# Dynamics of nonlocality of two-mode quantum vortex state under thermal environment

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## Abstract

Recently, Agarwal [New J. Phys., **13**, 073008(2011)] proposed the generation of a non-Gaussian two-mode quantum vortex state of a system by subtracting a photon from a squeezed vacuum. In the present paper, we study the dynamics of this state under thermal environment. We find strong violation of Bell-CHSH inequality by this state and study the effect of thermal environment on it.

**Key Words:** Non-Gaussian state, quantum vortex state, Bell-CHSH inequality, decoherence

## 1. Introduction

Non-Gaussian states are important resource in quantum information and communication because some quantum-information tasks are unlikely to be implemented only by using Gaussian operations. Quantized vortex state is an important two-mode non-Gaussian state. A number of studies have been made on the quantized vortex state. Agarwal et al. [1] first studied the generation of a circularly symmetric quantum vortex. An optical vortex is a beam of light the phase of which varies in a corkscrew-like manner along the beam's direction of propagation. Very recently, Agarwal [2] proposed that a *'two-mode quantized vortex state (TMQVS)'* may be prepared through a squeezed vacuum by subtracting a photon (a non-Gaussian operation). Two-mode squeezed vacuum state is defined by

$$\left|\xi\right\rangle_{ab} = \exp(\xi\hat{a}^{\dagger}\hat{b}^{\dagger} - \xi^{*}\hat{a}\hat{b})\left|0,0\right\rangle, \tag{1}$$

where $\xi = re^{i\varphi}$ is a complex squeezing parameter, r is the degree of squeezing, a and b represent two modes of the field with commutation relations $[\hat{a}, \hat{a}^{\dagger}] = [\hat{b}, \hat{b}^{\dagger}] = 1$, $[\hat{a}, \hat{b}^{\dagger}] = 0$, etc. After subtracting a photon from the two-mode squeezed state from idler mode *b* via a beam splitter with low reflectivity and detection of one photon by the avalanche photo diode (APD), or even better by a single-photon detector, the resulting TMQVS [2] of the output field is

$$\left|\xi\right\rangle_{ab}^{(s)} = N\hat{b}\exp(\xi\hat{a}^{\dagger}\hat{b}^{\dagger} - \xi^{*}\hat{a}\hat{b})\left|0,0\right\rangle = e^{i\varphi}\hat{a}^{\dagger}\left|\xi\right\rangle / (\cosh r), \tag{2}$$

where, $N = \left\langle\xi\right|\hat{b}^{\dagger}\hat{b}\left|\xi\right\rangle^{-1/2} = 1/\sinh r$. Wigner function of TMQVS is

$$W_{ab}^{(s)}(\alpha,\beta) = \frac{4}{\pi^2}\left(4\left|\tilde{\alpha}\right|^2 - 1\right)\exp\left[-2\left(\left|\tilde{\alpha}\right|^2 + \left|\tilde{\beta}\right|^2\right)\right], \tag{3}$$

where $\tilde{\alpha}$ and $\tilde{\beta}$ are given by $\begin{pmatrix}\tilde{\alpha}\\ \tilde{\beta}^{*}\end{pmatrix} = \begin{pmatrix}\cosh r & -\sinh r\, e^{i\varphi}\\ -\sinh r\, e^{i\varphi} & \cosh r\end{pmatrix}\begin{pmatrix}\alpha\\ \beta^{*}\end{pmatrix}$, with $\alpha = \left|\alpha\right|e^{i\theta_\alpha}$ and $\beta = \left|\beta\right|e^{i\theta_\beta}$. Due to their specific spatial character and associated orbital angular momentum, TMQVS can be used to generate higher-dimensional entangled states and may play a significant advantage in introducing the use of higher dimensional systems for encoding and manipulating the quantum information. This can be an important practical advantage, as it allows increasing the information content and this, in turn, may cut down substantially the noise and losses arising from the imperfect generation and detection efficiency. Understanding and controlling decoherence (i.e., loss of coherence of states of quantum systems due to the interaction with the environment) of quantum states is a key step in building quantum information processors. Hence, in the present paper, we investigate the dynamics of TMQVS under thermal environment. Study of violation of Bell-type inequalities is used to demonstrate the nonlocal nature of the quantum mechanics. We examine the possible violation of Bell-CHSH inequality and its dynamics under thermal environment by the TMQVS.

## 2. Dynamics of TMQVS under thermal environment

The Fokker-Planck equation (in Born-Markov approximation) describing the time evolution of the Wigner function of a quantum state under thermal environment can be written as [3]

$$\frac{\partial W_{ab}^{(s)}(\alpha,\beta,\tau)}{\partial\tau}=\frac{\gamma}{2}\sum_{\alpha_i=\alpha,\beta}\left[\frac{\partial}{\partial\alpha_i}\alpha_i+\frac{\partial}{\partial\alpha_i^*}\alpha_i^*+2(\tfrac{1}{2}+\bar{n})\frac{\partial^2}{\partial\alpha_i\partial\alpha_i^*}\right]W_{ab}^{(s)}(\alpha,\beta,\tau)\,. \quad (4)$$

Here $\gamma$ represents the dissipative coefficient and $\bar{n}$ is the average thermal photon number of the environment. By solving this equation, we get the time evolution of Wigner function of TMQVS in thermal environment at time $\tau$ to be given by the convolution of the Wigner function of the initial state ( $W_{ab}^{(s)}(\alpha,\beta)\equiv W_{ab}^{(s)}(\alpha,\beta,\tau=0)$ ) and that of the thermal environment [4]:

$$W_{ab}^{(s)}(\alpha,\beta,\tau)=\frac{1}{t(\tau)^4}\int d^2\zeta d^2\eta W_a^{th}(\zeta)W_b^{th}(\eta)W_{ab}^{(s)}\left(\frac{\alpha-s(\tau)\zeta}{t(\tau)},\frac{\beta-s(\tau)\eta}{t(\tau)},\tau=0\right), \quad (5)$$

where $s(\tau)=\sqrt{1-e^{-\gamma\tau}}$ , $t(\tau)=\sqrt{e^{-\gamma\tau}}$ , and $W^{th}(\zeta)=(2/(\pi(1+2\bar{n})))\exp\left(-2|\zeta|^2/(1+2\bar{n})\right)$.

Using the integral identity, $(1/\pi)\int d^2\alpha\exp(-a|\alpha|^2+b\alpha+b^*\alpha^*+c\alpha^2+c^*\alpha^{*2})=$ $\left(1/\sqrt{a^2-4|c|^2}\right)\exp\left[(b^2c^*+b^{*2}c+a|b|^2)/(a^2-4|c|^2)\right]$, if $a^2-4|c|^2>0$, we derived the expression of time evolution of TMQVS in thermal environment,

$$W_{ab}^{(s)}(\alpha,\beta,\tau)=\frac{A}{a^2}\left[\frac{4|\tilde{\alpha}|^2}{t(\tau)^2}\left(\frac{2s(\tau)^2}{at(\tau)^2}-1\right)^2+2\left(\frac{2s(\tau)^2}{at(\tau)^2}-1\right)+1\right]\exp\left[\frac{4s(\tau)^2}{at(\tau)^4}\left(|\tilde{\alpha}|^2+|\tilde{\beta}|^2\right)\right], \quad (6)$$

where $A=\dfrac{16}{\pi^2t(\tau)^4(1+2\bar{n})^2}\exp\left[-\dfrac{2}{t(\tau)^2}(|\tilde{\alpha}|^2+|\tilde{\beta}|^2)\right]$ , and $a=\dfrac{2s(\tau)^2}{t(\tau)^2}+\dfrac{2}{(1+2\bar{n})}$ .

With this expression we can examine the effect of squeezing (r), number of thermal photons ( $\bar{n}$ ) and intensity of either mode ( $|\alpha|^2=|\beta|^2=J$ ) on time evolution of TMQVS. We find different conditions where the Wigner function, $W_{ab}^{(s)}(\alpha,\beta,\tau)$, of the TMQVS is negative which is an indicator of the nonclassicality of the state. For example, for J = 0.02, r = 0.2, and $\bar{n}$=0.1, the negativity of $W_{ab}^{(s)}(\alpha,\beta,\tau)$ decreases as time $s(\tau)$ increases. Further, if we increase $\bar{n}$, the negativity vanishes more rapidly as $s(\tau)$ increases.

## 3. Violations of Bell-CHSH inequality with Photon Parity measurement scheme and its dynamics under thermal environment

An operational definition of the two-mode Wigner function for the state $|\psi\rangle$ can be given in terms of a correlated parity measurement by the following POVM operators [5]:

$$\hat{\Pi}^{+}(\alpha) = \hat{D}(\alpha)\sum_{k=0}^{\infty}\left|2k\right\rangle\left\langle 2k\right|\hat{D}^{\dagger}(\alpha), \tag{7}$$

$$\hat{\Pi}^{-}(\alpha) = \hat{D}(\alpha)\sum_{k=0}^{\infty}\left|2k+1\right\rangle\left\langle 2k+1\right|\hat{D}^{\dagger}(\alpha), \tag{8}$$

where $\hat{D}(\alpha) = \exp(\alpha\hat{a}^{\dagger} - \alpha^{*}\hat{a})$ is the displacement operator. Corresponding operator for the correlated measurement of the parity on modes 'a' and 'b' of two parties, say Alice and Bob, may be defined as

$$\hat{\Pi}_{ab}(\alpha,\beta) = [\hat{\Pi}_{a}^{(+)}(\alpha) - \hat{\Pi}_{a}^{(-)}(\alpha)] \otimes [\hat{\Pi}_{b}^{(+)}(\beta) - \hat{\Pi}_{b}^{(-)}(\beta)]. \tag{9}$$

The outcome of the measurements is either +1 or -1. Then the Bell-CHSH inequality is

$$\left|B(\alpha,\beta)\right| = \left|\left\langle \hat{\Pi}_{ab}(\alpha,\beta) + \hat{\Pi}_{ab}(\alpha,\beta') + \hat{\Pi}_{ab}(\alpha',\beta) - \hat{\Pi}_{ab}(\alpha',\beta')\right\rangle\right| \leq 2, \tag{10}$$

where we call $B(\alpha,\beta)$ as the Bell-CHSH function. Wigner function of the two-mode state $\hat{\rho}_{ab}$ is proportional to the mean of $\hat{\Pi}_{ab}$ such that $W_{ab}^{(s)}(\alpha,\beta) = (4/\pi^{2})\mathrm{Tr}[\hat{\rho}_{ab}\hat{\Pi}_{ab}(\alpha,\beta)]$. The time evolution of $B(\alpha,\beta)$ can be written in terms of the Wigner functions at different phase-space points,

$$B(\alpha,\beta,\tau) = (\pi^{2}/4)[W_{ab}^{(s)}(0,0,\tau) + W_{ab}^{(s)}(\alpha,0,\tau) + W_{ab}^{(s)}(0,\beta,\tau) - W_{ab}^{(s)}(\alpha,\beta,\tau)]. \tag{11}$$

Any violation of this inequality $\left|B(\alpha,\beta,\tau)\right| \leq 2$ confirms the local realistic theory. Using Eq. (3) in Eq. (11), we find strong violations of the inequality for $\tau = 0$. For example, if we see the variation of $B(\alpha,\beta,\tau = 0)$ with J and $\phi$, we find strong violations ($B(\alpha,\beta,\tau = 0) =$ -2.204) at r = 0.8, J = 0.02 and $\phi$ = 3.016. Further, to investigate the dynamics of Bell-CHSH inequality for TMQVS under thermal environment, we use Eq. (6) in Eq. (11). For example, for r = 0.2, $\bar{n}$ = 0.1, and J = 0.02, the variation of $B(\alpha,\beta,\tau)$ with $(\phi - \theta_{\alpha} - \theta_{\beta})$ and $s(\tau)$ shows that as time $s(\tau)$ increases, violation of Bell-CHSH inequality vanishes whatever be the value of r, $\bar{n}$ and J. We have analysed different conditions for the violations of Bell-CHSH inequality under thermal environment.

In summary, it is found that the transition of the Wigner function of TMQVS from negative (nonclassicality) to completely positive definite depend not only on the average number ($\bar{n}$) of thermal environment, but also on the average number of the TMQVS (J) and squeezing parameter (r). We have studied the dynamic behavior of the nonlocality for the TMQVS in the thermal environment and found different situations under which TMQVS violate the Bell-CHSH inequality strongly.

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