Realizing of Doubly Heavy Tetraquarks from Lattice QCD

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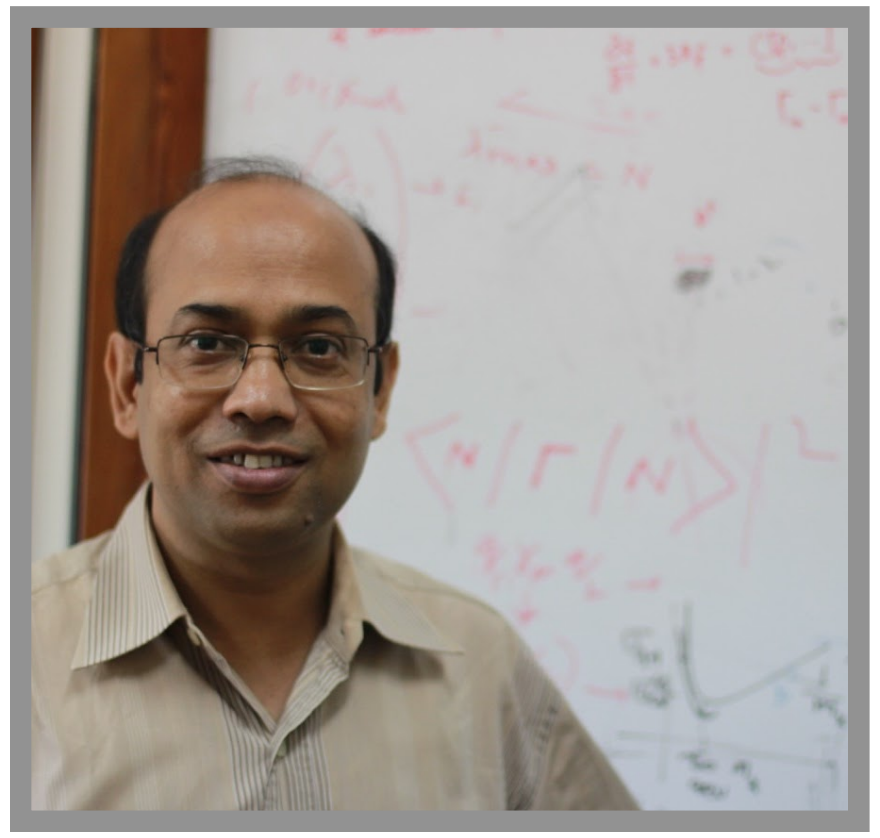
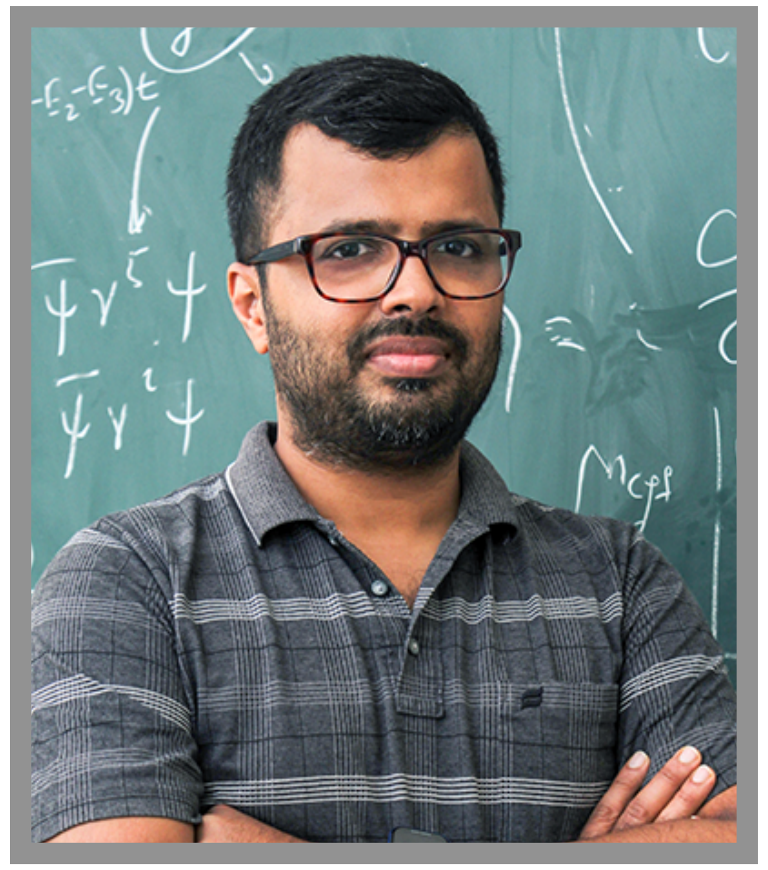
17 March 2025



Nilmani Mathur(TIFR)

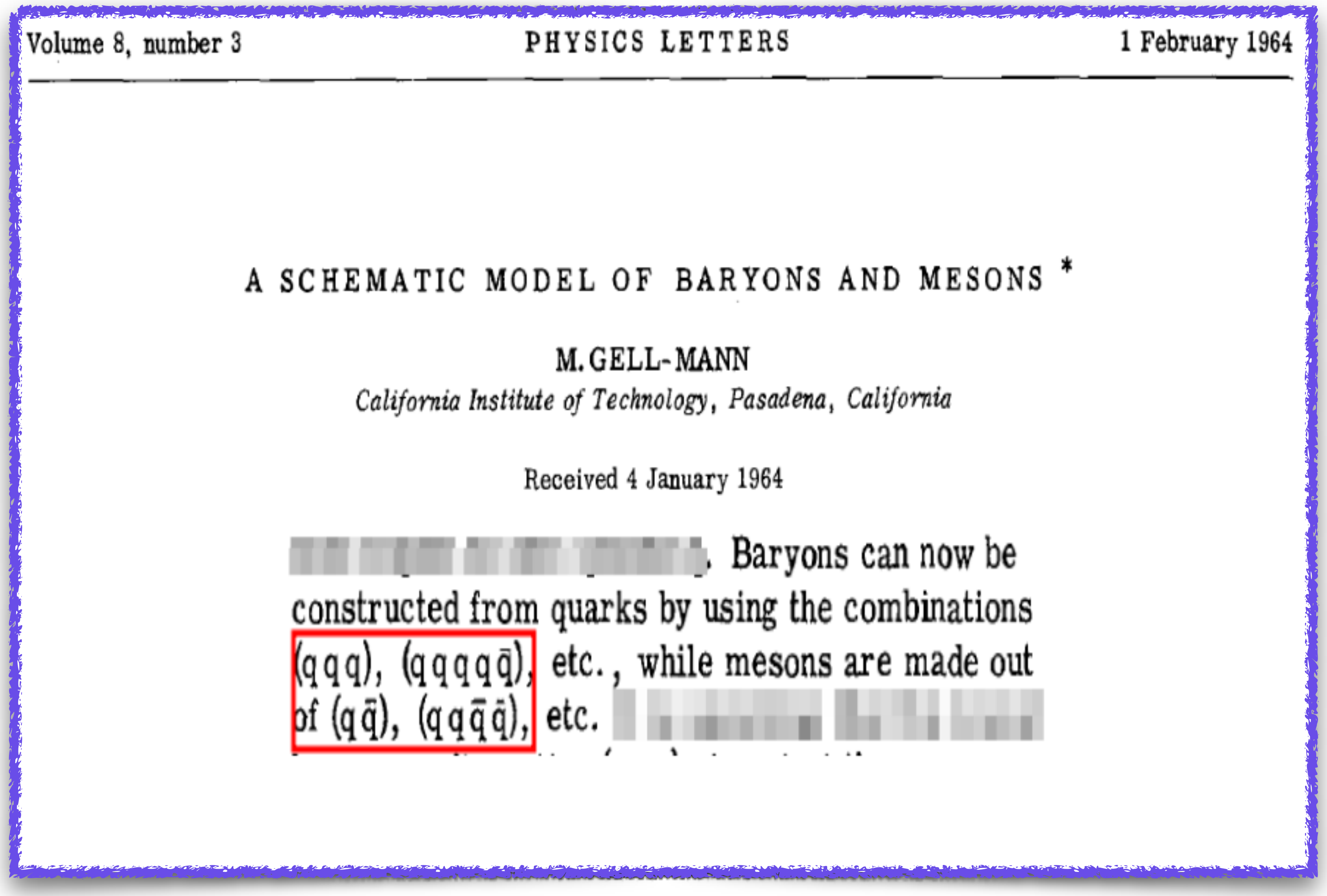
IMSc Physics Seminar



M. Padmanath(IMSc)

Based on arXiv:2503.09760

Introduction and Motivation



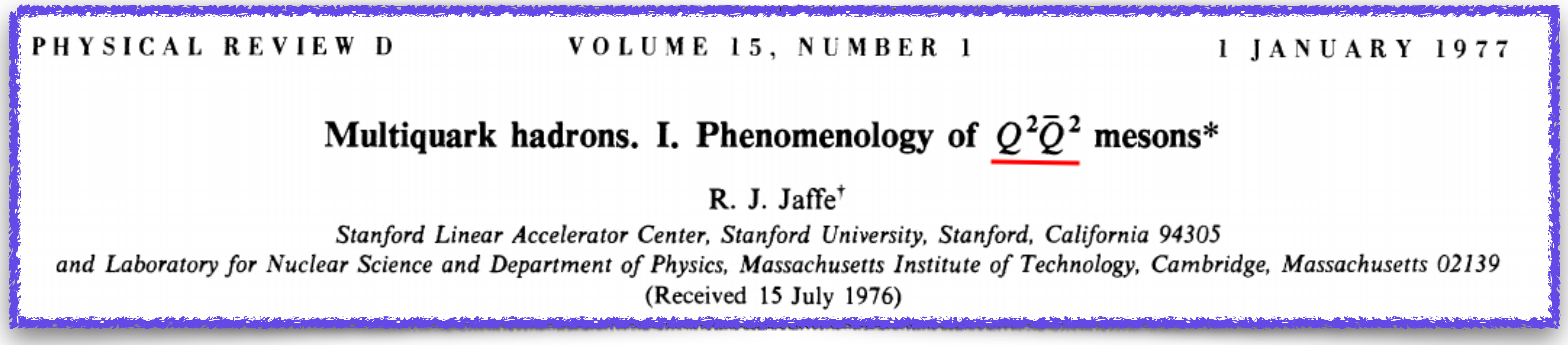
Volume 8, number 3 PHYSICS LETTERS 1 February 1964

A SCHEMATIC MODEL OF BARYONS AND MESONS *

M. GELL-MANN
California Institute of Technology, Pasadena, California

Received 4 January 1964

Baryons can now be constructed from quarks by using the combinations (qqq), $(qqqq\bar{q})$, etc., while mesons are made out of $(q\bar{q})$, $(qq\bar{q}\bar{q})$, etc.

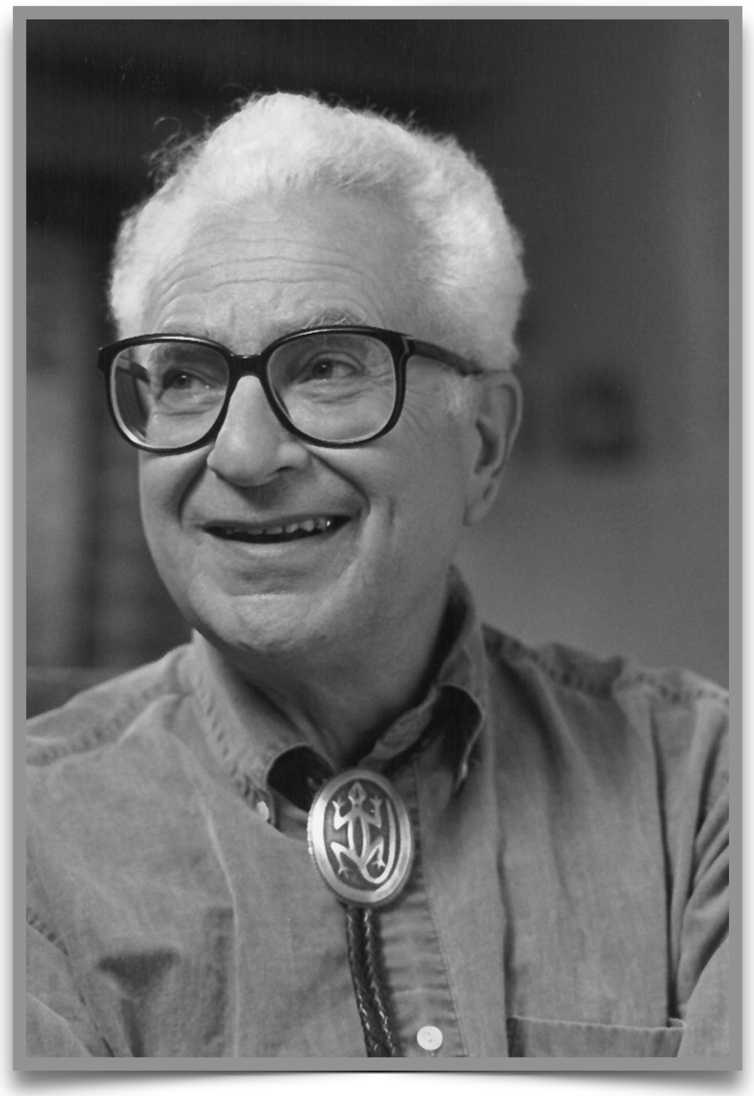


PHYSICAL REVIEW D VOLUME 15, NUMBER 1 1 JANUARY 1977

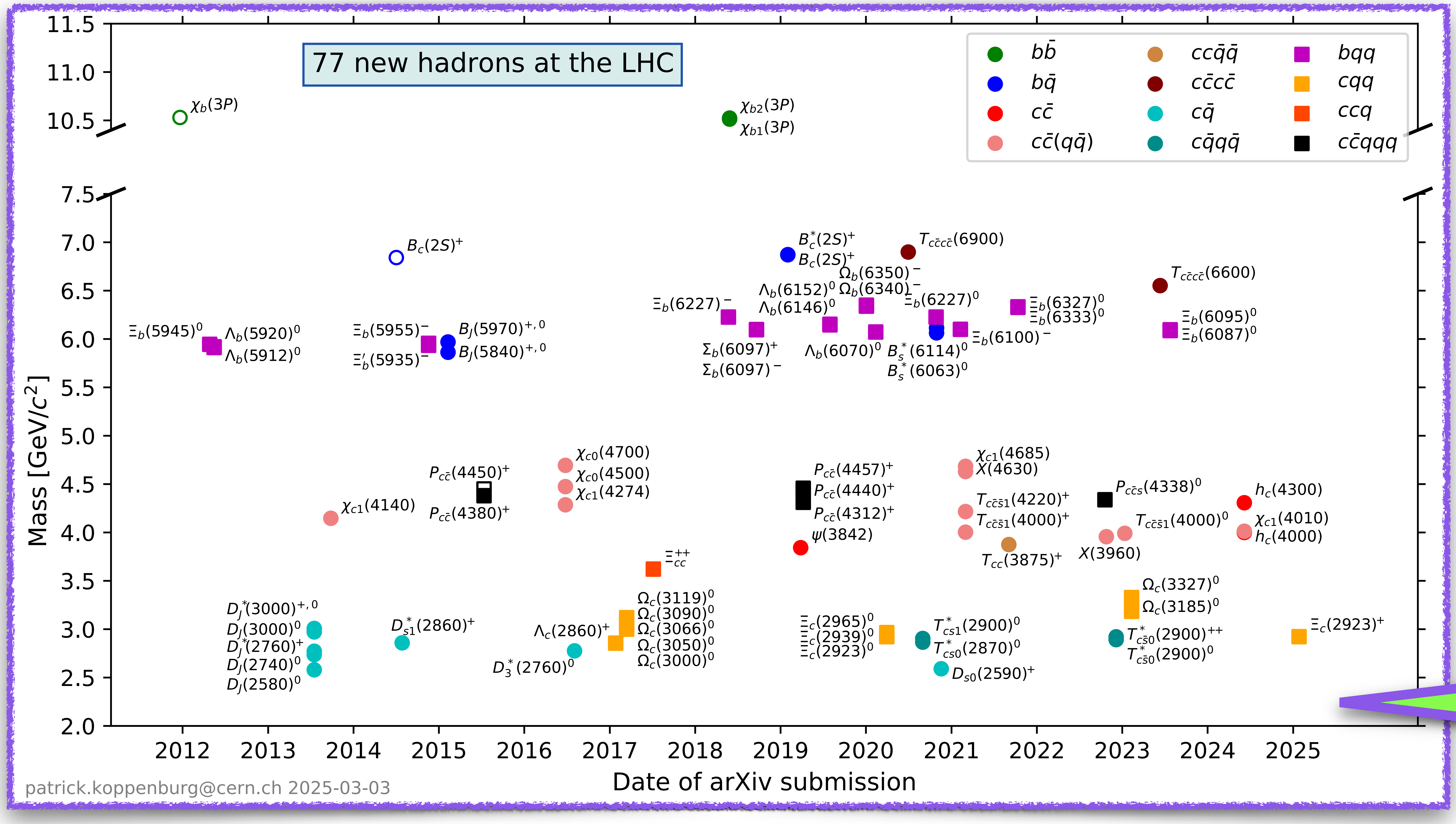
Multiquark hadrons. I. Phenomenology of $Q^2\bar{Q}^2$ mesons*

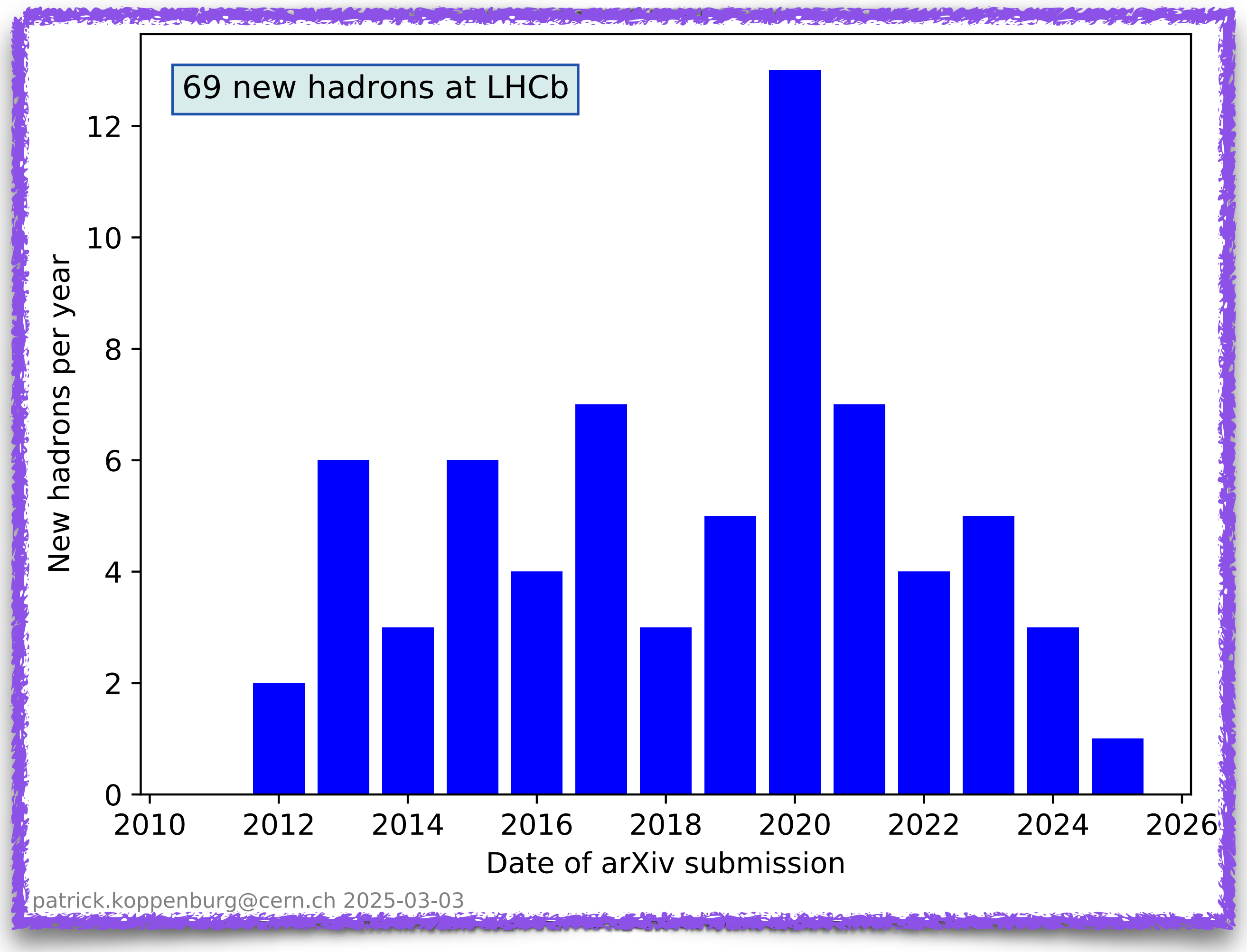
R. J. Jaffe†
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
and Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 15 July 1976)

- Murray Gellmann indicated the possibility of multiquark systems.
- Jaffe described it as color neutral states of diquark and antidiquark.
- Currently known as Tetraquarks and Pentaquarks.



Experimental Results in LHC

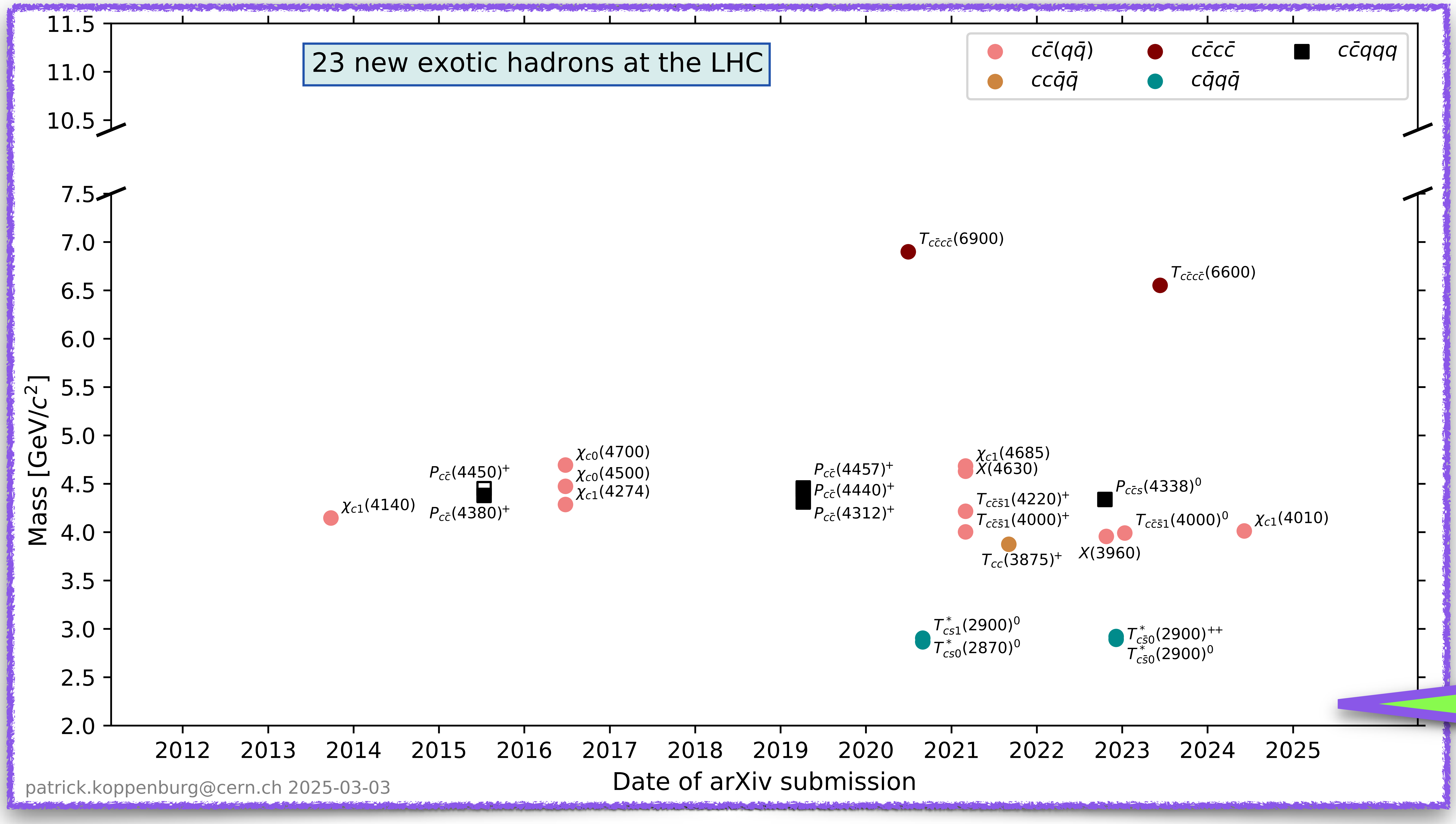


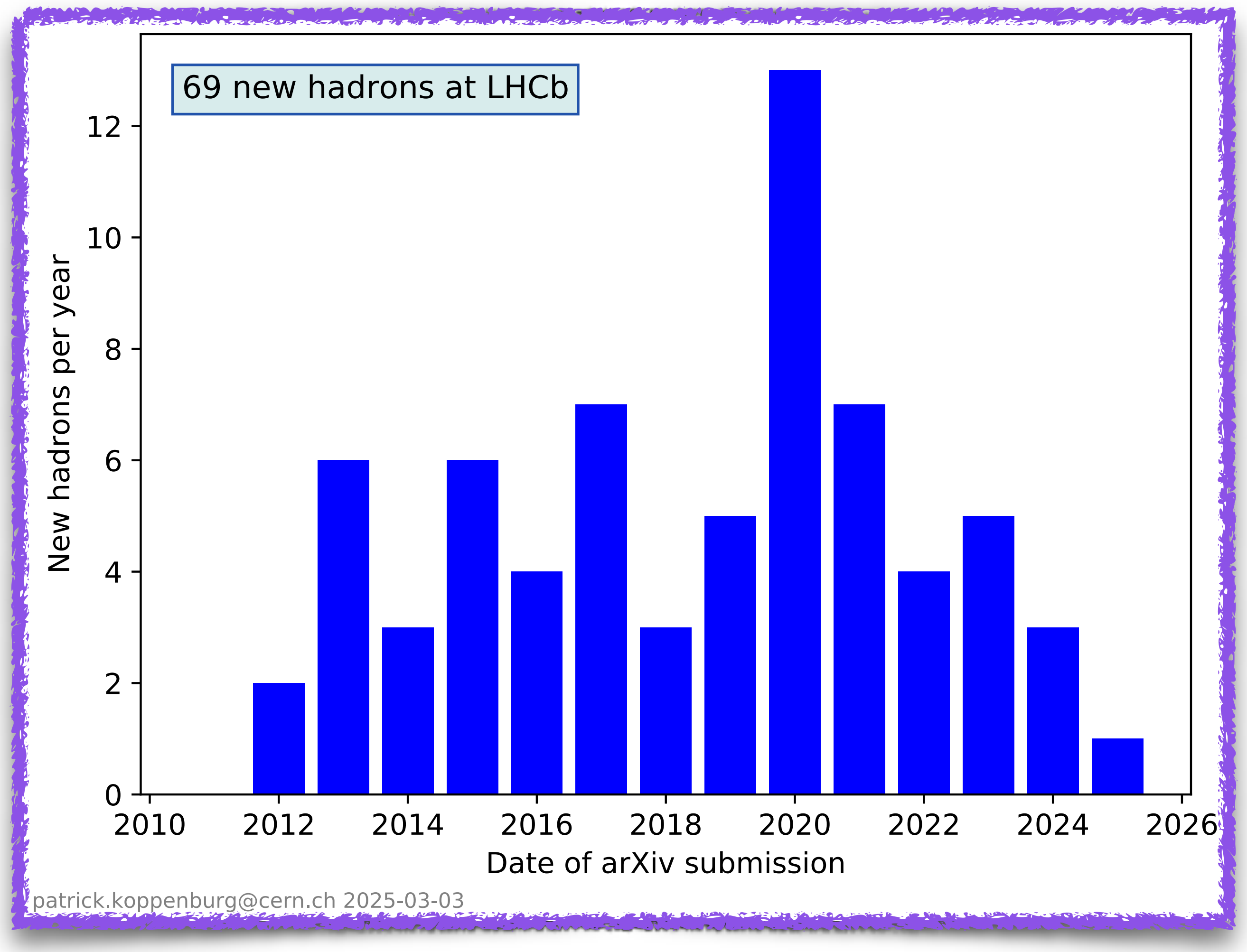


Particles discovered in the last decade including conventional heavy hadrons, open flavor mesons, exotic particles

Other research facilities like Belle, BES-III etc. have active spectroscopy programs.

Experimental Results in LHC



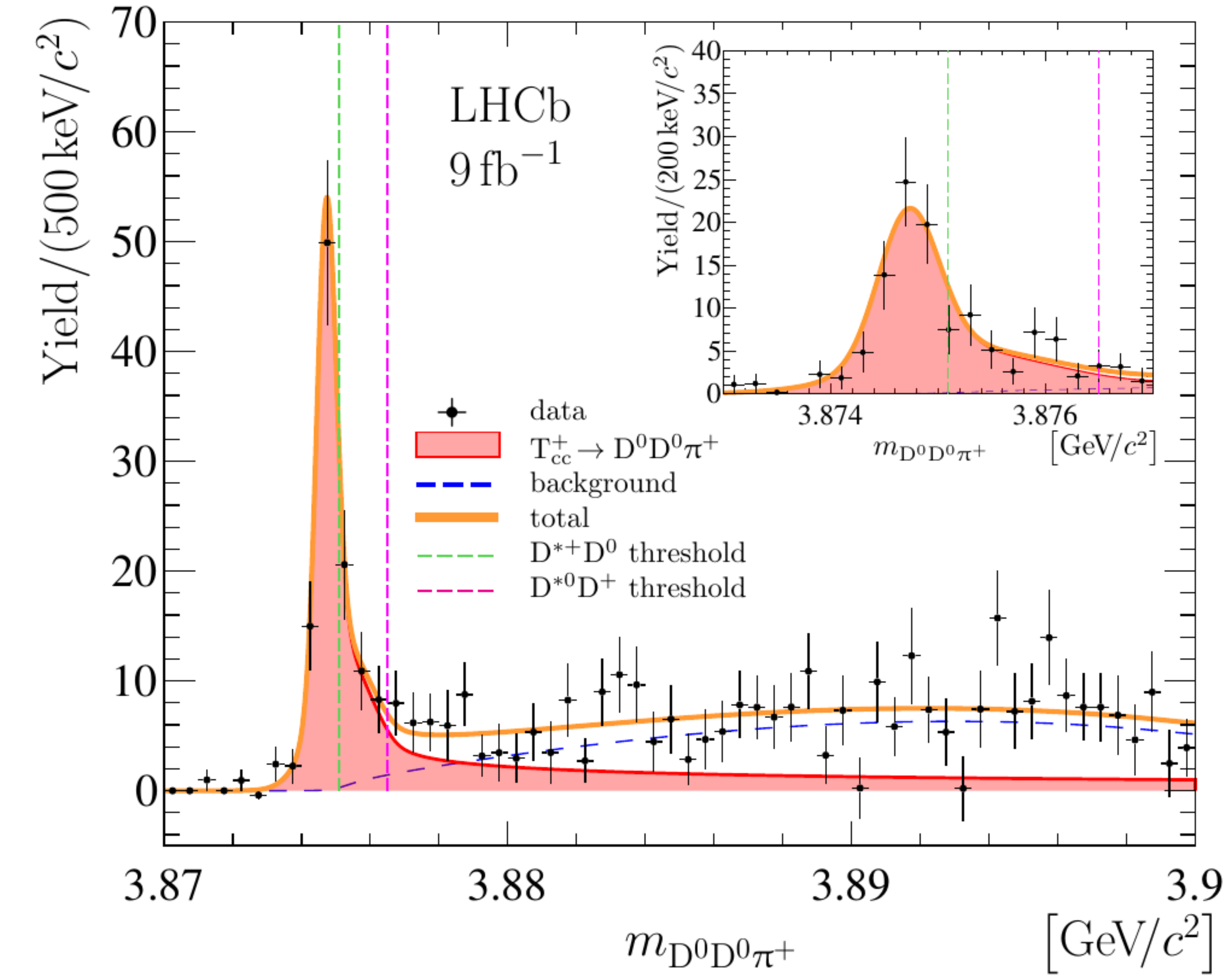


Collection of exotic particles

Other research facilities like Belle, BES-III etc. have active spectroscopy programs.

$T_{cc}(cc\bar{u}\bar{d})$ discovery at LHC

- In 2021, LHCb made headlines by discovering the longest-lived exotic state ever observed.
- It was observed in the channel $I = 0$, $J^P = 1^+$ below D^0D^{*+} threshold(in $D^0D^0\pi^+$).
- Many more exotic tetraquark discovered recently e.g. T_{cs}, $T_{c\bar{s}}$, Z_c and so on. Scope for T_{bc}, T_{bs} in near future.



Nature phys: https://rdcu.be/dNMRV
Arxiv:2109.01038

$$\delta M = M_{T_{cc^+}} - (M_{D^{*+}} + M_{D^0})$$

$$\delta M_{pole} = -360 \pm 40(^{+4}_{-0}) keV/c^2$$

$$\Gamma_{pole} = 48 \pm 2(^{+00}_{-14}) KeV$$

Long History of $T_{bb}(bb\bar{u}\bar{d})$

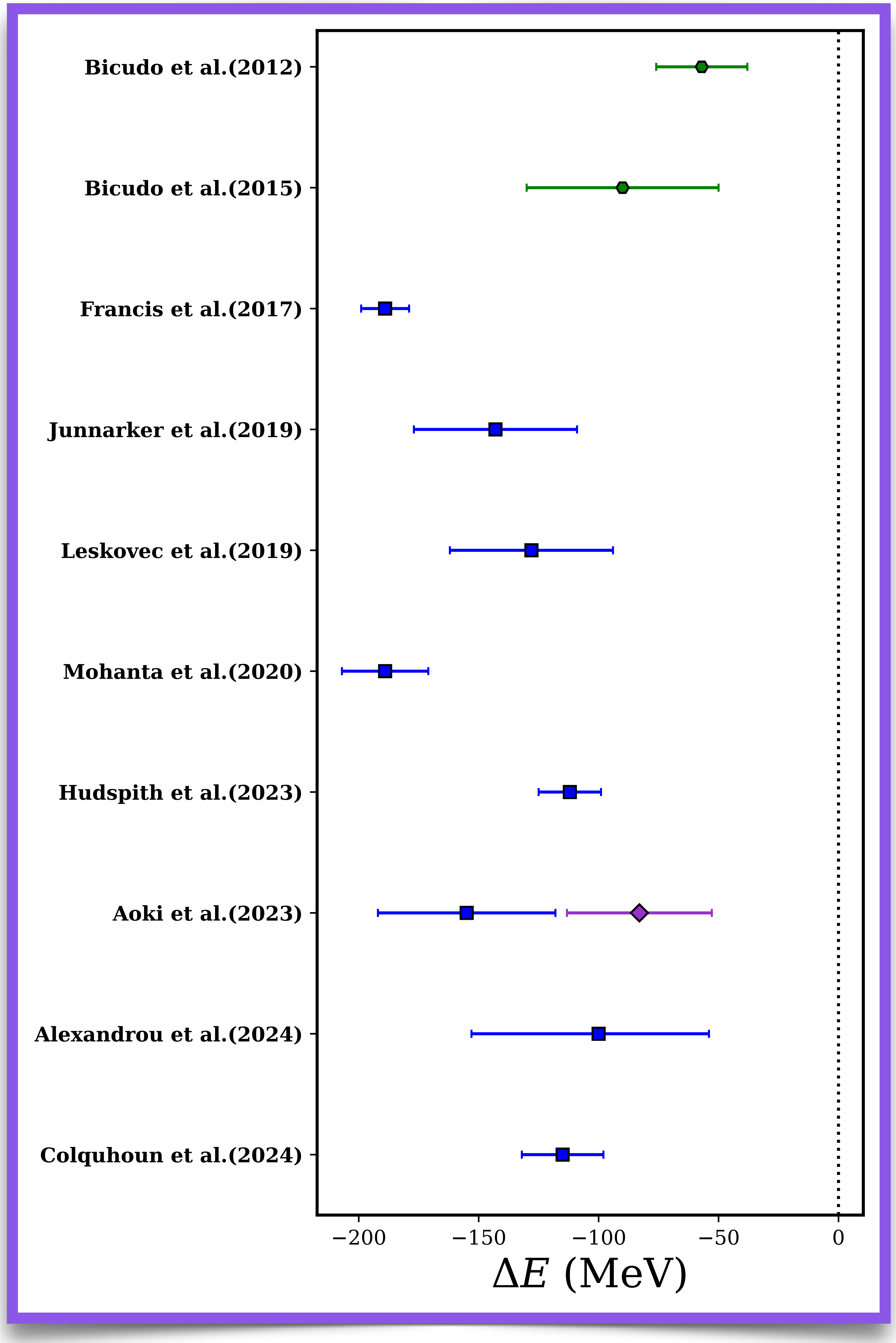
- Phenomenological calculation of $I(J^P) = 0(1^{1+})\ T_{bb}$ can be date back to the early 80's.

- Prediction of deeply bound state in the heavy quark limit.

Nucl.Phys.B 399 (1993)

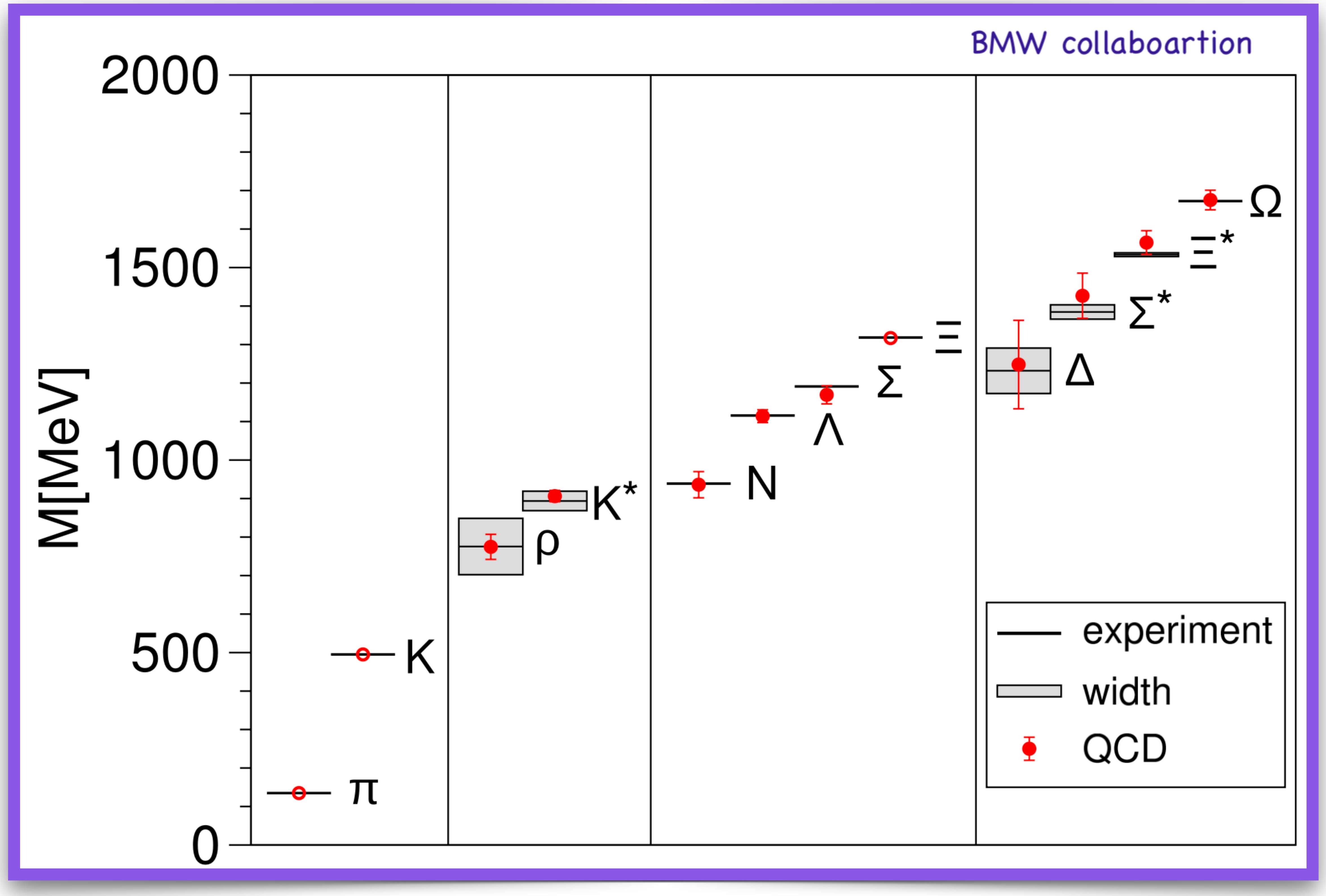
- Results from various phenomenological studies suggest possibility of deeply bound state.

- Previous lattice calculations on $bb\bar{u}\bar{d}\ I = 0$ shows deep bound state upto systematics.

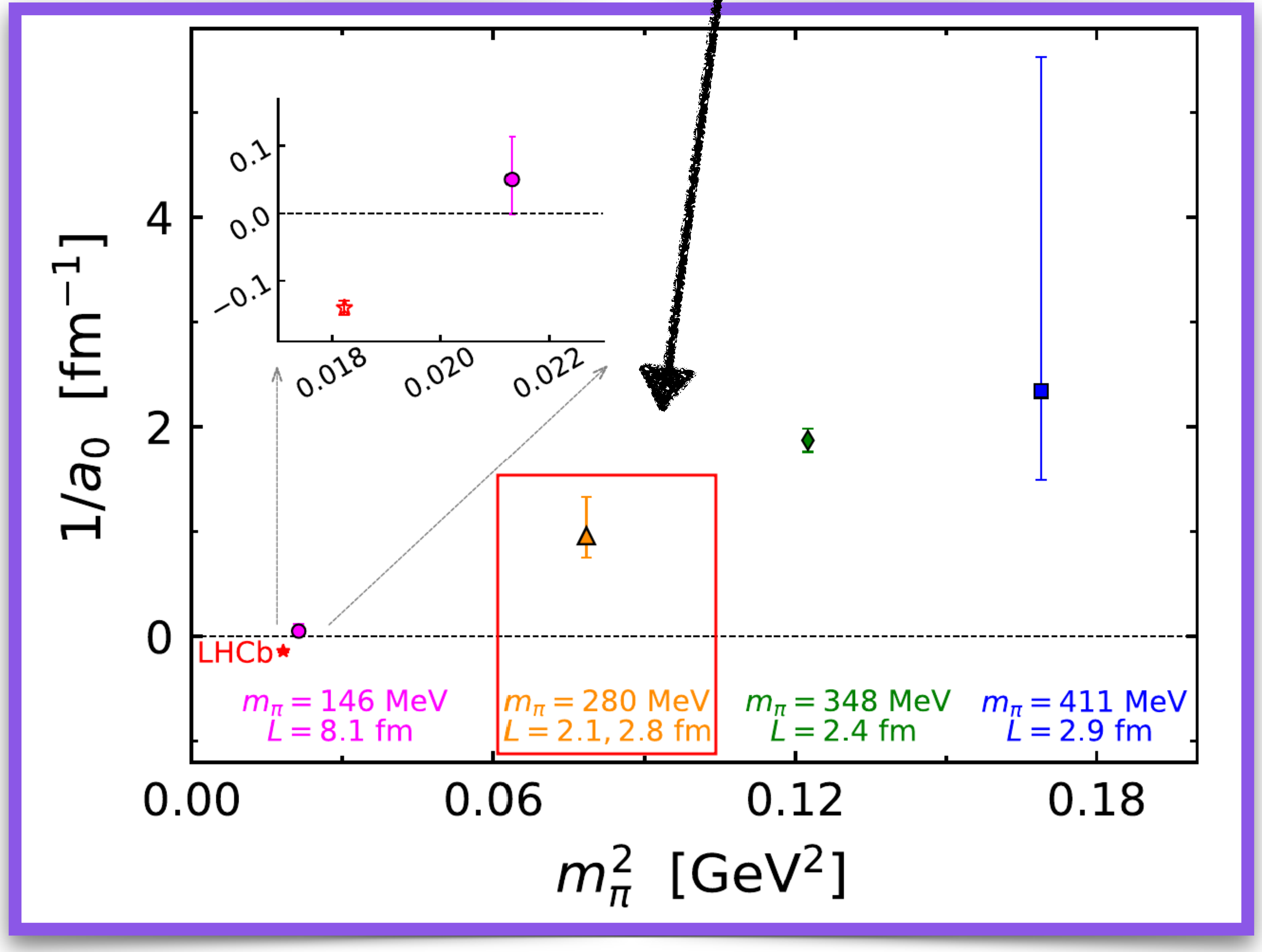


Lattice Predictions

Phys. Rev. Lett. 129, 032002 Padmanath, Prelovsek



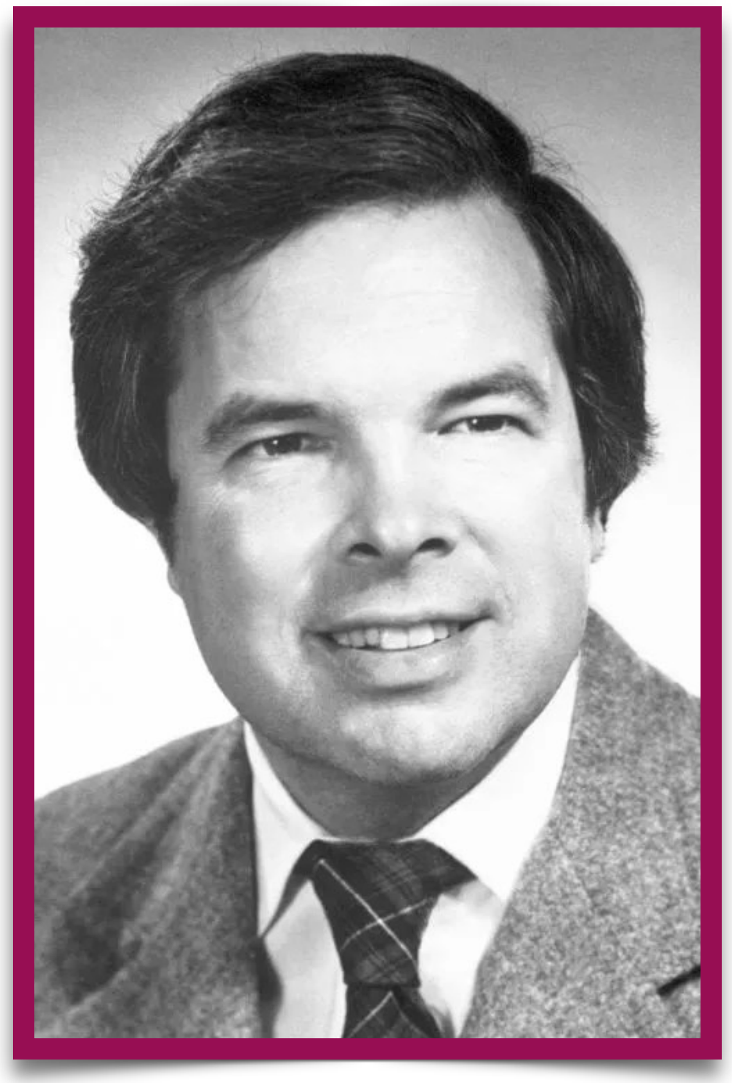
Science 322 (2008) 1224-1227



Yan Lyu et al. Phys Rev Lett.131.161901

- **Early Lattice calculations accurately predicts masses of known hadrons.**
- T_{cc} **lattice results matches with that of the experimental results as pion mass decreases.**

How Lattice QCD Works

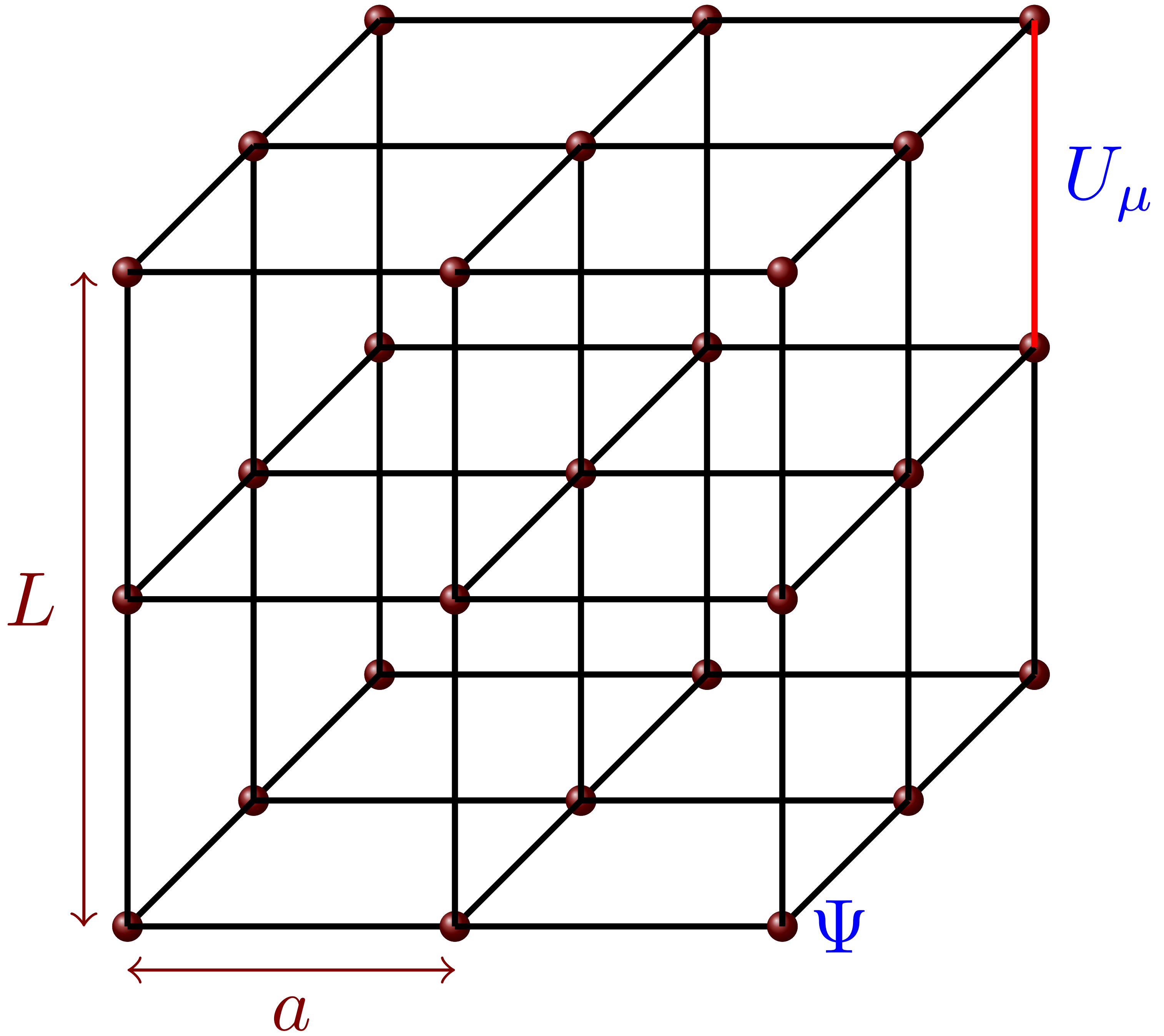


- To understand low energy physics from first principles.
- Fermions (Ψ) in lattice point, gluon field (A_μ) in links.

$$U_\mu = e^{igA_\mu}$$

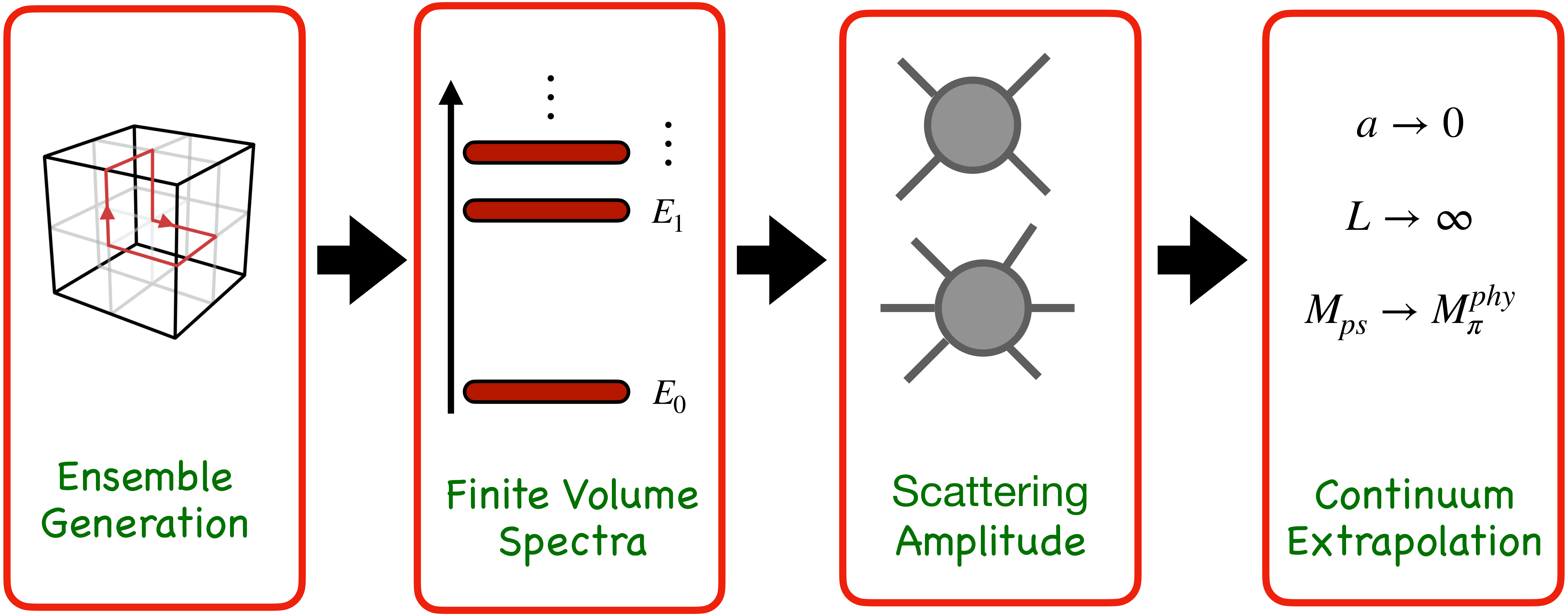
- UV regularized with lattice spacing a, IR regularised with Lattice extent L.
- Define Action of the theory.
- Sample phase space using Markov chain Monte Carlo Method.

$$< \mathcal{O} > = \int D\phi \mathcal{O}[\phi] e^{-S[\phi]}$$



Pedagogical lectures by ND Haridass and M. padmanath at IMSc spring School available at IMSc YT Channel

Hadron Spectroscopy in a Nutshell



Ensemble Generation

Finite Volume Spectra

Scattering Amplitude

$$a \to 0$$

$$L \to \infty$$

$$M_{ps} \to M_{\pi}^{phy}$$

Continuum Extrapolation

Configuration Generation

- Need huge computational resources.
- Cost(€) increases multifold by adding fermions.
- Naive discretisation creates doublers.
- Lattice Fermions:-

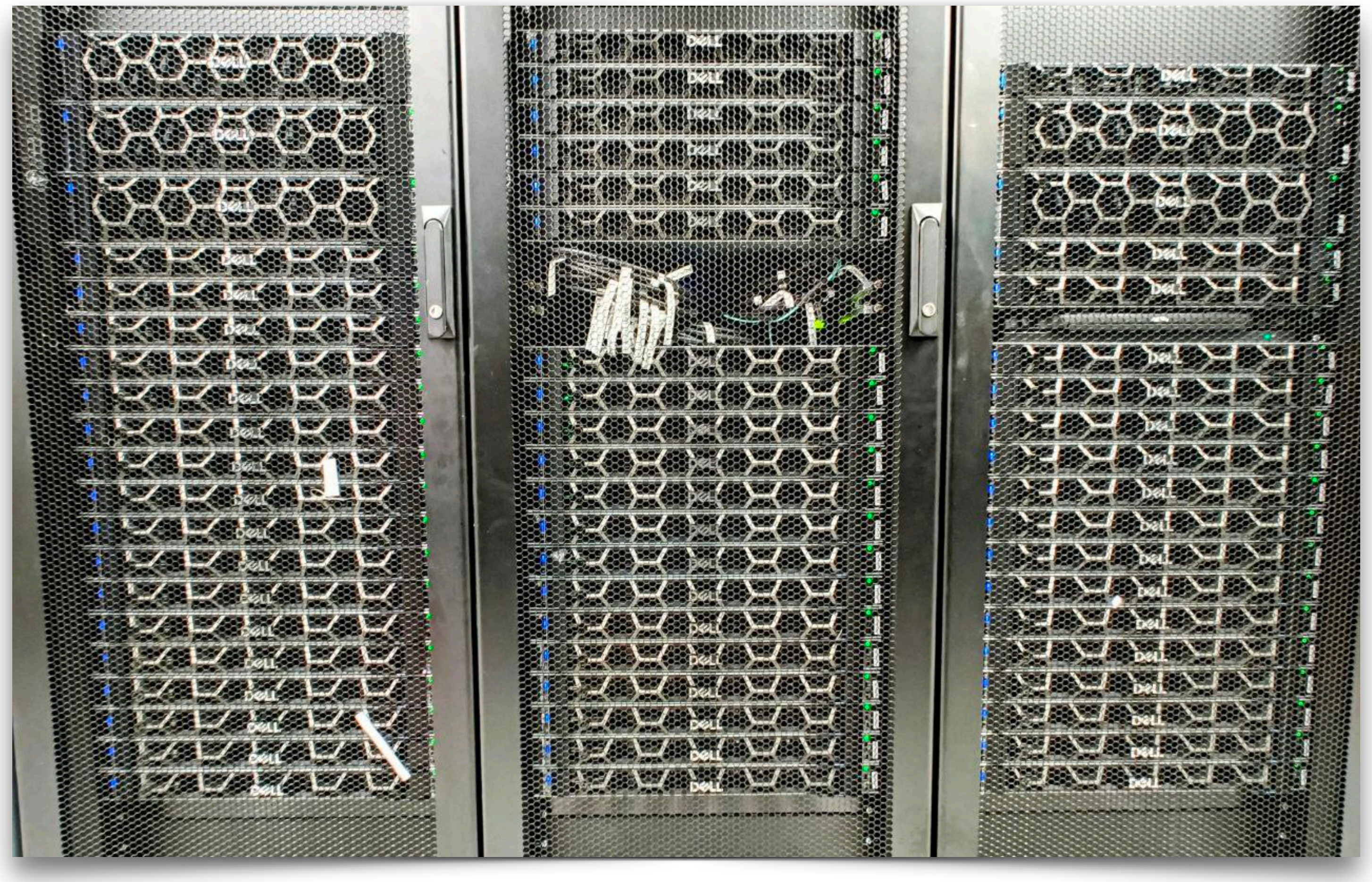
€

i) Staggered fermion (MILC, Simulated QCD)

ii) Wilson fermions (openQCD)

iii) Domain Wall (Grid)

iv) Overlap



Kamet hpc, IMSc

€€ $\uparrow$ as $a \downarrow L \uparrow M_\pi \downarrow$

Configuration Genration

- For fermions,

$$Z = \int dU \prod_f d\Psi^f d\bar{\Psi}^f e^{-S_g - \Psi^f M \bar{\Psi}^f}$$

$$Z = \int dU \prod_f det(M_f) e^{-S_g}$$

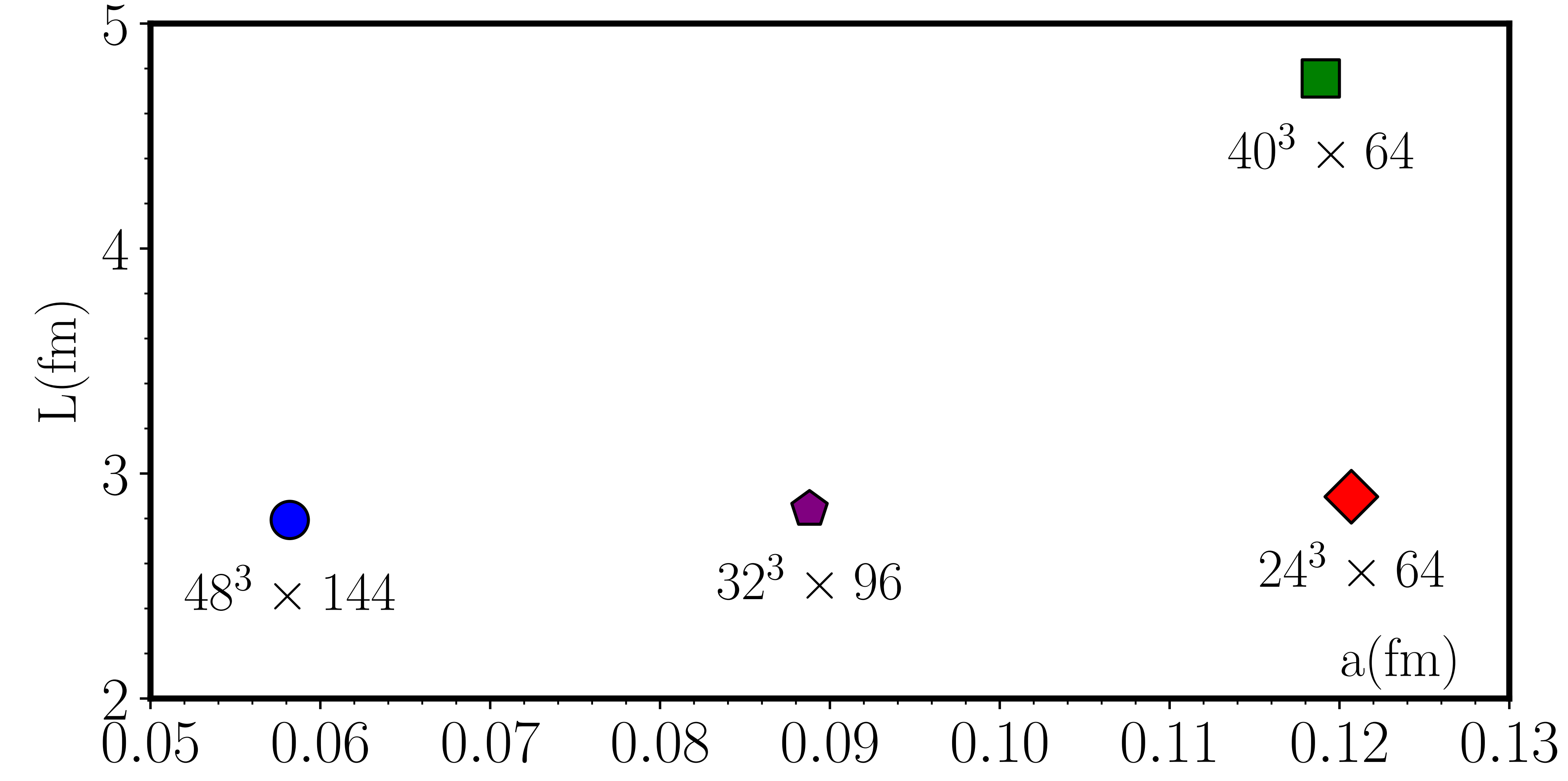
- For e.g. $N_f = 2 + 1$,

Highly Non-local Object €

$$Z = \int dU det(M_u) det(M_d) det(M_s) e^{-S_g}$$

Lattice Setup

- We are interested in doubly bottom tetra quarks $bb\bar{u}\bar{d}$ with $I(J^P) = 0(1^+)$.
- Worked with 4 MILC ensembles with $N_f = 2 + 1 + 1$ using HISQ action.
- Ensembles were generated at unphysical light quarks and physical charm and strange quarks.
- Light quark propagators were constructed using Overlap action. For heavy(bottom) quark, we used NRQCD action.
- Wall-source smearing setup.
- Used multiple volumes, box-sink correlators to reduce systematic effects.



NRQCD formalism

- Relativistic propagator not possible as $am_b > > 1$ in our setup. Require bigger lattice size.

- NRQCD becomes most suitable candidate.

- Energy scales here,

$$m_b > > m_b v > > m_b v^2$$

- We exclude rest mass term to make momentum as highest energy scale allow for calculation in larger lattice spacing.

- Offset correction accounted at the time of Analysis.

Building Hadrons

- Goal is to construct interpolating fields coupled with ground state of desired quantum number.

- Properties:-

   1. Flavor Sturture:- Correct combination of quark fields.

   2. Spin and Parity:- Appropriate Dirac bilinear Γ and quadrilinear(Tqs) to match desired spin and parity.

- Lattice breaks O(3) symmetry, operators must transform according to irreps of the cubic group.

- e.g. Pion:- $\Phi_\pi = \bar{d}^\alpha_c(\gamma_5)_{\alpha\beta}u^\beta_c$, c -> color index

Pseudo scalar

Extracting Finite Volume Spectrum

- To extract spectrum we need good interpolating operators.

- Here we are using two types of operators.

Meson-Meson :

$$\Phi_{\mathscr{M}_{BB^*}}(x) = [\bar{u}(x)\gamma_i b(x)][\bar{d}(x)\gamma_5 b(x)] - [\bar{u}(x)\gamma_5 b(x)][\bar{d}(x)\gamma_i b(x)]$$

Diquark-antidiquark:

$$\Phi_{\mathscr{D}}(x) = [(\bar{u}(x)^T C\gamma_5 \bar{d}(x)) - (\bar{d}(x)^T C\gamma_5 \bar{u}(x)) \times (b^T(x) C\gamma_i b(x))]$$

- Finite volume spectrum can be calculated using Euclidean $\mathscr{C}_{ij}(t)$, between $\Phi's$

$$\mathscr{C}_{ij}(t) = \sum_X \left\langle \Phi_i(\mathbf{x}, t)\tilde{\Phi}_j^\dagger(0) \right\rangle \sim e^{-E_n t}$$

Wall Sources

- Instead of using a single point source, a source is placed at every spatial point on the source time slice.

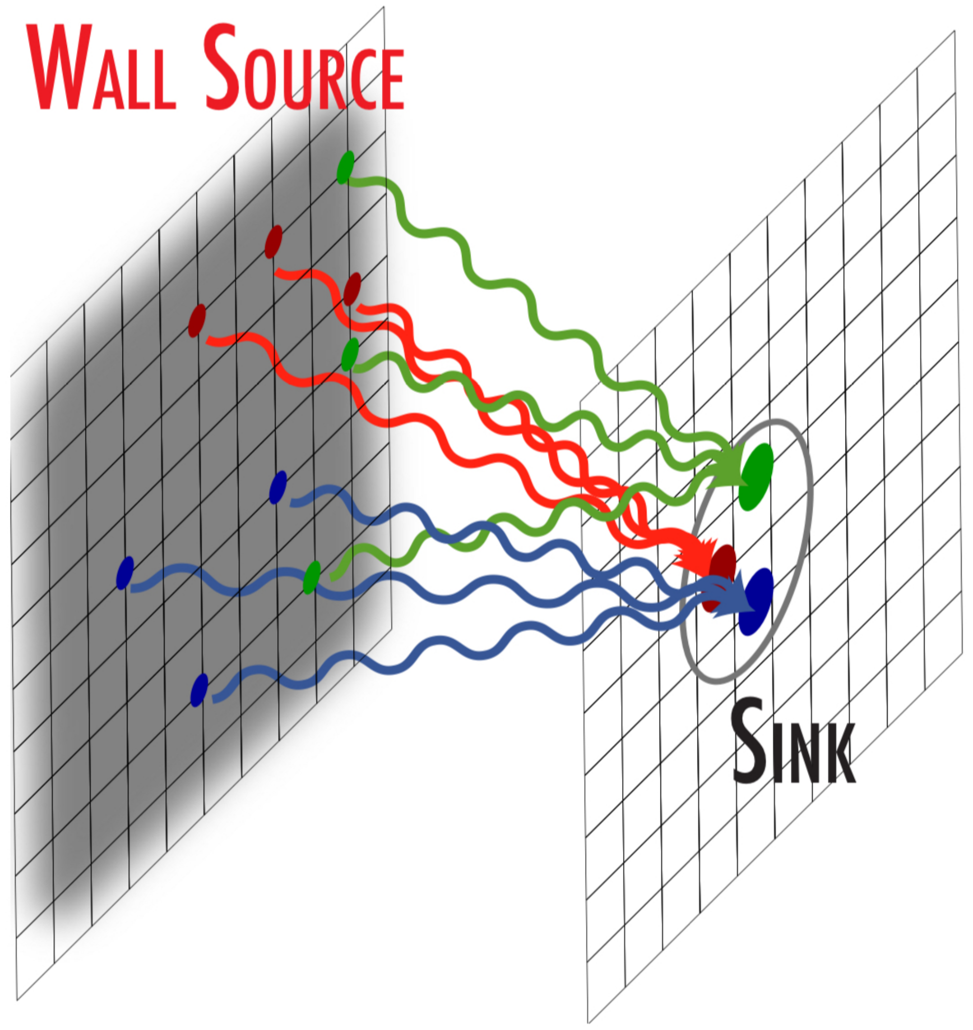


Image Credit:- S. Aoki

$$Q(\bar{x}, t; t') = \sum_{\bar{x}'} Q(\bar{x}, t; \bar{x}', t')$$

- ADVANTAGES:- Better signals in the ground state.
- DISADVANTAGE:-
  1. Assymetric Correlation Function, Non-Hermitian GEVP Needed.
  2. False Plateau encounter, careful with fitting time window.
- Why not Wall source wall sink correlator? -> Very Noisy signal.

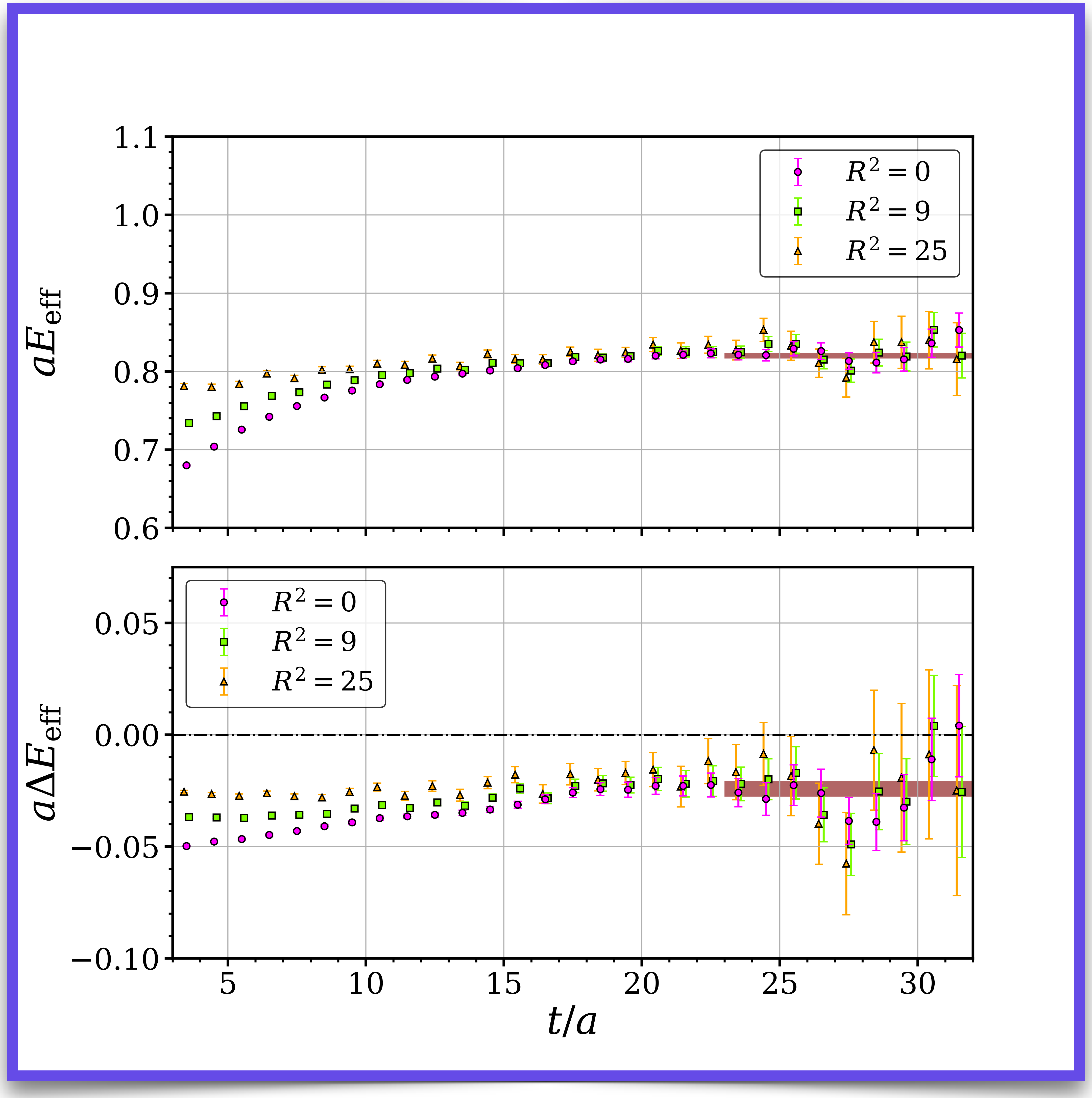
Wall source- Box Sink Correlator

- Instead of wall-sink, we build box-sink correlator.

Phys. Rev. D **102**, 114506

$$Q(\bar{x}, t; t') = \sum_{|\bar{y}-\bar{x}|<R} Q(\bar{y}, t; , t')$$

- As we increase box radius R, it approaches to symmetric correlator.
- Used to make comparative study of the asymptotic signals.
- Increases robustness of the calculations.



arXiv:2503.09760 BST, Mathur, Padmanath

Finite Volume Spectrum cont.

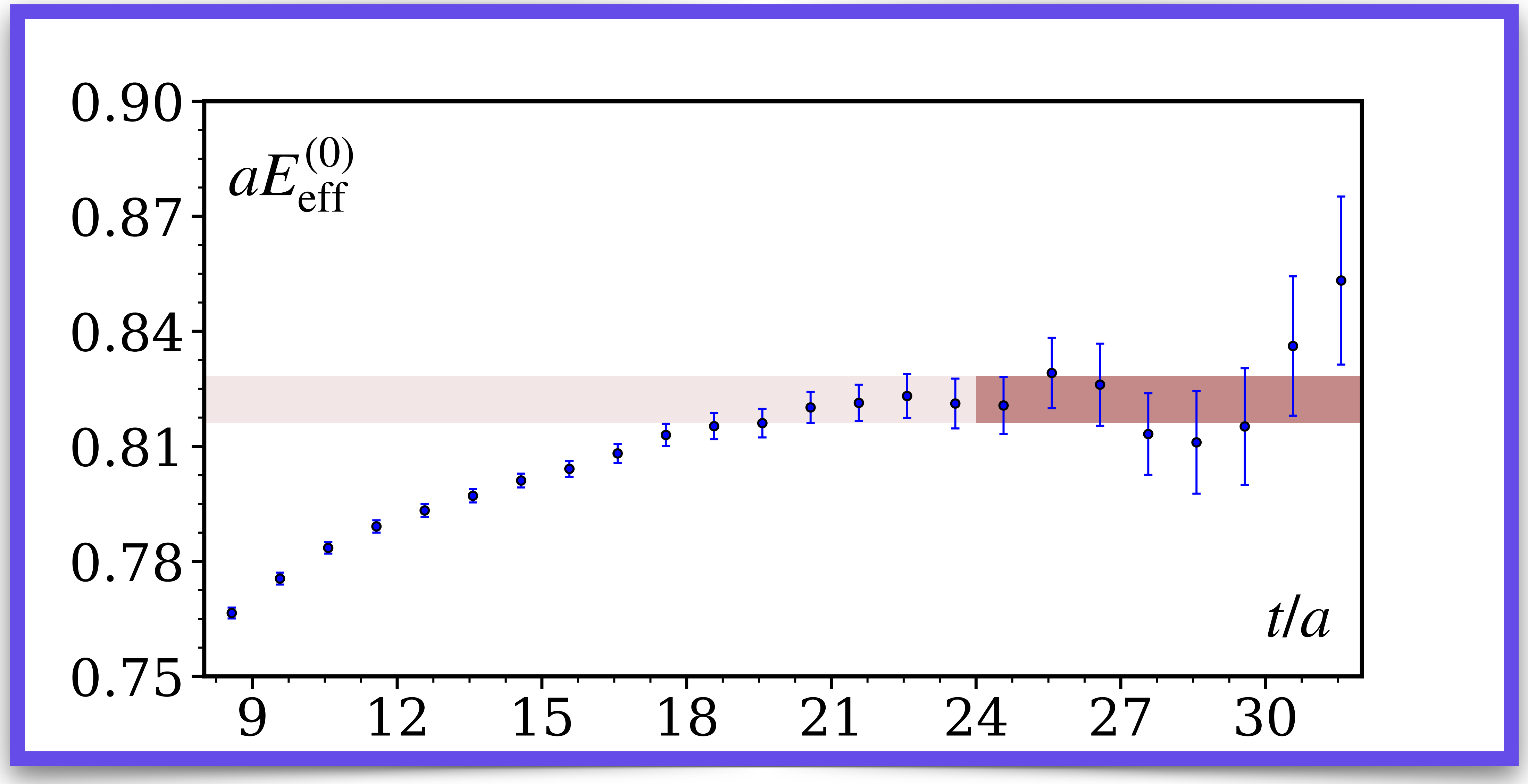
- We use GEVP to extract finite volume spectrum from correlation-matrix variationaly.

$$\mathscr{C}(t)v^{(n)}(t) = \lambda^{(n)}(t, t_0)\mathscr{C}(t_0)v^{(n)}(t)$$

- Fitting the leading exponential of $\lambda^n(t)$, yields the energy Eigen states E_n.

$$\lambda^n(t, t_0) = |A_n|^2 e^{-E_n(t-t_0)}[1 + \mathcal{O}(e^{-\Delta_n(t-t_0)})]$$

- Excited states can be determined with this method.
- Repeated for B and B^* mesons.



Non-Hermitian GEVP

- Modification of GEVP to account non-hermiticity.

- Solving asymmetric correlation matrix with left and right Eigenvactor with same Eigenvalue.

$$\mathscr{C}(t_d)v_r^{(n)}(t_d) = \tilde{\lambda}^{(n)}(t_d)\mathscr{C}(t_0)v_r^{(n)}(t_d)$$

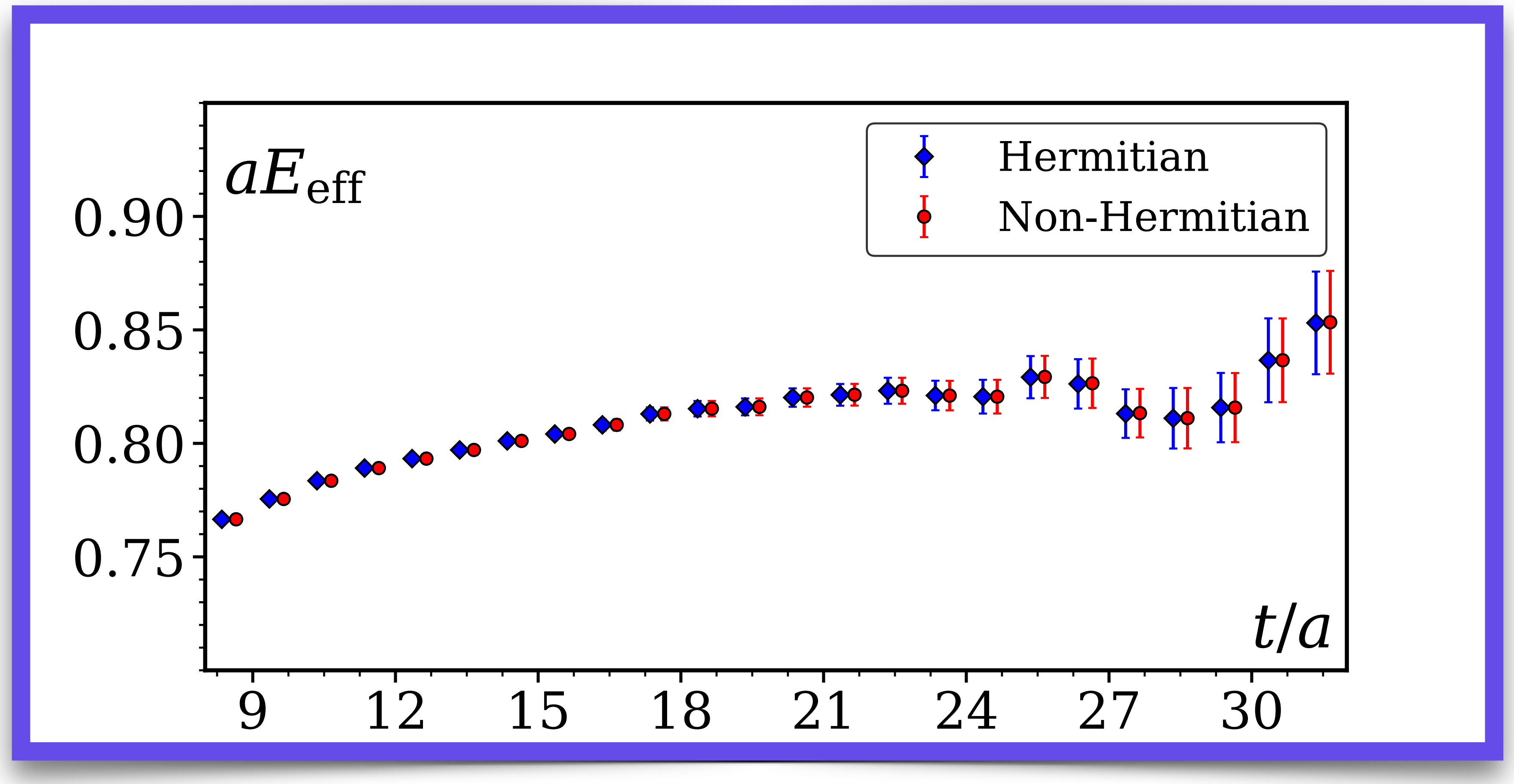
$$v_l^{(n)\dagger}(t_d)\mathscr{C}(t_d) = \tilde{\lambda}^{(n)}(t_d)v_l^{(n)\dagger}(t_d)\mathscr{C}(t_0)$$

t_d -> diagonalization time slice

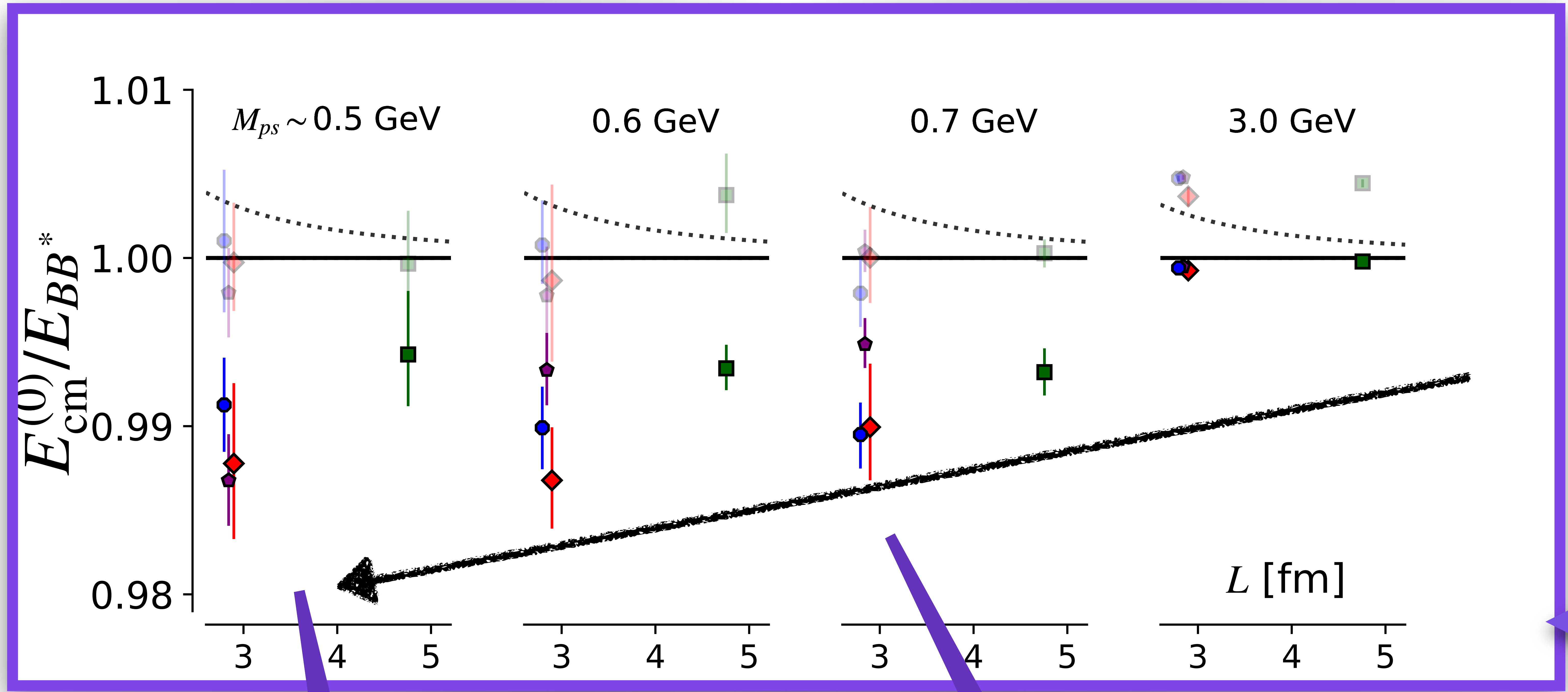
- Solve it for other time slices with $v_l(t_d)$ and $v_r(t_d)$ using

$$\tilde{\lambda}^{(n)}(t) = v_l^{(n)\dagger}(t_d)\mathscr{C}(t)v_r^{(n)}(t_d)$$

- Found $\dfrac{Im(\tilde{\lambda}^{(n)}(t))}{Re(\tilde{\lambda}^{(n)}(t))} < 0.01$ for all correlators used.



Finite Volume Spectrum cont.



- Spectrum extraction repeated for every M_{ps} and every ensemble.

Spectrum were calculated In units of nearest Two body decay threshold $BB*$.

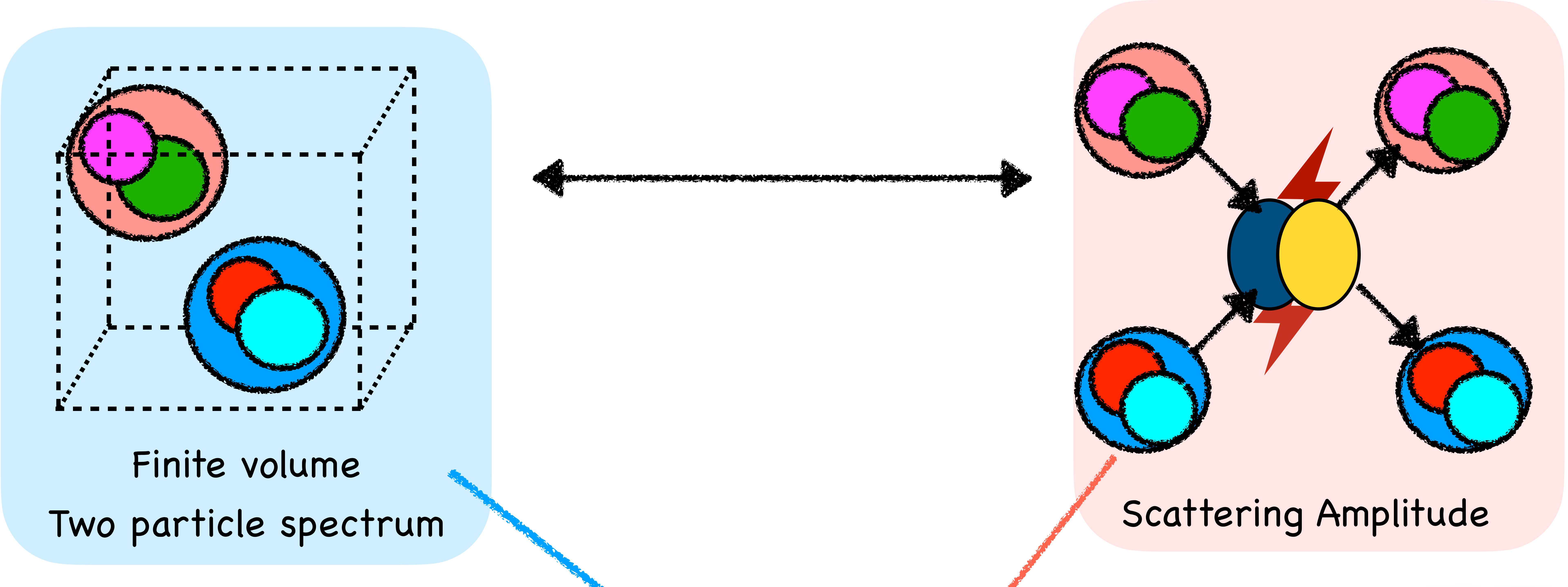
NRQCD Offset corrected

A decreasing trend can be observed

We need continuum extrapolation to have results in physical limit

Amplitude Analysis

Lüscher based quantization condition(1991)



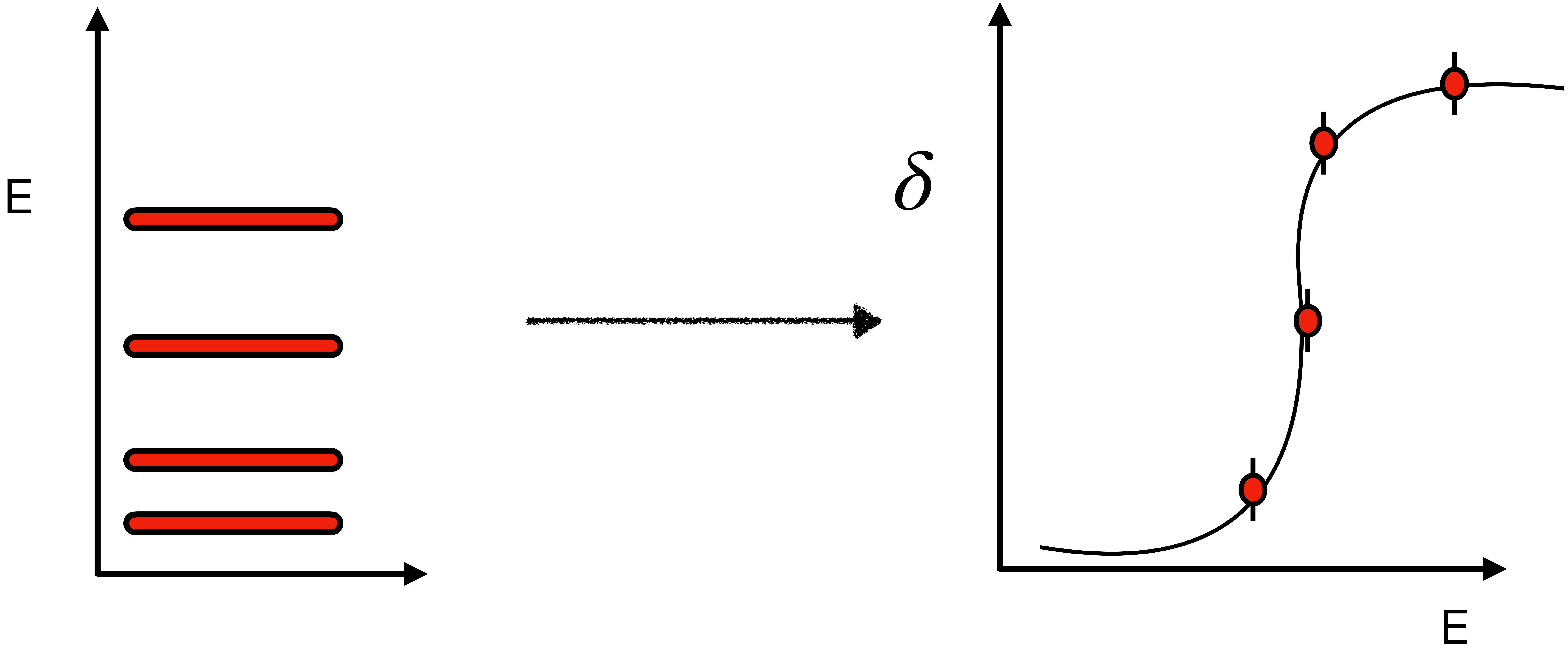
Finite volume

Two particle spectrum

Scattering Amplitude

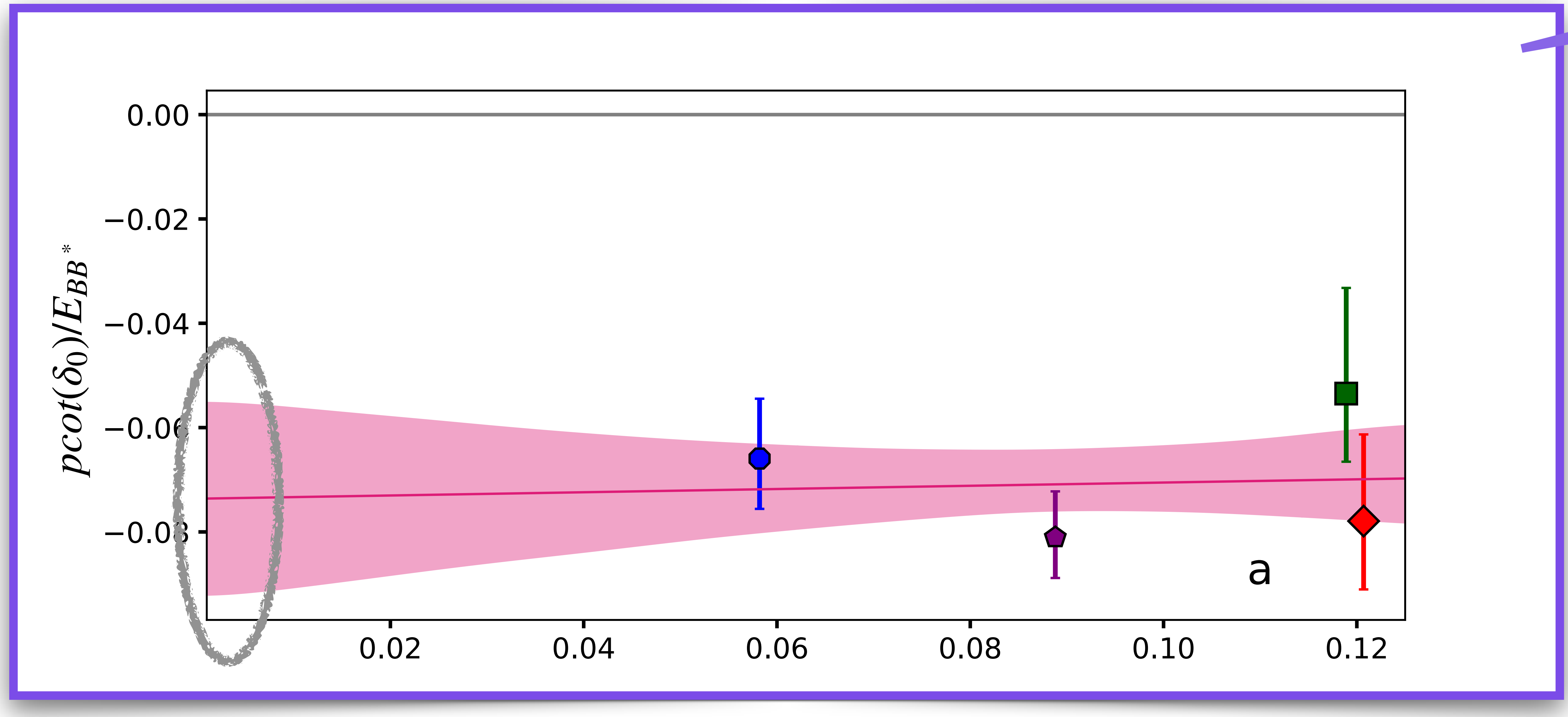
$$\boxed{det\left[1+i\mathscr{G}(E)\ \mathscr{M}(E)\right]=0}$$

Amplitude Analysis



- Energy is discrete.
- Energy dependence is required.

Continuum Extrapolation



$M_{ps} = 0.5$ GeV

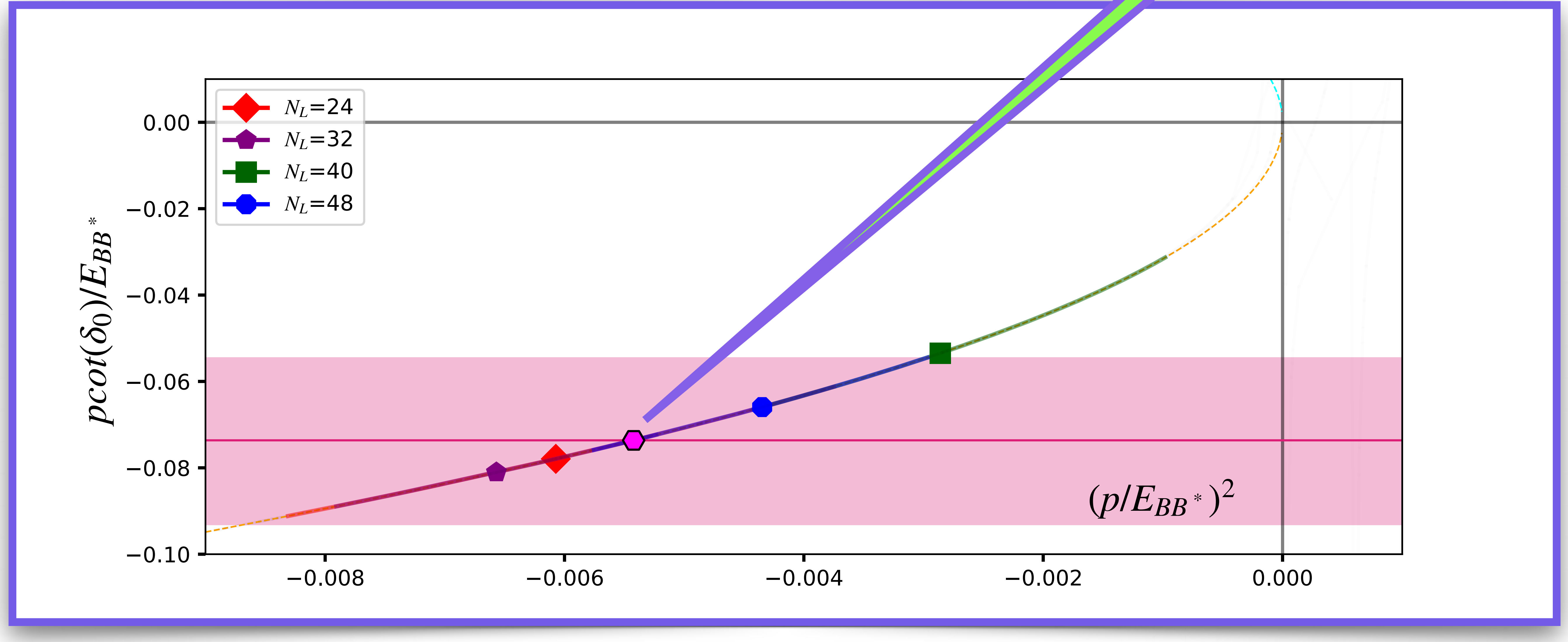
- **Scattering Amplitude is given as**

$$T \propto (pcot\ \delta - ip)^{-1}$$

- **It is parametrised as**

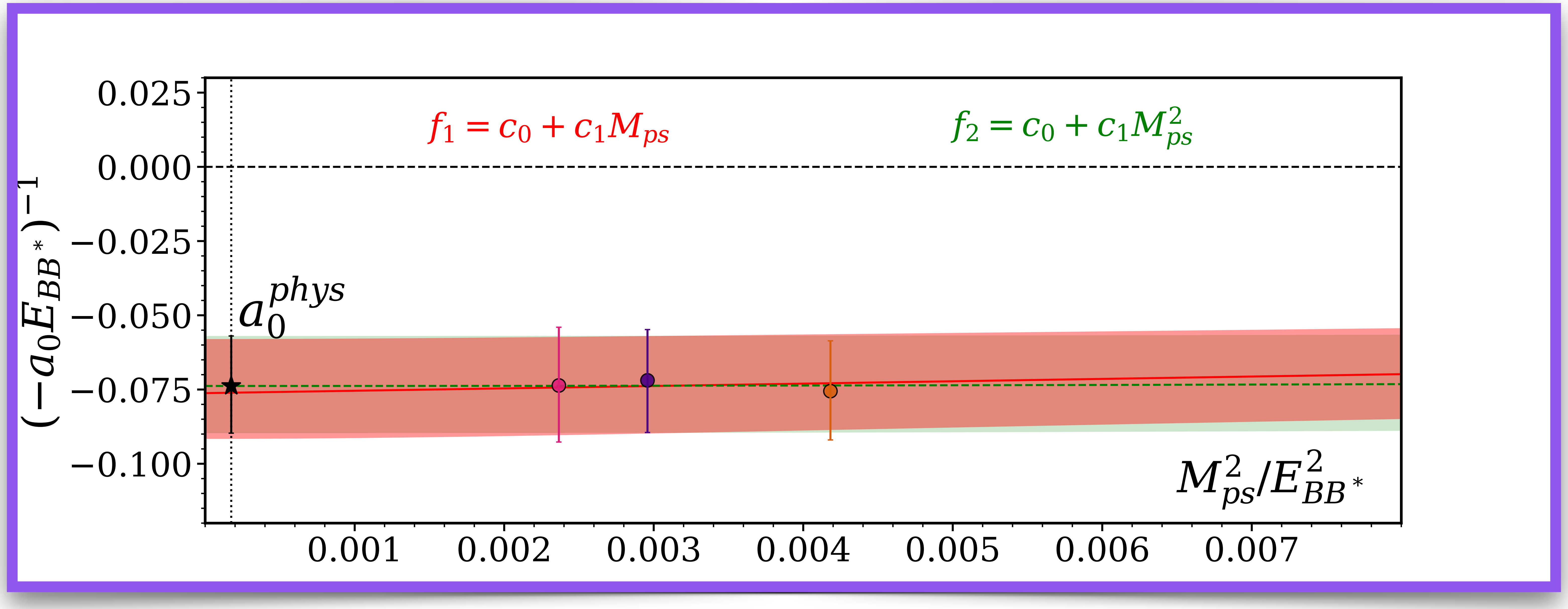
$$pcot\ \delta = -\frac{1}{a_0} + B \cdot a$$

Real Bound State



- **Same repeated for other M_{ps}.**
- **Consistent Negative values for other M_{ps} as well as real bound state.**

Chiral Extrapolation

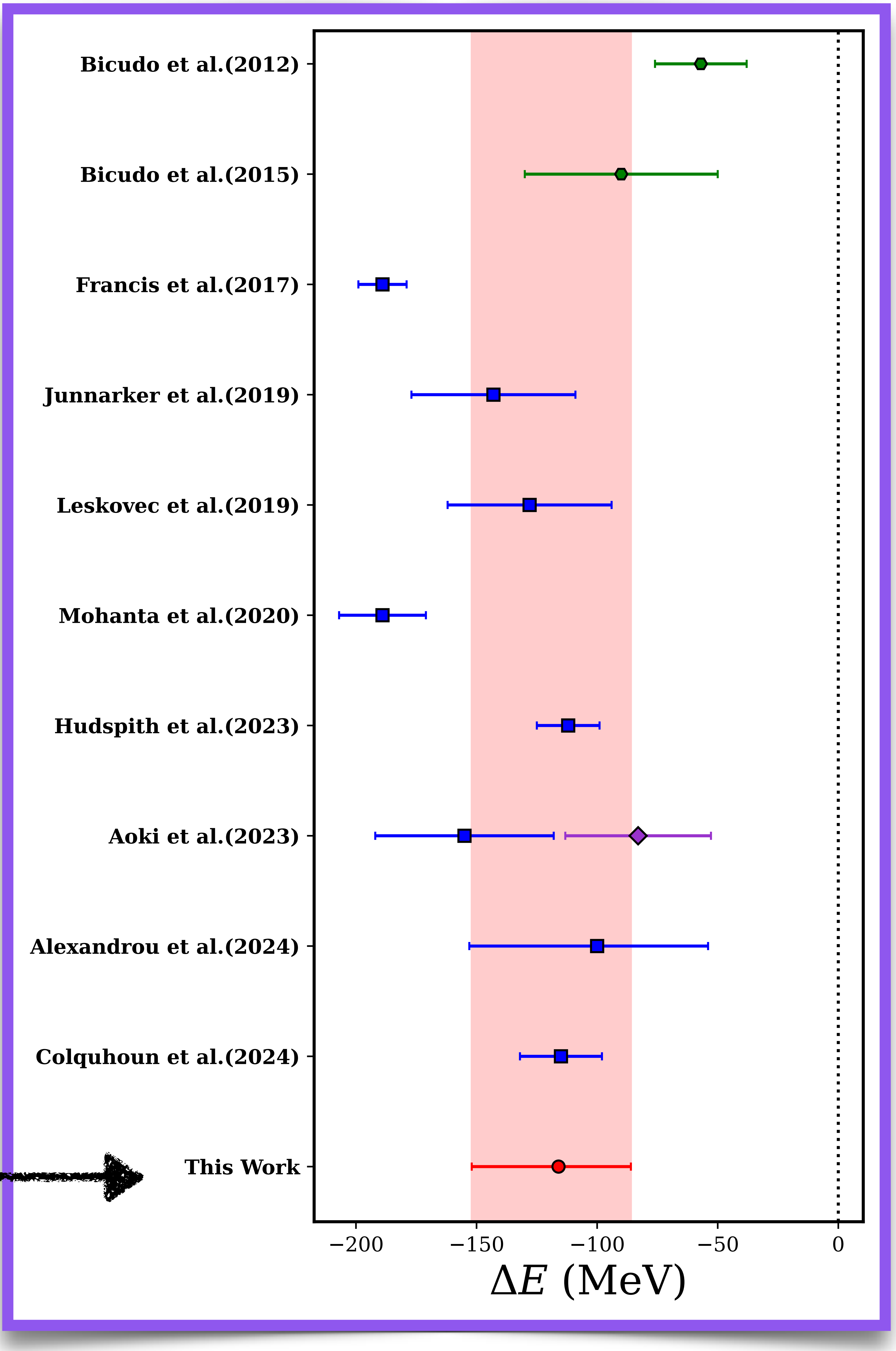


Result:

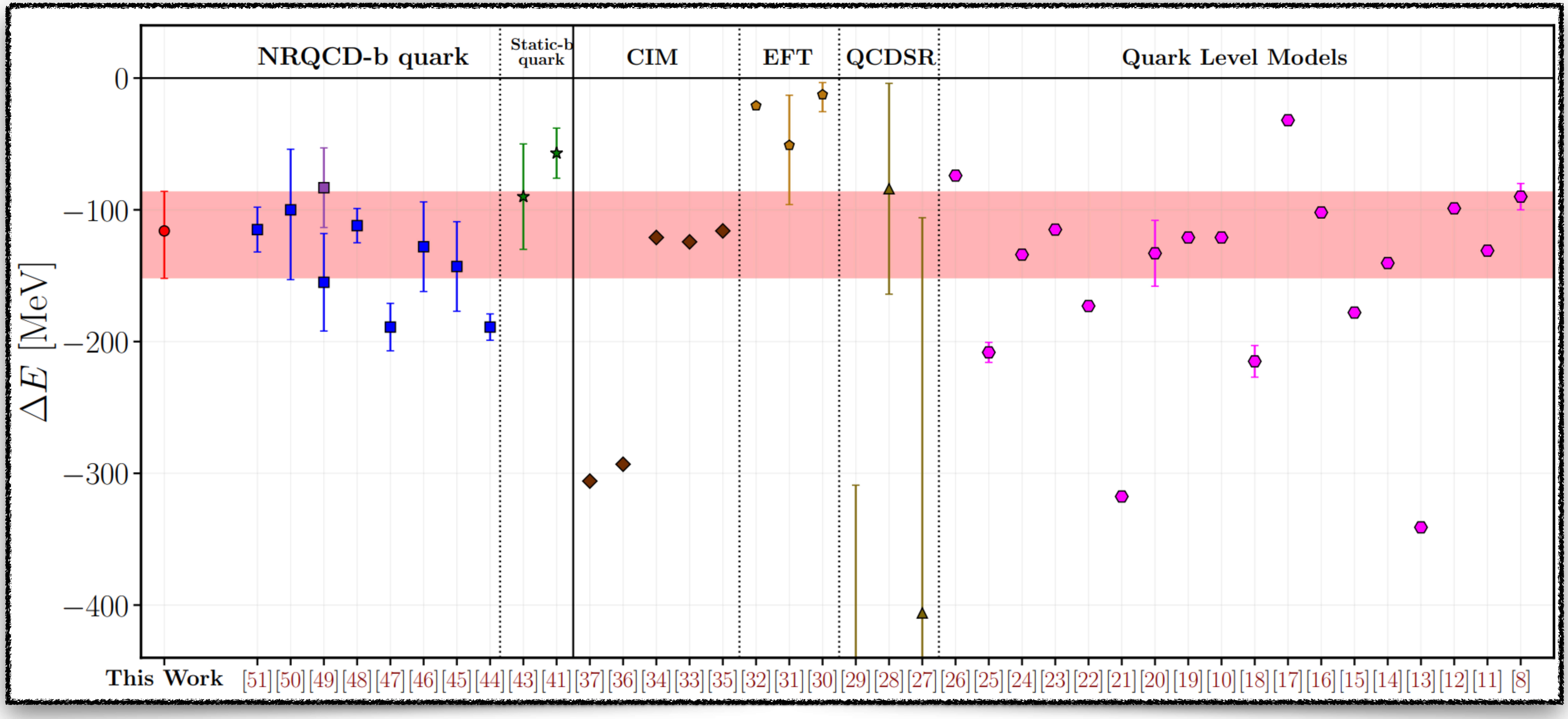
Scattering length at physical limit

$$a_0^{phy} = 0.25(^4_3)\ fm$$

Corresponds to binding energy $\Delta E = -116(^{+30}_{-36})$ **MeV.**

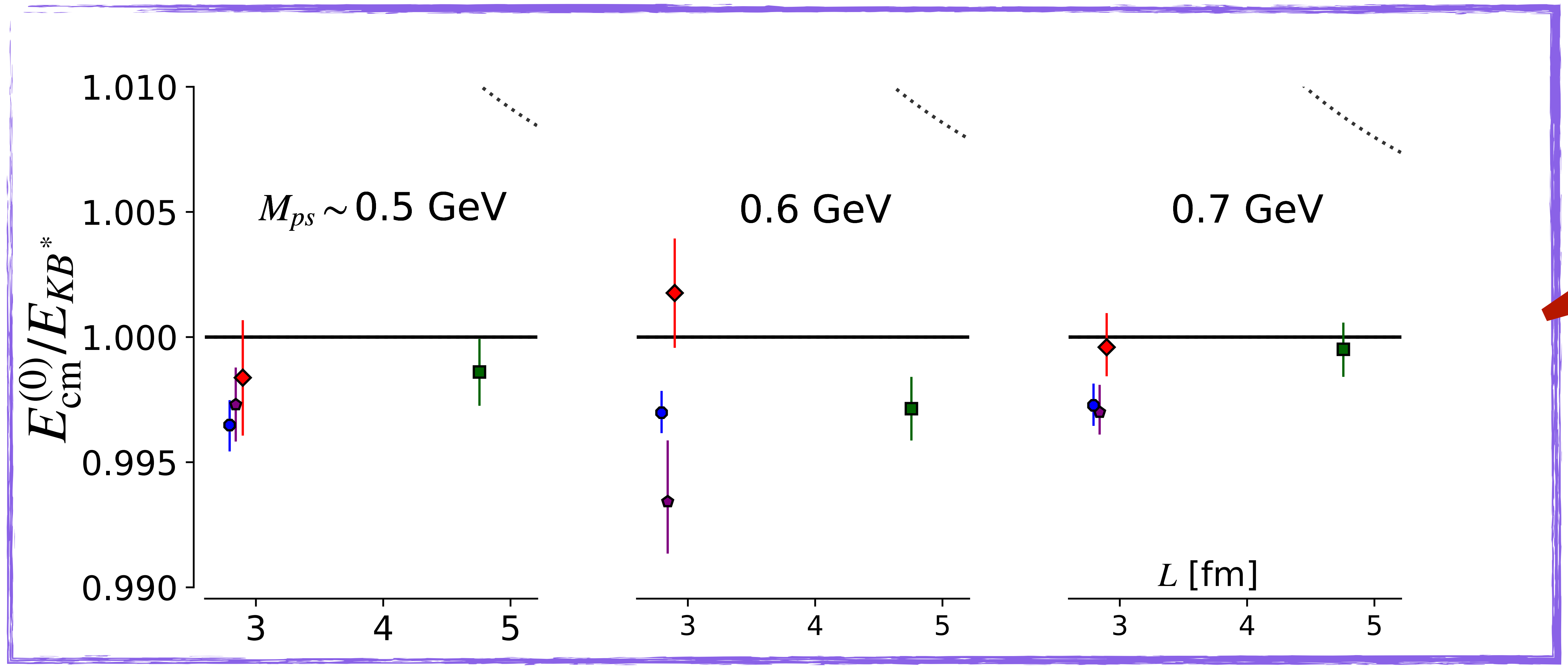


Summary T_{bb}



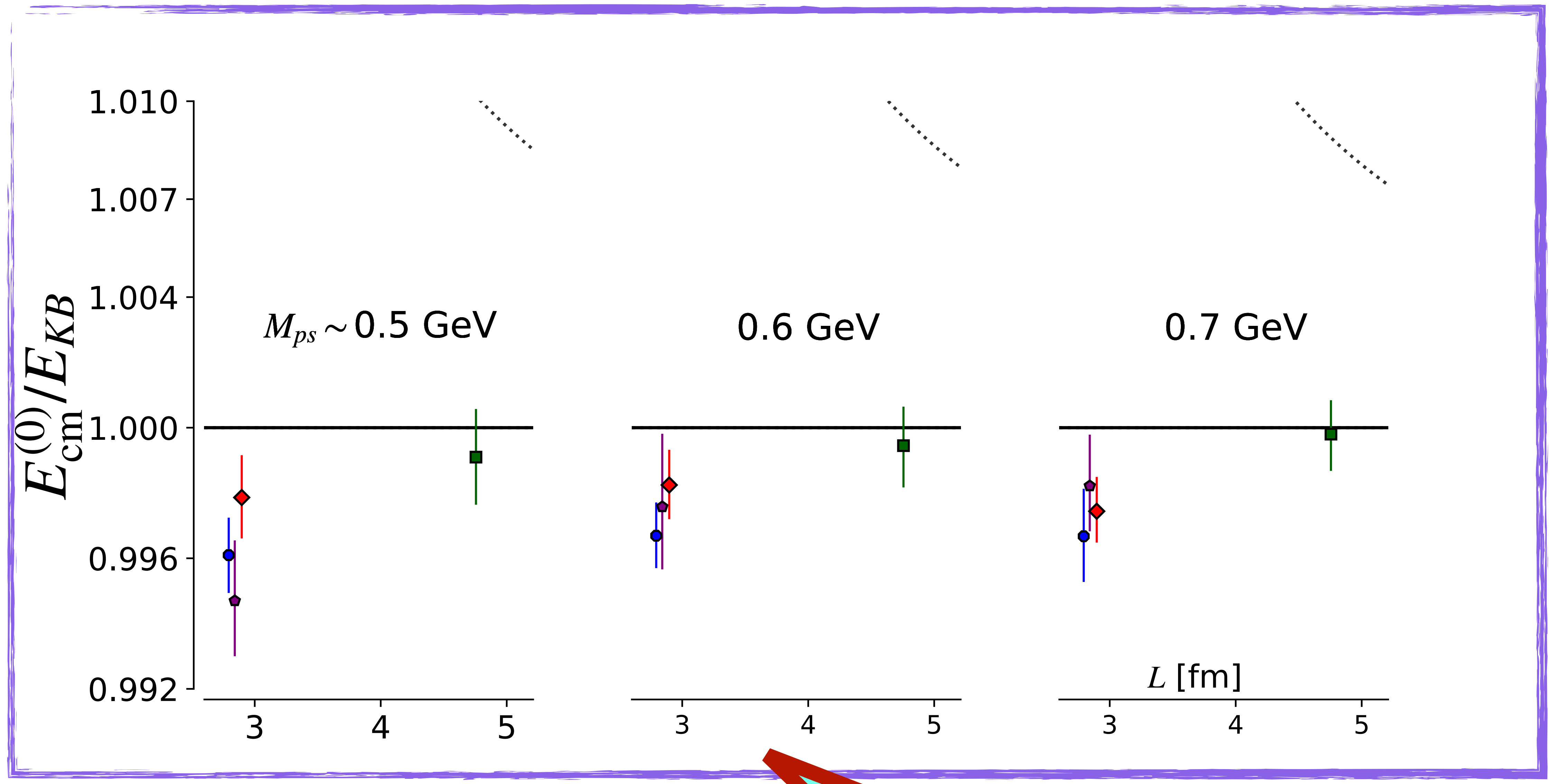
arXiv:2503.09760 BST, Mathur, Padmanath

Results of T_{bs}



Axial vector T_{bs}

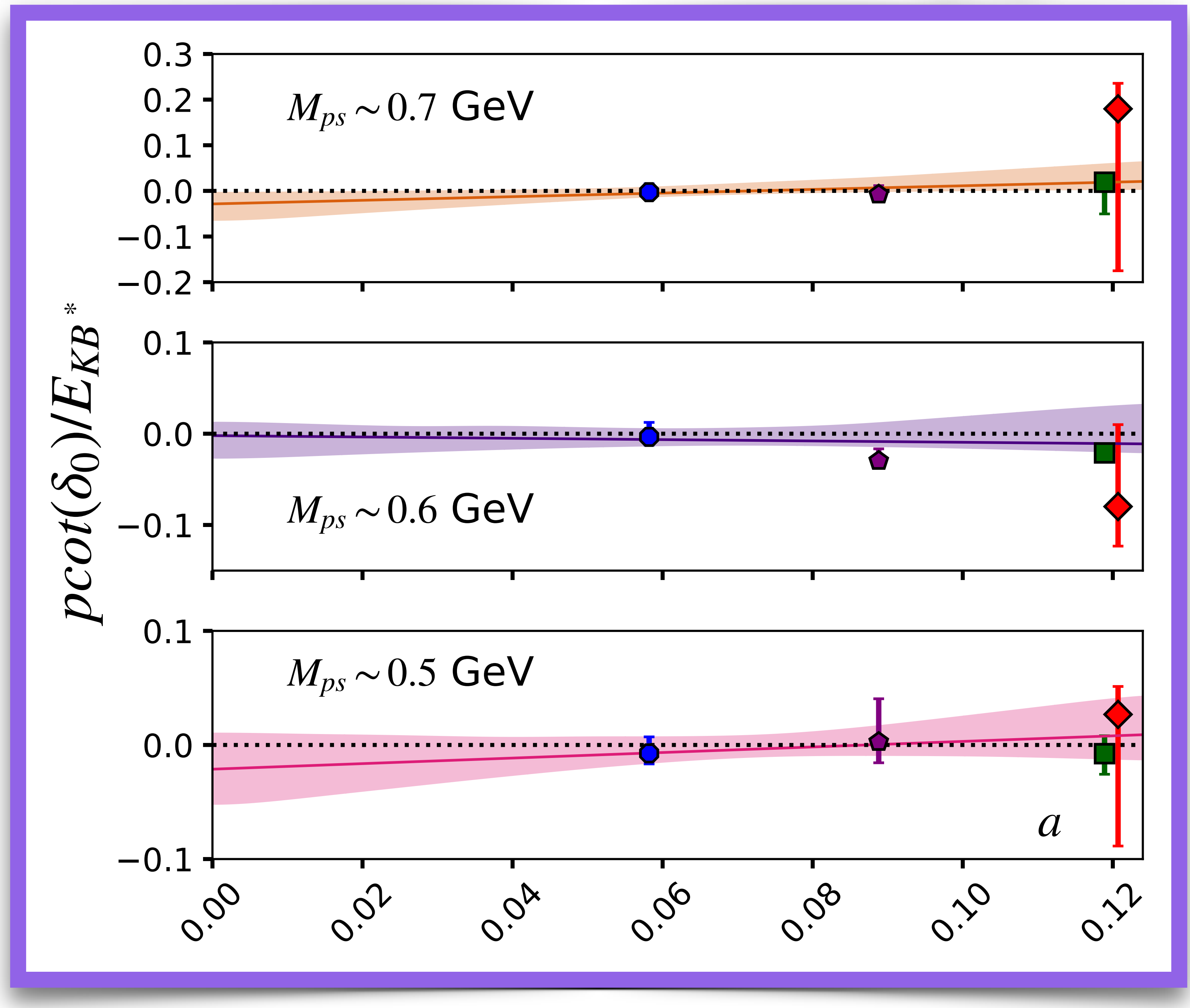
- Results are consistent with threshold in both the cases for larger volume.
- Need low M_{ps} datas for better results.



Scalar T_{bs}

Results of T_{bs}

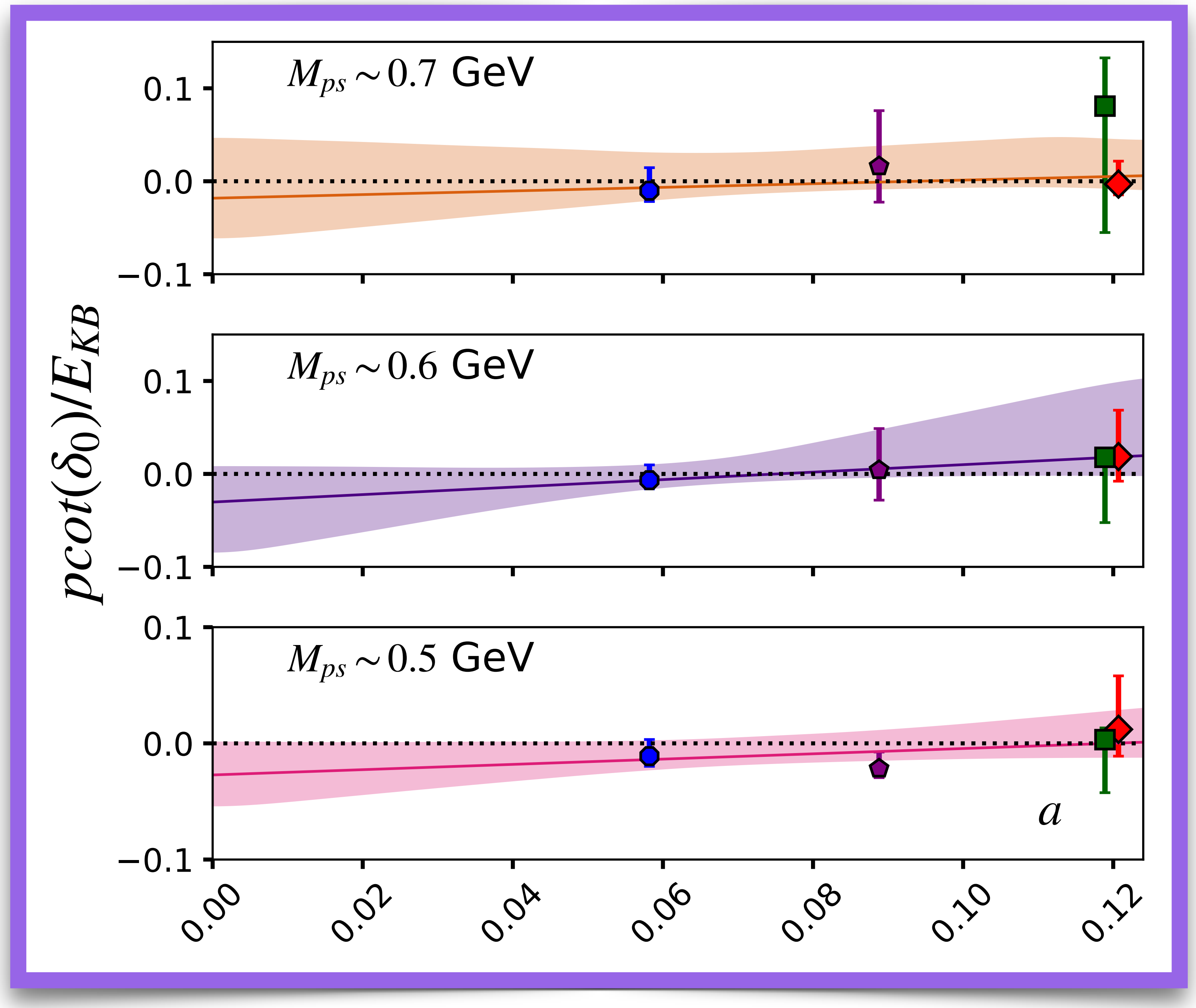
arXiv:2503.09760 BST, Mathur, Padmanath

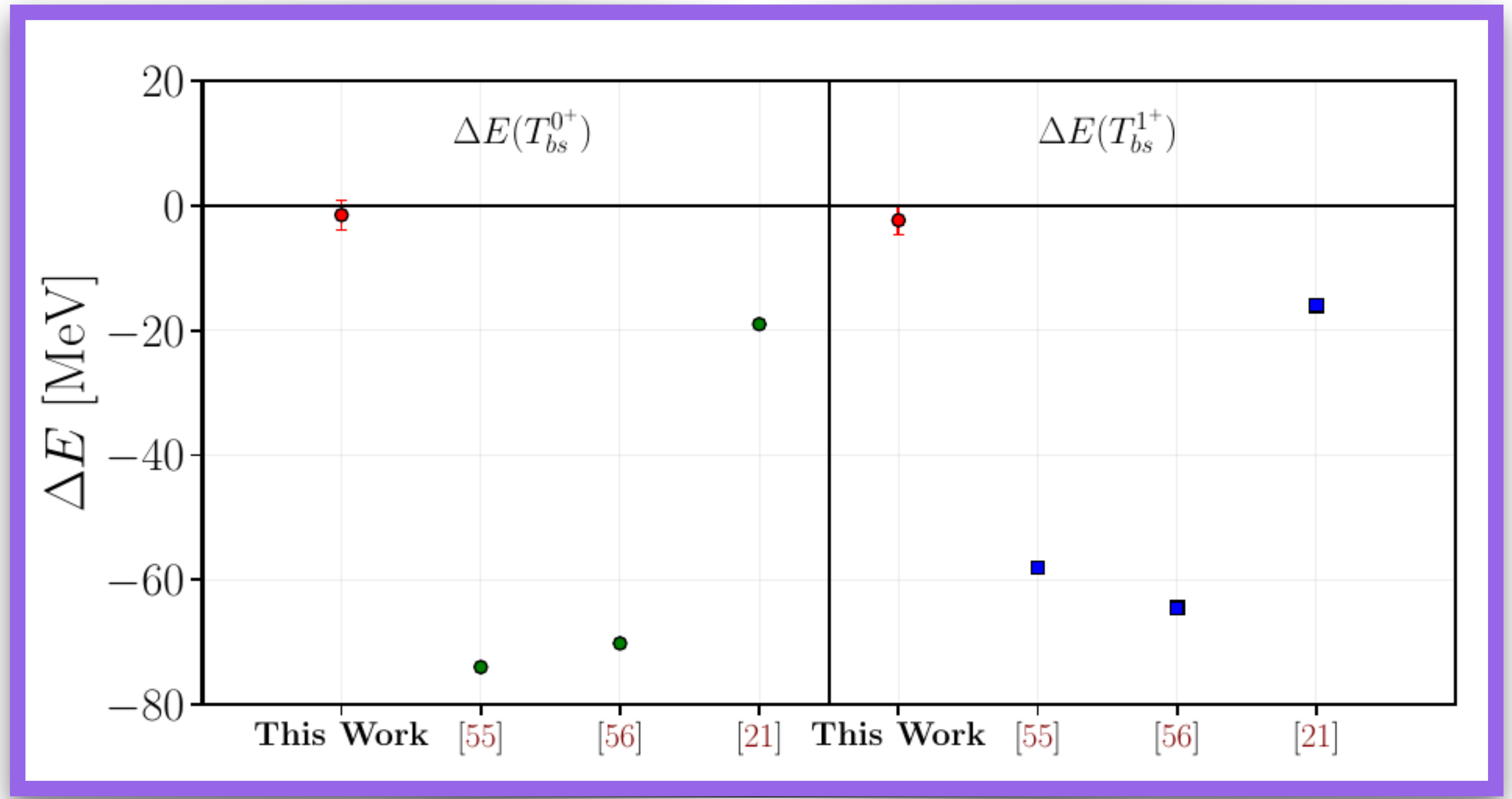


Axial vector T_{bs}

Summary T_{bs}

Scalar T_{bs}





Can we understand phenomenological why this is the case??

Summary and Outlook

- Various work widely predicted deep binding in isoscalar axial-vector T_{bb}.
- Rigorous spectrum analysis were done for T_{bb} tetraquark with $I(J^P) = 0(1^+)$.
- We worked with multiple lattice spacing, two volumes to control systematics.
- Finite volume spectrum indicates negative energy shift with respect to BB^* threshold.
- Found a possible deeply bound state for T_{bb} not such exciting results in T_{bs}.

Slides will be available at my website https://www.imsc.res.in/~bhabanist/

THANK YOU

Our Group @ IMSc

Tanishk

Navdeep

Priyajit