Quantum back-action in a two-component Bose-Einstein condensate

Ebubechukwu O. Ilo-Okeke[1,2] and Tim Byrnes[2]

[1] *Department of Physics, School of Science, Federal University of Technology, P. M. B. 1526, Owerri, Imo State 460001, Nigeria.*
[2] *National Institute of Informatics, 2-1-2 Hitosubashi, Chiyoda-ku, Tokyo 101-8430, Japan.*

We investigate a non-destructive homodyne measurement of atomic condensates using off-resonant optical laser beam that interacts with atomic Bose-Einstein condensate (BEC). The interaction between laser beam and the atomic BEC creates entangled states of atom and light that are destroyed in a homodyne detection of light. The destruction of the entangled states causes back-action on the atomic states. We derive expression for the measurement operator on the atomic states due to the photon measurement. We characterize the back-action by calculating quantities such as the Q-function and observe the variation of the measured state with the strength of the light-BEC interaction.

PACS numbers:

I. INTRODUCTION

Information about a property of a system is acquired by performing measurement repeatedly on identically prepared systems or performing measurement repeatedly on a single system. For a quantum system like BEC, projective measurement or dispersive measurement that uses optical techniques are used to acquire information about the condensate. In projective measurement such as absorption imaging [1, 2], optical probe beam that is nearly resonant with atomic transition is used to image the cloud. Because of high optical density of the BEC in atom traps that results in absorption of the probe beam, the BEC are released from the atom trap before projective measurement could be applied. As such, BEC is destroyed in the measurement process therefore requiring identical samples of the condensates to be prepared.

On the contrary, dispersive measurement like the phase contrast imaging [3, 4] uses off resonant beam to measure the properties of small and dense atomic condensates *in situ*. The atoms coherently scatter photons in the probe beam as described in Fig 1. Because dispersive measurement does not destroy the BEC, it is applied repeatedly on the same sample and has been used to image BEC in a number of experiments [5–7]. Such technique is a useful resource for information read-out in several applications proposing the use of ultra-cold atoms and atomic condensates in metrology [8, 9], quantum information and processing [10], and quantum computation.

The scattered photons in the probe beam carry away information about the atomic condensates. The information is accessed by performing homodyne detection on light beam. Here we analyse dispersive imaging of a two component atomic condensates using off resonant laser pulses. The laser pulses are strongly detuned from atomic resonance such that atomic population is conserved. A light beam of well-known polarisation couples the ground states of the atomic condensates to the excited states as shown in Fig. 1. For sufficiently large detuned light, the ground states follow the light fields adiabatically and results in the entanglement of the light and atomic states.

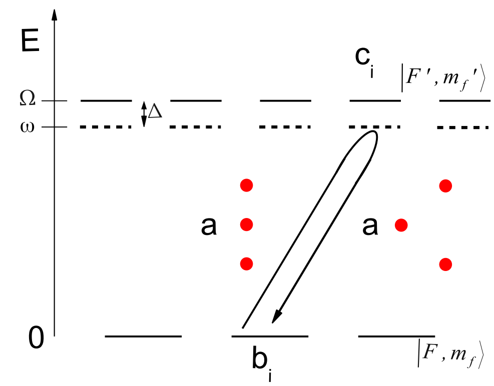

FIG. 1: The energy level diagram showing the atomic non-resonant transition during phase contrast imaging. The incident light of frequency ω is detuned from atomic resonance by Δ. The atom in the ground state $|F, m_f\rangle$ absorbs a photon (red ball) from the beam, and makes transition to the quasi-excited state (the dashed lines). It emits the same photon via stimulated emission and transitions back to the same ground state. The emitted photon lags behind the unabsorbed photons thereby introducing a phase shift in the wave-front of light.

Also, the interaction between atom and light causes the light states to undergo a phase rotation. However, not all incident light beam pass through the condensate. The beam not deflected and the beam deflected by atomic condensates are interfered at the detector in a homodyne measurement of the photons.

By projecting unto the photon subspace, we derive expression for the measurement operator that captures the effect of homodyne measurement on atomic condensates and is central to the results of this work. The measurement operator is used to derive an expression for the signal obtained in the measurement. We plot the signal obtained from the measurement as a function of the initial amplitude of the atomic states for different coupling strength in Fig. 2. The signal diminishes with increased

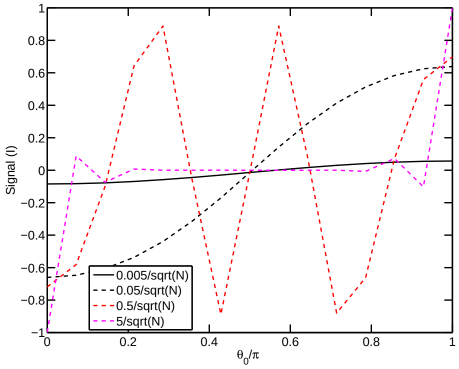

FIG. 2: The inferred signal I from the measurement of photon number as a function of θ_0 for $N = 200$, $|\gamma_u| = \sqrt{135}$, and $|\gamma_p| = 2$.

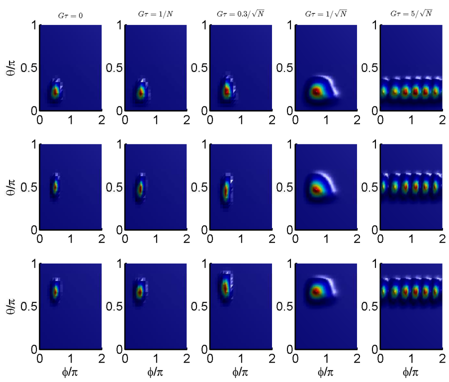

FIG. 3: The Husimi Q-distribution plots for different initial conditions at different values of $G\tau$. Each column is plotted at the value of $G\tau$ specified at the column head. The first row is plotted for $\theta_0 = 40°$, $\phi_0 = 90°$, the second row is plotted for $\theta_0 = 90°$, $\phi = 90°$, and the third row is plotted for $\theta_0 = 120°$, $\phi_0 = 90°$. For these plots $|\gamma_u| = \sqrt{80}$, $|\gamma_p| = 2$, $N = 300$, $g\tau = 0.001/\sqrt{N}$, and the phase-plate angle $\theta_u = \pi/3$.

correlation between the atom and light. For a total population of N atoms in the atomic condensates, we show that signal is lost when the coupling strength is of the order $1/\sqrt{N}$ [see Fig. 2]. We calculated the sensitivity of homodyne measurement to the signal and showed that it depends on the coupling between atom and light, the population of atoms in the atomic condensate and the initial noise of the probe beam.

We also study the disturbance induced by the measurement on the atomic condensates using the Husimi Q-distribution and present the results in Fig. 3. We show that in the regime of small coupling between atom and light, the Q-distribution is Gaussian, with a width that scales roughly as $1/\sqrt{N}$. However, in the large coupling limit the Q-distribution is no longer Gaussian. Instead the width grows and there emerge several satellite peaks in the phase plane. To quantify the amount of information gained from measurement and assess how close the estimated atomic states are to the actual states, we calculate the Euclidean spherical distance on a Bloch sphere. Our results show that in the weak coupling limit, little information is gained with negligible disturbance of the BEC states. At the same time, the error in the estimated atomic states is very large. The implication of this is that one measurement is not sufficient to give information on the property of the atomic condensate. This is to be expected since the coupling between atom and light is weak, thus requiring more measurements to be carried out on the same sample in order to get more information. Increasing the coupling improves the estimate at the expense of increasing the disturbance experienced by the atomic condensate.

[1] M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell, Science **269**, 198 (1995).
[2] M. R. Andrews, C. G. Townsend, H.-J. Miesner, D. M. K. D. S. Durfee, and W. Ketterle, Science **275**, 637 (1997).
[3] M. R. Andrews, M.-O. Mewes, N. J. van Druten, D. S. Durfee, D. M. Kurn, and W. Ketterle, Science **273**, 84 (1996).
[4] C. C. Bradley, C. A. Sackett, and R. G. Hulet, Phys. Rev. A **55**, 3951 (1997).
[5] M. R. Andrews, D. M. Kurn, H.-J. Miesner, D. S. Durfee, C. G. Townsend, S. Inouye, and W. Ketterle, Phys. Rev. Lett. **79**, 553 (1997).
[6] J. M. Higbie, L. E. Sadler, S. Inouye, A. P. Chikkatur, S. R. Leslie, K. L. Moore, V. Savalli, and D. M. Stamper-Kurn, Phys. Rev. Lett. **95**, 050401 (2005).
[7] R. Meppelink, R. A. Rozendaal, S. B. Koller, J. M. Vogels, and P. van der Straten, Phys. Rev. A **81**, 053632 (2010).
[8] Y. Shin, M. Saba, T. A. Pasquini, W. Ketterle, D. E. Pritchard, and A. E. Leanhardt, Phys. Rev. Lett. **92**, 050405 (2004).
[9] Y. Wang, D. Z. Anderson, V. M. Bright, E. A. Cornell, Q. Diot, T. Kishimoto, M. Prentiss, R. A. Saravanan, S. R. S. R., and S. Wu, Phys. Rev. Lett. **94**, 090405 (2005).
[10] T. Byrnes, K. Wen, and Y. Yamamoto, Phys. Rev. A **85**, 040306 (2012).