Ergodicity Transitions of Quantum Correlations in Many-Body Systems

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Quantum correlations – being one of the most important physical resources in quantum information science [1] – has led to the development of technologies based on quantum principles. Understanding quantum correlations theoretically and their robust experimental generation remain important goals, although significant advances have been achieved in recent years on both fronts. The characterization, quantification and realization of quantum correlations in many-body systems are some of the main challenges in quantum information theory.

Quantum correlation concepts in multiparty systems can broadly be classified into two categories – entanglement-separability paradigm measures [2] and information-theoretic ones [3]. Quantum correlations of the first kind are established to be useful resources for many quantum information tasks which include quantum dense coding, quantum teleportation, and secure quantum cryptography. Recently however, several non-classical phenomena have been discovered in which entanglement is absent To understand and quantify the resource necessary for exhibiting such non-classicality, information-theoretic quantum correlation measures like quantum discord [4, 5] and quantum work-deficit [6] have been proposed.

On the other hand, the importance of studying the time-evolved state cannot be over emphasized in both many-body physics and quantum information. In particular, important processes in quantum information, like the one-way quantum computer crucially depends on the time-evolved state and the possibility to control it in experimental set-ups [7]. The changes happening in several physical parameters, including statistical mechanical properties of such parameters, during evolution, is an important aspect in many-body physics, to understand, for example, the phenomena of decoherence in the system. However, despite its vital importance, the complexity of the time-dynamics in physically interesting models, has led to a limited amount of study of such phenomena [8].

The validity of a statistical mechanical description of a physical quantity depends on the behavior of that quantity in the time-evolution of the system. Ergodicity is a necessary condition for the validity of a statistical mechanical description of a physical quantity. A physical property is said to be ergodic if the time average of the quantity matches its ensemble average.

Here we compare the ergodicity behaviors of classical and quantum correlations. Under the umbrella of quantum correlation measures, for our analysis we consider the entanglement-separability paradigm measures as concurrence [9] and logarithmic negativity [10] and the information-theoretic measures as quantum discord and quantum work deficit. For our analysis, here we consider (i) the quantum XY spin system and (ii) Heisenberg spin systems in a transverse field in one-dimensional, ladder, and 2-dimensional lattice arrangements [11, 12, 13].
(i) Quantum XY spin system: We find that the two-body classical correlations along with the magnetization in the system, with a suitably chosen magnetic field, cannot equilibrate, and hence remain nonergodic, while the bipartite entanglement in the system with the same magnetic field can be ergodic [11]. There exists an interplay between temperature and the transverse field: For high fields, the quartet of physical quantities consisting of the transverse magnetization and the three diagonal classical correlations are all nonergodic, because there is no temperature at which the time average matches the ensemble average, while entanglement is ergodic. The thesis is true for the infinite chain, which is exactly diagonalizable, and low dimensional finite systems – ladder and 2D models with PBC – mimics the 1D infinite one. We are therefore led to the conclusion that although entanglement is mathematically constituted out of classical correlations and magnetizations, it does not necessarily imply that statistical mechanical properties of entanglement will be inherited from those of classical correlations and magnetizations.

By comparing the statistical-mechanical behaviors of the two major paradigms in which quantum correlation measures are defined, viz., the entanglement-separability paradigm and the information-theoretic one, we show that entanglement-separability measures are ergodic in such models, while the quantum correlation measures defined from an information-theoretic perspective can be nonergodic.

(ii) Heisenberg spin system: Quantum Heisenberg models have created lot of interest due to their rich physical properties and the possibility of realizing such systems in artificial materials as well as in inorganic compounds. However, investigations into the dynamics of such models, for example, under the influence of time-dependent magnetic fields, are limited by the fact that the system cannot be diagonalized analytically. Here, we have studied the behavior of quantum correlations, both from the entanglement-separability paradigm and the information-theoretic one, of the equilibrium state as well as the evolved state of the quantum Heisenberg anisotropic XYZ model, by numerical simulations [13] (see Fig. 1).

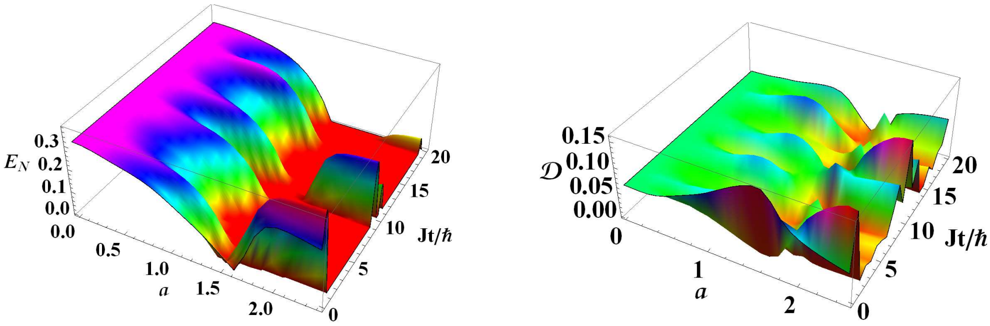

Figure 1: Quantum correlations of the time-evolved states. We plot quantum correlation measures of nearest-neighbor reduced states of the canonical equilibrium states, for a system of 8 quantum spin-$\frac{1}{2}$ particles arranged as a ring and described by the Heisenberg Hamiltonian. The evolution is assumed to begin in the equilibrium state at $t = 0$ and at an exemplary value of the temperature given by $J\alpha = 20$. Logarithmic negativity (left plot) and quantum discord (right plot) of the nearest-neighbor reduced states of the time-evolved states, are plotted against the initial magnetic field, a, and $\frac{Jt}{\hbar}$, for different values of δ. Here we choose $\gamma = 0.8$ and $\delta = 0.8$. All axes correspond to dimensionless quantities except those for quantum discord, which is measured in bits.

In particular, we found that although entanglement measures are ergodic irrespective of the system parameters, information-theoretic measures exhibit a rich picture, with respect to their statistical mechanical properties. Specifically, we find that the zz-interaction strength has a cross-over value, for a given xy-anisotropy and a given information-theoretic quantum correlation measure, that indicates a transition from nonergodic to ergodic behavior for that measure. The qualitative features of the measures in the entanglement-separability paradigm and the information-theoretic one are the same in the one-dimensional, ladder, and two-dimensional square lattices. However, in the square lattice, the information-theoretic measures are more sensitive to the change of the zz-interaction strength than in other dimensions. Such dimension-dependent change of ergodic behavior is absent for entanglement measures.

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