Separability criteria of $k-$separable $n-$partite quantum states

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The genuine multipartite entanglement is an important concept in quantum information theory, which can be used for various quantum information tasks such as quantum teleportation, quantum secret sharing and superdense coding [1–5]. The characterization of multipartite entangled states becomes complicated due to its rich structure [6, 7]. In particular, detecting the entanglement in multipartite states is nontrivial even for three qudit states. To detect the entangled multipartite states several separability criteria were proposed. To characterize $k-$separable multiqubit states certain experimentally accessible conditions have also been proposed [8–12].

We present the k-separability criteria of n-partite states. To begin with we recall the definition of an $n-$partite pure quantum state $|\psi_{k-\text{sep}}\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes \ldots \otimes |\psi_k\rangle$, called $k-$separable ($k = 2, 3, \ldots, n$), iff it can be written as a product of k substates. A mixed state $\rho_{k-\text{sep}}$ is called $k-$separable iff it has a decomposition into $k-$separable pure states $\rho_{k-\text{sep}} = \sum_i p_i \ \rho^i_{k-\text{sep}}$ [12]. In this direction, recently Gühne and Seevinck have proposed separability criteria for different classses of $3-$qubit and $4-$qubit entanglement, in particular genuine $3-$qubit and $4-$qubit entanglement [13]. Later T. Gao and Y. Hong have derived the separability criteria of biseparable and fully separable $n-$qubit and $n-$qudit states and proved each criterion from general partition [14]. We extend the criteria generalized by Gao and Hong to the $k-$separable $n-$partite states. Our criteria is able to detect any $k-$separable $n-$qubit and $n-$qudit states under any possible partitions. Our results validates the case, $k = 2$ and $k = n$ as biseparable and fully separable respectively. If a state violates our condition for the value of $k(1 < k \leq n)$ then it is genuinely entangled. The inequalities are given in terms of density matrix elements which provides the experimental accessibility [15].

References

[1] S. Bose, V. Vedral and P. L. Knight, Phys. Rev. A **57**, 822 (1998)

[2] M. Hillery, V. Bužek and A. Berthiaume, Phys. Rev. A **59**, 1829 (1999)

[3] S-B Zheng, Phys. Rev. A **74**, 054303 (2006)

[4] A. Karlsson and M. Bourennane, Phys. Rev. A **58**, 4394 (1998)

[5] P. Agrawal and A. Pati, Phys. Rev. A **74**, 062320 (2006)

[6] R. Horodecki, P. Horodecki, M. Horodecki, K. Horodecki, Rev. Mod. Phys. **81**, 865-942 (2009)

[7] O. Gühne and G. Tóth, Phys. Rep. **474**, 1-75 (2009)

[8] W. Laskowski and M. Zukowski, Phys. Rev. A **72**, 062112 (2005)

[9] M. Seevinck and G. Svetlichny, Phys. Rev. Lett. **89**, 060401 (2002)

[10] D. Collins, N. Gisin, S. Popescu, D. Roberts and V. Scarani, Phys. Rev. Lett. **88**, 170405 (2002)

[11] M. Seevinck and J. Uffink, Phys. Rev. A **78**, 032101 (2008)

[12] A. Gabriel, B. C. Hiesmeyr and M. Huber, Quantum Inf. Comput. **10** 0829 (2010)

[13] O. Gühne, M. Seevinck, New J. Phys. **12**, 053002 (2010)

[14] T. Gao, Y. Hong, Eur. Phys. J. D **61**, 765 (2011)

[15] N. Ananth, M. Senthilvelan, Separability criteria of $k-$separable $n-$partite quantum states (Under preparation)