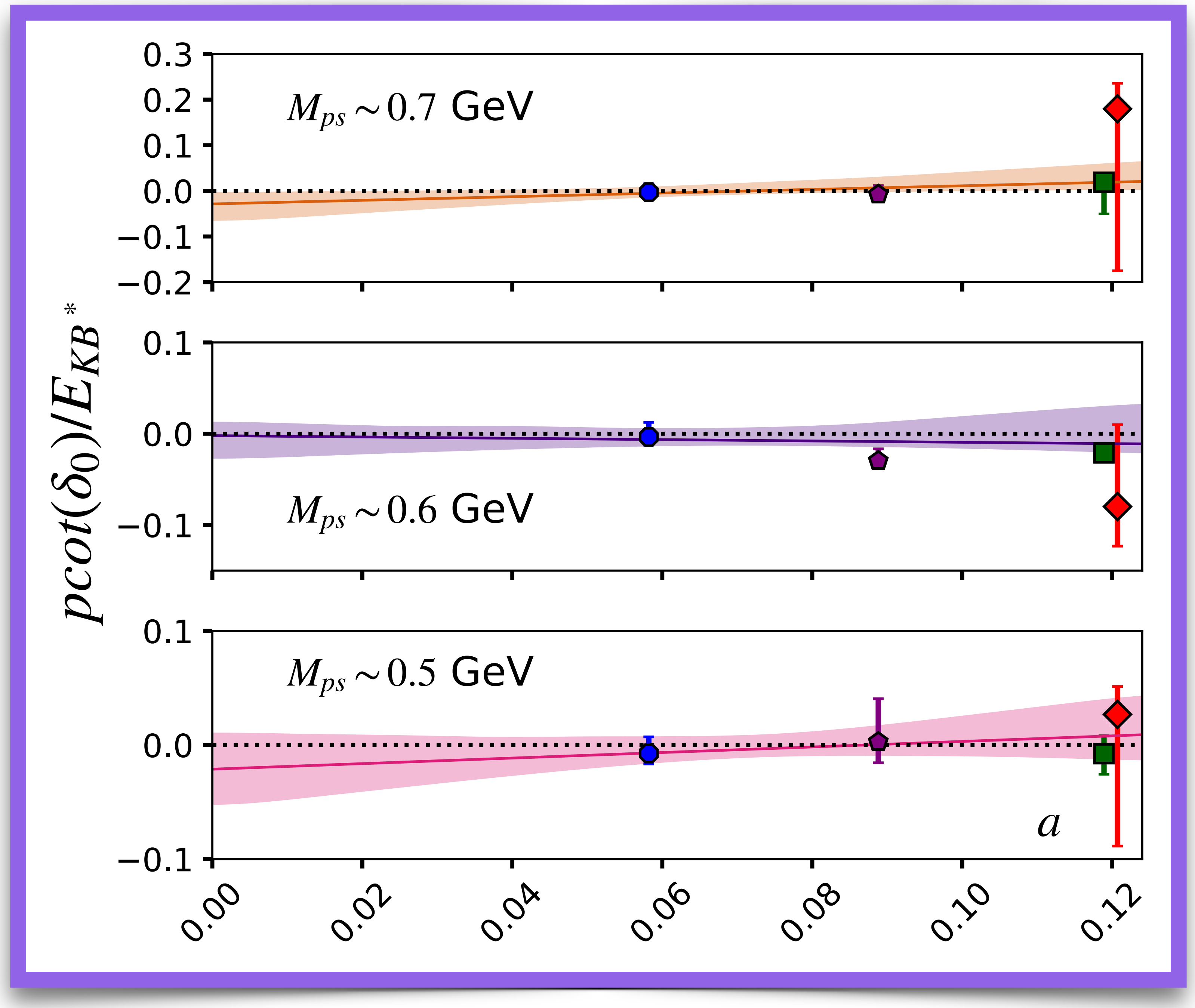
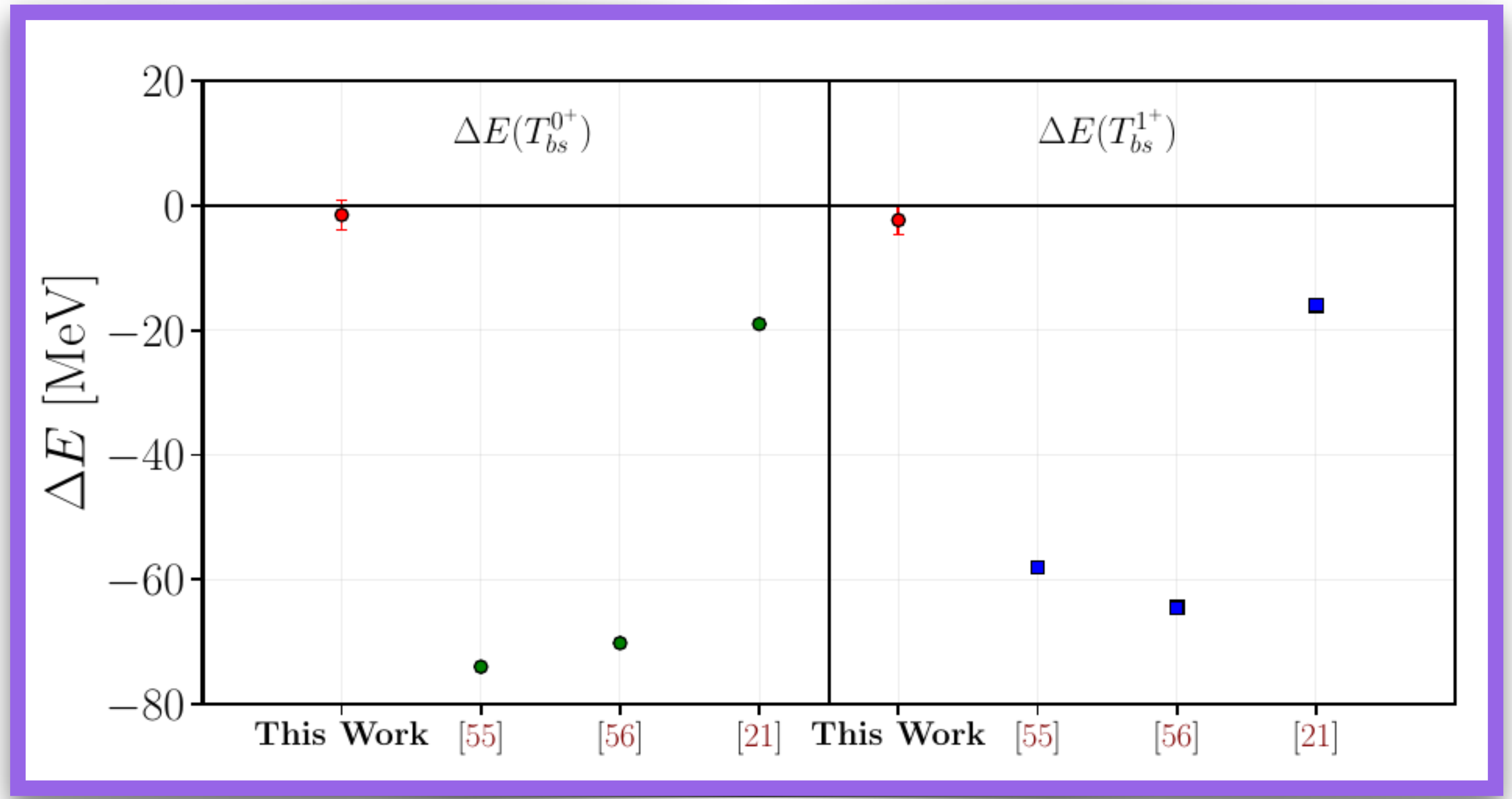
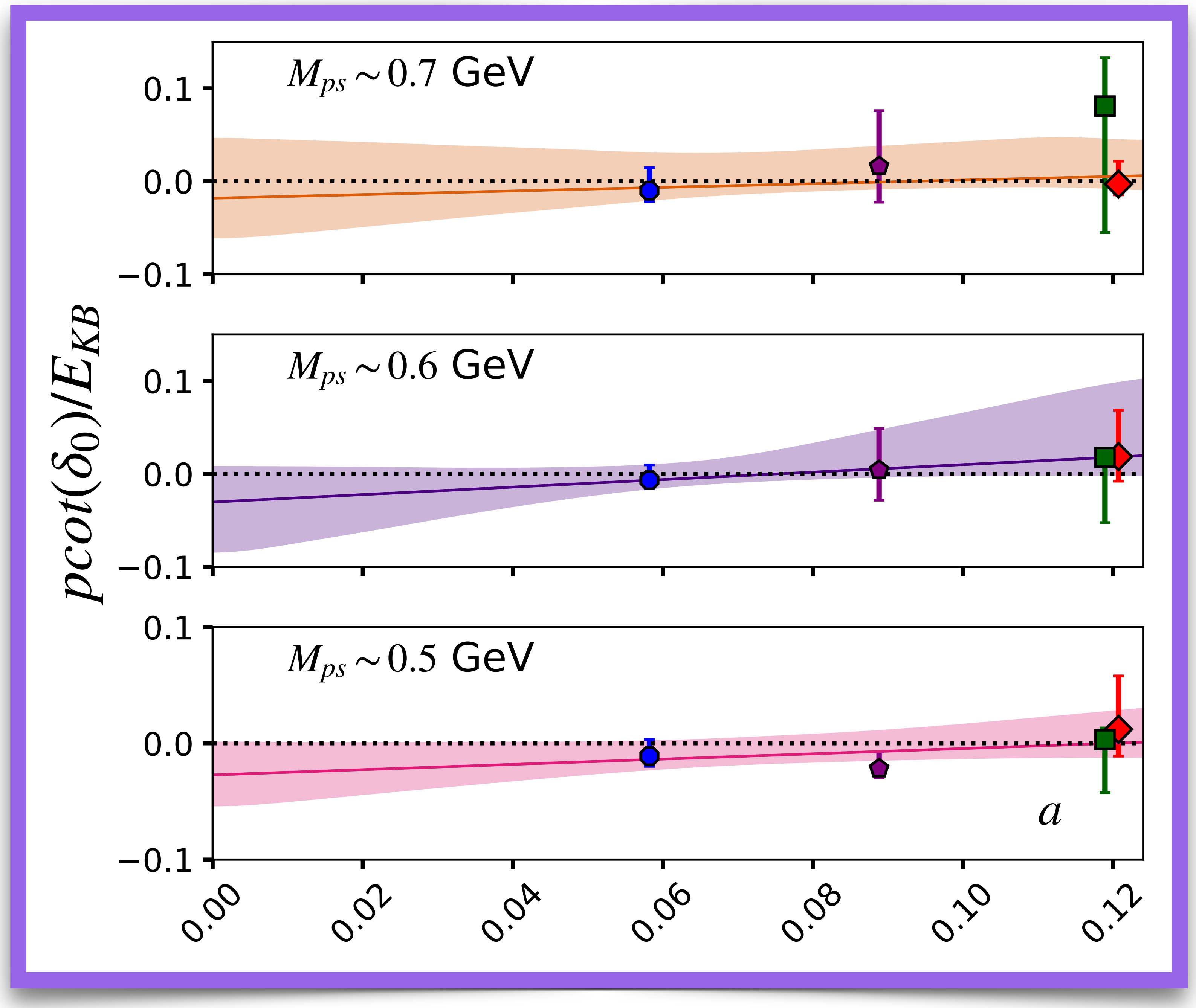



Axial vector $T_{bs}$

Summary $T_{bs}$




Scalar $T_{bs}$

# Summary and Outlook



- We worked with isoscalar axial vector $T_{bb}$ and both scalar and axial-vector $T_{bs}$
- Various work widely predicted deep binding in isoscalar axial-vector $T_{bb}$.
- Rigorous spectrum analysis were done for $T_{bb}$ and $T_{bs}$ tetraquark.
- We worked with multiple lattice spacing, two volumes to control systematics.
- Finite volume spectrum indicates negative energy shift with respect to $BB^*$ threshold.
- Found a possible deeply bound state for $T_{bb}$ not such exciting results in $T_{bs}$.

# THANK YOU

# BACKUP SLIDES

# Wall Sources Point Sink

- Instead of using a single point source, unique source is placed at every spatial point on the source time slice.

$$Q(\bar{x}, t; t') = \sum_{\bar{x}'} Q(\bar{x}, t; \bar{x}', t')$$

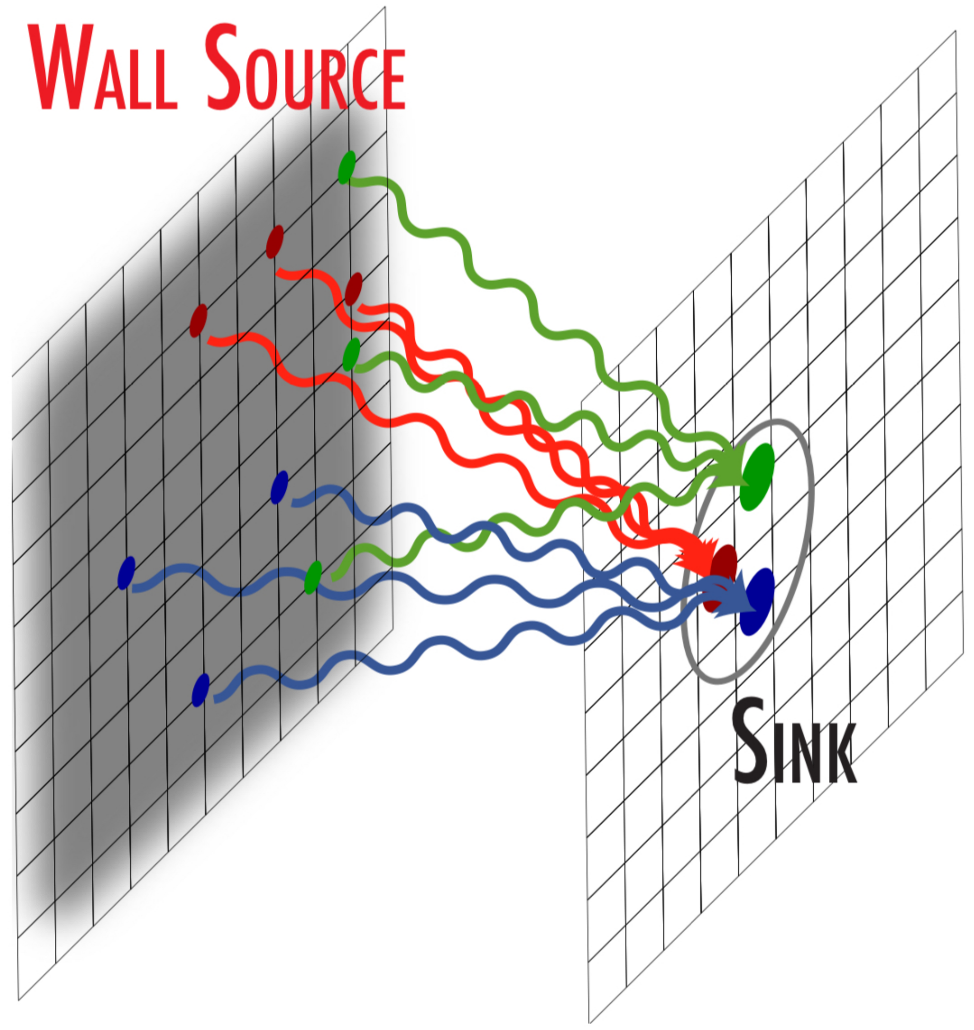



Image Credit:- S. Aoki

- ADVANTAGES:- Better signals in the ground state.

- DISADVANTAGE:-

  1. Assymetric Correlation Function, Non-Hermitian GEVP Needed.

  2. False Plateau encounter, careful with fitting time window.

- Why not Wall source wall sink correlator? -> Very Noisy signal.

# Wall source- Box Sink Correlator

- **Instead of wall-sink, we build box-sink correlator.**

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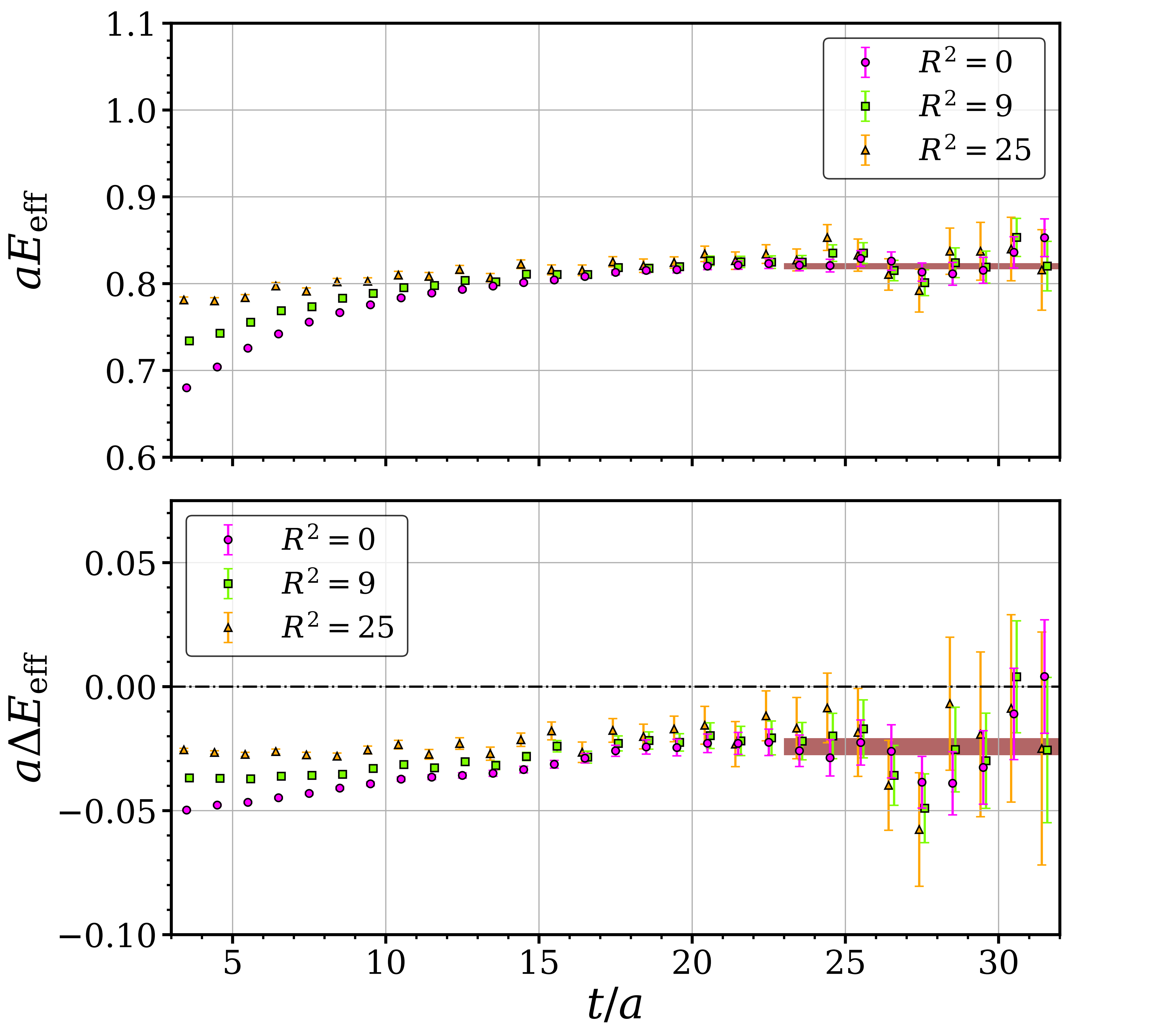
$$Q(\bar{x}, t; t') = \sum_{|\bar{y}-\bar{x}|<R} Q(\bar{y}, t; , t')$$

- **As we increase box radius $R$, it approaches to symmetric correlator.**
- **Used to make comparative study of the asymptotic signals.**
- **Increases robustness of the calculations.**






arXiv:2503.09760 BST, Mathur, Padmanath