ESTIMATION OF QUANTUM STATES BY WEAK AND PROJECTIVE MEASUREMENTS: A COMPARATIVE STUDY

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1. Introduction

The problem of quantum state tomography has been dealt with in many ways in the past. The various methods that have been used include von Neumann measurements or projective measurements, POVMs, etc. For the case of a qubit, the simplest method involves dividing the ensemble into three equal sub-ensembles and carrying out projective measurements in three orthogonal directions, taking one sub-ensemble at a time. The best possible state estimate is obtained when the ensemble is of an infinite size. However, in practice the ensemble is finite. In certain cases the ensemble size could even be very small. The quantum state estimation using the above procedure for such an ensemble is of extremely low fidelity.

The problem stems from the following: A projective measurement destroys the state of the system and an individual member of the ensemble cannot be re-used a second time [5, 4]. This can be circumvented by using an alternate method we have developed, which replaces the von Neumann measurements by weak measurements. Weak measurements were introduced by Aharonov, Alberti and Vaidman in 1988 [1]. These measurements yield very little information but at the same time, leave the system nearly unchanged. Weak measurements are obtained either by making the strength of interaction between the quantum system and the measurement device very small [1, 2] or by preparing the device in an initial state given by a distribution with a large standard deviation [3]. Interaction between the system and the device takes the joint state to an entangled state which consists of two overlapping distributions (for a qubit), centered over the spin component eigenvalues of $+\frac{1}{2}$ and $-\frac{1}{2}$. The weaker a measurement, the greater the overlap and lesser the change in the original state. We have used these properties in our recipe for quantum state estimation.

2. Method and Results

We use a regime of values of a weakness parameter, ϵ, in which there is only a partial overlap between the two distributions. Firstly a measurement of σ_z, defined by a parameter ϵ_1, is performed on all the members of the ensemble. With the resulting ensemble, a measurement of σ_x, defined by a parameter ϵ_2, is made. Finally on this ensemble, we perform a projective measurement of σ_y. The method is summarized in the *Fig.*1.

In the first two steps, if the device pointer shows a value where there is an overlap, the result is ambiguous and we discard the reading. For this we define a region of width $2a$, centered

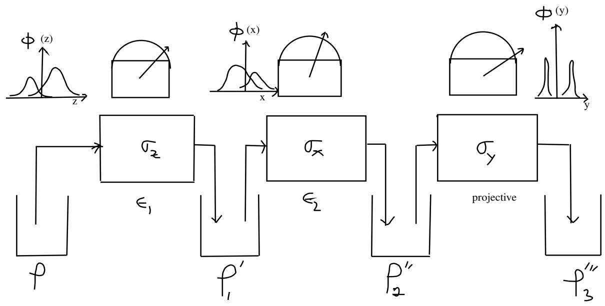

Figure 1. *Setup*

at the origin, in which the readings are assumed to be lying in the overlapping region. The state is estimated using the remaining readings which lie either to the right or to the left of this region. A fidelity measure f related to the distance between the estimated and the original states, defined as

$$f = (x - x_{est})^2 + (y - y_{est})^2 + (z - z_{est})^2 \tag{1}$$

averaged over 1000 runs and 100 states generated randomly is plotted by varying ϵ (for this we take $\epsilon_1 = \epsilon_2 = \epsilon$) and compared with the earlier method described using von Neumann measurements. It is seen that while the method is better than projective measurement method for some states, for a range of ϵ values on the average, it fails (*Fig.2*).

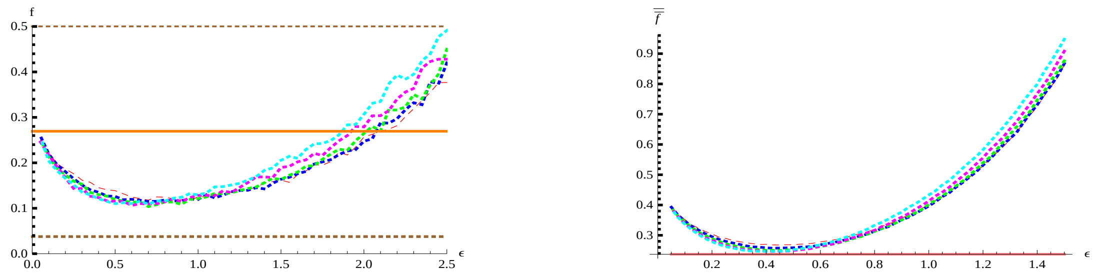

Figure 2. *(2a.)Weak measurement method as compared to projective measurement (orange line) for a particular state. (2b.)Weak measurement method as compared to projective measurement (pink line) averaged over 100 states. Red broken:a=0,blue:a=0.2,green:a=0.4,purple:a=0.6,cyan: a=0.8, brown broken:projective fidelity* $\pm\sigma$. *Ensemble size=30.* $\epsilon_1 = \epsilon_2 = \epsilon$.

The interesting part is that the method is suitable for certain states, only when the ensemble size is small, as in the above case with 30 members. For larger ensembles, the projective measurement method is better. Also a greater amount of discarding i.e. larger a gives a better estimate.

In another study in which $\epsilon_1 = \epsilon_2 - \delta$, a plot of average fidelity against ϵ_2 reveals that our method is actually better for a range of ϵ_2 (*Fig.3*).

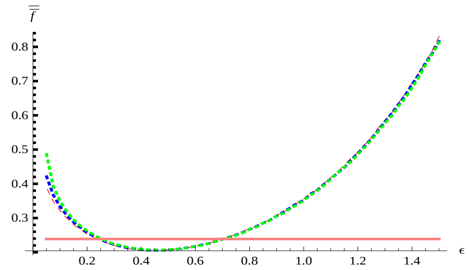

FIGURE 3. *Weak measurement method as compared to projective measurement (pink line) averaged over 100 states with* $\epsilon_1 = \epsilon_2 - \delta$. *Red broken:*$\delta = 0.01$,*blue:*$\delta = 0.02$, *green:*$\delta = 0.03$. *Ensemble size=30.*

3. Conclusions

In this work, we have explored how measurements based on weak probes and state recycling can be used for state reconstruction. We have compared the weak measurement scheme with projective measurements and shown that under certain circumstances the weak measurement scheme performs better. We have randomly sampled the one-qubit Hilbert space to remove any biases. This is the beginning of a programme wherein we want to explore the possibilities of using different types of measurements to gain information from a quantum system.

References

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