Maximal Average Fidelity in Quantum Teleportation of Single Qubit Mixed Information State by Using Two Qubits X-State as Resource

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Quantum teleportation (QT) is based on the well-known concept of quantum entanglement. Entanglement plays a very important role in quantum information processing. Quantum entangled states have become the key ingredient in the rapidly expanding field of quantum information science, with remarkable prospective applications such as quantum computation [1], quantum teleportation [2-5], super dense coding [6], quantum cryptographic schemes [7-9], entanglement swapping [10-12], and remote states preparation [13-14]. Quantum teleportation, which describes the transmission of an unknown quantum state from a sender to a spatially distant receiver via classical information, is one of the most astonishing features in quantum information and communication. In 1993, Bennett et al. [2] investigated that it was possible to transmit one-qubit state via Einstein–Podolsky–Rosen (EPR) state by sending two classical bits of information. Probabilistic teleportation [15-16] and probabilistic superdense coding [17-18] were also investigated via non-maximally entangled.

A pure separable state is represented by the product of states for both systems. A general pure state is a superposition of factorizable states. Maximally entangled states are usually considered to have the highest amount of entanglement. The nature of the teleportation channel is dependent on both the entangled state resource and the particular LOCC protocol [19-20]. H. Prakash and V. Verma [21] have shown that perfect QT is possible only when maximally entangled states are used as quantum channel. The maximal fidelity for separable states is indeed given by F=1/2. A natural question arises concerning teleportation, whether states which violate the Bell-CHSH inequalities are suitable for teleportation. Horodecki *et al* [20] showed that any mixed two spin-1/2 state which violates the Bell-CHSH inequalities is suitable for teleportation.

We considered the standard quantum teleportation protocol of mixed (impure) single qubit information state by using a general bipartite X-state as entanglement resource. A general two qubits X-state [22-25], which in the most general cases are mixed, is given by the density operator

$$\rho = \begin{aligned} & A|00\rangle\langle 00| + B|01\rangle\langle 01| + C|10\rangle\langle 10| + D|11\rangle\langle 11| + \\ & F|00\rangle\langle 11| + G|01\rangle\langle 10| + G^*|10\rangle\langle 01| + F^*|11\rangle\langle 00| \end{aligned}$$

In matrix form $X = \begin{pmatrix} A & 0 & 0 & F \\ 0 & B & G & 0 \\ 0 & G^* & C & 0 \\ F^* & 0 & 0 & D \end{pmatrix}$

where A, B, C, D are real. X describes a quantum state provided the unit trace and positivity conditions: $A+B+C+D=1$, $|F| \le \sqrt{AD}$ and $|G| \le \sqrt{BC}$ are fulfilled. X-states are entangled if and only if either $|F| > \sqrt{AD}$ or $|G| > \sqrt{BC}$. Both conditions cannot hold simultaneously [23]. X states contain many important states like the four maximally entangled Bell states, the maximally mixed state, mixtures of maximally entangled and maximally mixed states like Werner states [26]. The purity will be only if either $A=D=F=0$ and $|G| = \sqrt{BC}$ or $B=C=G=0$ and $|F| = \sqrt{AD}$. We address the following basic question: for a mixed X-state of two qubits, what is the maximal teleportation fidelity that can be obtained when this state is used as quantum channel for the teleportation of single qubit information state. We obtained an expression for maximal average fidelity (the fidelity averaged over all input states and maximized over general unitary operations done by receiver) in each case of sender's four Bell States Measurement (BSM) results as

$$F_{av.max} = \frac{1}{2} + \frac{\gamma^2}{2}(|F|+|G|)I_3 + \frac{1}{2}\left|\frac{1}{2}\{(A-B)(1-x)+(D-C)(1+x)\}I + 2\gamma^2(|F|-|G|)I_3\right|$$

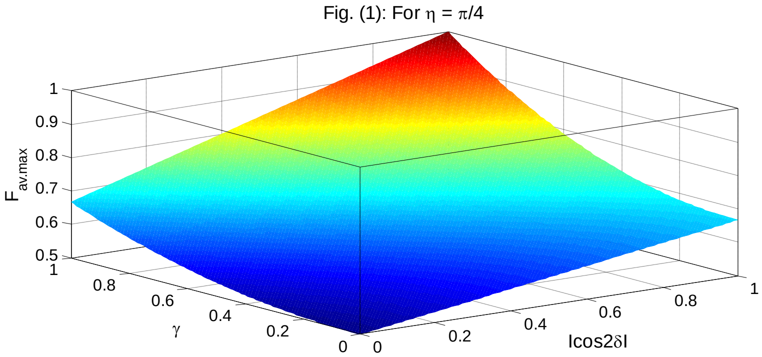
or, $$F_{av.max} = \frac{1}{2} + \frac{\gamma^2}{2}(|F|+|G|)I_3 + \frac{1}{2}\left|(AD-BC)I + \gamma^2(|F|-|G|)I_3\right|,$$

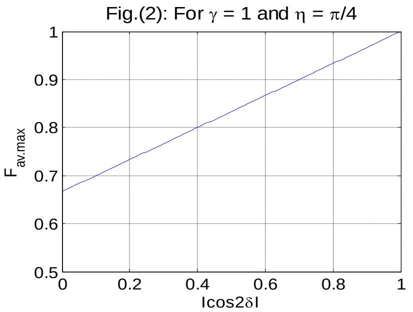
where $I = \frac{4}{x^2}[\frac{1}{2x}\log\left\{\frac{1+x}{1-x}\right\} - 1]$ and $I_3 = 1 - \frac{1}{4}(1-x^2)I$ with x = (A+B) - (C+D), $0 \le x < 1$.

It is also shown that, in a special case (x = 0), this expression for fidelity is same as that given by R. Horodecki et.el. [20]. Horodecki et al [20] also showed that any mixed two spin-1/2 state which violates Bell-CHSH inequality is useful for teleportation. Recently Ming [22] also showed that any X-states which violate the Bell-CHSH inequality can also be used for non-classical teleportation if receiver can only perform the identity or Pauli rotation operations. But in this paper we have seen that this is not true always when the information is mixed (impure). If we consider $A = \cos^2\delta\cos^2\eta$, $B = \sin^2\delta\cos^2\eta$, $C = \sin^2\delta\sin^2\eta$, $D = \cos^2\delta\sin^2\eta$, $|F| = \frac{1}{2}\cos^2\delta\sin 2\eta$ and $|G| = \frac{1}{2}\sin^2\delta\sin 2\eta$ with $0 \le \delta \le \frac{\pi}{2}$ and $0 < \eta < \frac{\pi}{2}$. In this case, $M(\rho) = \max.\{(1+\cos^2 2\delta)\sin^2 2\eta, \cos^2 2\delta + \sin^2 2\eta\}$, $x = \cos 2\eta$ and the expression of the fidelity becomes

$$F_{av.max} = \frac{1}{2} + \frac{\gamma^2}{4}\sin 2\eta\ I_3 + \frac{1}{8}|\cos 2\delta \sin 2\eta|\left|\sin 2\eta . I + 2\gamma^2 I_3\right|.$$

At $\eta = \frac{\pi}{4}$, $M(\rho) = 1 + \cos^2 2\delta \geq 1$, $x = 0$ and then the above fidelity becomes $F_{av.max} = \frac{1}{2} + \frac{\gamma^2}{6} + \frac{1}{6}(1+\gamma^2)|\cos 2\delta|$ and the variation of this fidelity is shown in Fig.(1). For γ = 1 (pure information) and $\eta = \frac{\pi}{4}$, $F_{av.max} = \frac{2}{3} + \frac{1}{3}|\cos 2\delta| \geq \frac{2}{3}$ and the variation of the fidelity is shown in Fig.(2). Since at $\eta = \frac{\pi}{4}$, $M(\rho) > 1$ provided $\delta \neq \frac{\pi}{2}$ and therefore Bell-CHSH inequality [25] is violated. The Fig.(2) shows that in case of pure information state, all the X-state that violates Bell-CHSH inequality is suitable for non-classical teleportation. But from Fig.(1), it is clear that when information is mixed state, then, all the X-state which violates Bell-CHSH inequality is not suitable for non-classical teleportation. At $\delta = 0, \frac{\pi}{2}$, X-state would be pure maximally entangled state, the non-classical teleportation occurs for any (pure or mixed) information states. Thus, we have seen that the expression for maximal average fidelity in QT of single qubit mixed information state using X-state as resource is more general and in a special case (x=0) it reduces to that given by Horodecki et. al. [20].





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