Bound on tri-partite Hardy's nonlocality respecting all bi-partite principles

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Study of certain class of tripartite correlations under a number of recently proposed bi-partite physical principles has produced important insight regarding such principles. We find a lower bound on success probability of tri-partite Hardy's correlation respecting all bi-partite physical principles. The no-signaling principle does not reveal any gap in Hardy's maximum success probability for bi-partite and tri-partite system, whereas quantum mechanically there is such a gap. Interestingly, we show that, Information causality principle is successful in qualitatively revealing this quantum feature.

Motivation–The outcome of local measurements on spatially separated parts of a composite quantum system can be non-classically (nonlocally) correlated. Violation of the Bell-CHSH inequality [1, 2] is a witness of this nonlocal feature in such correlations. The value of Bell-CHSH expression, exceeding the classical bound 2, then qualifies as a measure of nonlocality. This nonlocality within the quantum mechanics is limited by the Cirel'son bound $2\sqrt{2}$ [3]. On the other hand, on considering a larger set of generalized correlations (possibly non-quantum) which are compatible with *no signaling* (NS) principle, nolocality underlying these correlations can achieve any value up to the algebraic maximum 4 for the Bell-CHSH expression (e.g. PR-box correlation achieves the value 4 [4]). So the natural questions arises; what are the physical principles, other than NS, that can distinguish quantum correlations from post-quantum no-signaling correlations? This fundamental question has been addressed in several recent works proposing novel physical principles, like, no nontrivial communication complexity [5, 6], macroscopic locality [7] and information causality [8], for explaining the boundary defining quantum correlations. In particular, the application of *the principle of noviolation of information causality* (IC) has produced very interesting results, like explaining the Cirel'son's bound and showing that in a bipartite scenario any correlation going beyond the Cire'lson bound is unphysical. IC principle is a generalization of no-signaling condition—while relativistic causality (the no-signaling principle) states that a party cannot extract more information then the communicated (say, m) number of cbits; information causality further restricts *free choice to decode deterministically* a single m-cbit information, from

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different possible m-cbit informations potentially encoded within the communicated amount of m-cbit message. Applications of the IC principle in the study of both bi-partite and tri-partite correlations has produced some interesting results [9–12]. On the other hand, IC or any other bi-partite principle has been shown to be insufficient for witnessing all multipartite post-quantum correlations [13, 14]. Thus, multipartite generalization of IC, or some other genuinely multipartite physical principle(s) are necessary to characterize all quantum correlations. Studying some simple class of multipartite correlations, like Hardy's set [15, 16, 18, 19], can give useful insight about the strength and weakness of such principles [10, 11, 14].

Like Bell-CHSH nonlocality test, Lucian Hardy first proposed [15] an elegant argument for witnessing nonlocal correlations without any use of inequality. For two qubit states subjected to local projective measurements, the maximum success probability of Hardy's nonlocality argument has been shown to be $(5\sqrt{5}-11)/2$ (≈ 0.09)[16, 17]. Recently, it has also been shown that this value is 'device independent', i.e, the maximum success probability of Hardy's argument for any bipartite quantum state is $(5\sqrt{5}-11)/2$ [18]. Again, on extending Hardy's argument to three qubit systems subject to local projective measurements, the maximum success probability of the argument is shown to be $\frac{1}{8}$ ($= 0.125$) [19]—this value also holds for any tripartite state subjected to arbitrary measurements, i.e., this value is also device independent [14]. Thus, in quantum mechanics, the maximum success probability of Hardy's nonlocality argument for tri-partite system is greater than that for bi-partite system.

Hardy's nonlocality has been studied in the broader framework of general probabilistic theories by invoking physical principles like NS condition and IC condition. Under the NS condition, optimal success probability for Hardy's nonlocality is $\frac{1}{2}$, both for bi-partite system and tri-partite system [19]. Thus, in contrast to the quantum mechanical feature, under the NS condition there is no gap between Hardy's maximum success probability for bi-partite and tri-partite system. On the other hand, under the IC principle, it has been shown that the maximum success probability of Hardy's argument for bi-partite system is bounded above by 0.207 [10]. The study for the bound on Hardy's success probability for tri-partite system under IC condition has not yet been studied. The problem is very intriguing as information causality is a bi-partite principle and it is highly nontrivial to exhaust the IC condition under all bi-partitions with all possible wirings.

Results–Motivated along this line of research, in our work [23], we show that the maximum value of Hardy's success for tri-partite correlation satisfying every bi-partite principle cannot go below $\frac{1}{4}$. Then, we argue that in particular IC principle successfully reveals a quantum feature viz. a gap between Hardy's maximum success probability for bi-partite and tri-partite systems.

Moreover, the gap between two bounds is decisive, as for tri-partite system we achieve a lower bound through a probability distribution which is *time-ordered-bi-local* (TOBL) [20–22] and hence it not only satisfies IC but satisfies any bi-partite information principles discovered or not discovered. On the other hand, for the bipartite case, the upper bound on maximum success probability was derived by applying a necessary condition for respecting the IC principle. Thus, even though IC principle may not reproduce various quantum features quantitatively like maximum success probability for Hardy's nonlocality in quantum mechanics, still it could reproduce some qualitative interesting quantum features like revealing a gap between bi-partite and tri-partite cases for the maximum success probability of Hardy's nonlocality argument.

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