Quantum state cloning using Deutschian closed timelike curves

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Abstract. We show that it is possible to clone quantum states to arbitrary accuracy in the presence of a Deutschian closed timelike curve (CTC), with a fidelity converging to one in the limit as the dimension of the CTC system becomes large—thus resolving an open conjecture from [Brun *et al.*, Physical Review Letters **102**, 210402 (2009)]. This result follows from a CTC-assisted scheme for producing perfect clones of a quantum state prepared in a known eigenbasis, and the fact that one can reconstruct an approximation of a quantum state from empirical estimates of the probabilities of an informationally-complete measurement. Our results imply more generally that every continuous, but otherwise arbitrarily non-linear map from states to states can be implemented to arbitrary accuracy with Deutschian CTCs.

Keywords: no-cloning theorem, closed timelike curves, open timelike curves

The possible existence of closed timelike curves (CTCs) in certain exotic spacetime geometries [1, 2, 3] has sparked a significant amount of research regarding their ramifications for computation [4, 5, 6] and information processing [7, 8]. One of the well known models for CTCs is due to Deutsch [9], who had the keen insight to abstract away much of the space-time geometric details and use the tools of quantum information to address physical questions about causality paradoxes. One consequence is that quantum computers with access to "Deutschian" CTCs would be able to answer any computational decision problem in PSPACE [6], a powerful complexity class containing the well-known class NP, for example. Also, quantum information processors with access to Deutschian CTCs could distinguish non-orthogonal states perfectly [7], thus leading to the strongest violation of the uncertainty principle that one could imagine. From the perspective of Aaronson [10], we might take these results to be complexity- and information-theoretic evidence against the existence of CTCs that behave according to Deutsch's model.

In order to avoid "grandfather-like" paradoxes, Deutsch's model imposes a boundary condition in which the density operator of the CTC system before it has interacted with a chronology-respecting system should be equal to the density operator of the CTC system after it interacts. More formally, let ρ_S denote the state of the chronology-respecting system and let σ_C denote the state of the CTC system before a unitary interaction U_{SC} (acting on systems S and C) takes place. The first assumption of Deutsch's model is that the state of the chronology-respecting system S and the chronology-violating system C is a tensor-product state, since presumably they have not interacted before the CTC system comes into existence. Furthermore, Deutsch's model imposes the following self-consistency condition:

$$\sigma_C = \Phi_\rho\left(\sigma_C\right) \equiv \mathrm{Tr}_S\left\{U_{SC}\left(\rho_S \otimes \sigma_C\right)U_{SC}^\dagger\right\}, \quad (1)$$

so that potential grandfather paradoxes can be avoided. Computationally, one can take the view that nature is finding a fixed-point of the map Φ_ρ [9, 6], which depends on the state ρ_S of the chronology-respecting system. The chronology-respecting system's state evolves by

$$\rho_S \to \rho_{\mathrm{out}} = \mathrm{Tr}_C\left\{U_{SC}\left(\rho_S \otimes \sigma_C\right)U_{SC}^\dagger\right\},$$

where the partial trace is over the CTC system. Since σ_C depends on ρ_S, such an evolution is nonlinear and as a result is a non-standard quantum evolution. If one views a density operator as a statistical ensemble or as a state of knowledge, then Deutsch already realized that his model still leads to grandfather-like paradoxes [9], as was elaborated further in later work [11]. However, if one considers a density operator to be the fundamental object which characterizes a quantum state, then Deutsch's model indeed resolves these paradoxes.

Since quantum processors with access to CTCs can perfectly distinguish pure quantum states [7], one might conclude that such CTC-assisted processors could also approximately clone any pure quantum state, in violation of the celebrated no-cloning theorem [12, 13]. In fact, Deutsch suggested that quantum cloning should be possible when one has access to CTCs behaving according to (1) [9], and Brun *et al.* conjectured that "a CTC-assisted party can construct a universal cloner with fidelity approaching one, at the cost of increasing the available dimensions in ancillary and CTC resources" [7].

In this paper, we give an approach to quantum state cloning with CTCs that is conceptually simple and appealing. We show how to clone any quantum state, such

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that the fidelity of each clone approaches one as the dimension of the assisting CTC system becomes large. Details of our argument appear in Ref. [14].

One can quickly grasp the main idea behind our construction by taking a glance at the circuit in Figure 1. The first step is to perform an informationally-complete measurement on the incoming state ρ_S. Such a measurement is well known in quantum information theory [15, 16, 17]—the probabilities of the outcomes are in one-to-one correspondence with a classical density operator description of the quantum state. (I.e., if one knew these probabilities, or could estimate them from performing this kind of measurement on many copies of the given state, then one could construct a classical description of the state.) Let ω denote the state resulting from the measurement:

$$\omega \equiv \sum_{x=0}^{d-1} \text{Tr}\left\{M_x \rho\right\} |x\rangle \langle x| , \tag{2}$$

where each M_x is an element of the informationally-complete measurement (so that $M_x \geq 0$ for all x and $\sum_x M_x = I$), d is the number of possible measurement outcomes, and $\{|x\rangle\}$ is the standard computational basis.

Next, we feed the state ω into a circuit that cyclically permutes it with N CTC systems that each have the same dimension as ω. Such an operation on its own (after tracing over all systems except for the N CTC systems) has as its unique fixed point the state $\omega^{\otimes N}$, so that, in some sense, the cyclic shift produces N "temporary" clones.

Finally, we copy the value of x from each of the N CTC systems to one of a set of ancillary systems in order to "read out" N copies of the state ω. In Figure 1 we've drawn this as a sequence of controlled-not (CNOT) gates, but in fact it will generally be a higher-dimensional analogue of a CNOT, like a modular addition circuit:

$$|x\rangle|y\rangle \to U(|x\rangle|y\rangle) = |x\rangle|x + y \mod d\rangle. \tag{3}$$

The fixed point of the overall circuit, after tracing over all systems except for the N CTC systems, is still $\omega^{\otimes N}$, because these modular addition gates do not cause any disturbance to the CTC systems. As a result, the reduced state on the N ancillas is equal to $\omega^{\otimes N}$, and we can then estimate the eigenvalues of ω simply by counting frequencies—the estimates become better and better as N becomes larger due to the law of large numbers. Since these eigenvalues result from an informationally-complete measurement, we can construct a classical description of the state ρ and produce as many approximate copies of it as we wish.

Our results imply more generally that every continuous, but otherwise arbitrarily non-linear map f from states to states can be implemented to arbitrary accuracy with Deutschian CTCs. This follows because we can estimate the incoming state ρ to arbitrary accuracy and then prepare $f(\rho)$ at will.

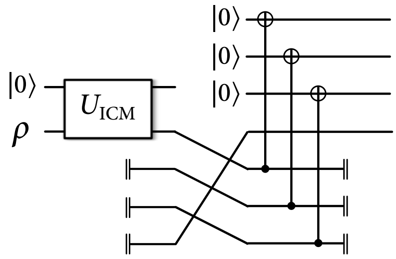

Figure 1: An example circuit for quantum state cloning using $N = 3$ CTC systems. The unknown state ρ is fed into a unitary operation U_{ICM}, whose effect is to implement an informationally-complete measurement with measurement operators $\{M_x\}$ such that $M_x \geq 0$ and $\sum_x M_x = I$. The resulting state $\omega = \sum_x \text{Tr}\{M_x\rho\}|x\rangle\langle x|$ is combined with N CTC systems and cyclically permuted with them. (For the CTC systems, the past mouth of the wormhole on the left, indicated by vertical double lines, is identified with the future mouth on the right.) Finally, modular addition circuits (depicted here as CNOT gates) "read out" N copies of the state ω, from which we can estimate the original state ρ to arbitrarily good accuracy as the number N of CTC systems becomes large (of course, one would require N to be much larger than three).

References

[1] K. Gödel, Rev. Mod. Phys. **21**, 447 (1949).

[2] J. R. Gott, Physical Review Letters **66**, 1126 (1991).

[3] W. B. Bonnor, J. Phys. A **13**, 2121 (1980).

[4] T. A. Brun, Found. Phys. Lett. **16**, 245 (2003).

[5] D. Bacon, Phys. Rev. A **70**, 032309 (2004).

[6] S. Aaronson and J. Watrous, Proceedings of the Royal Society A **465**, 631 (2009), arXiv:0808.2669.

[7] T. A. Brun, J. Harrington, and M. M. Wilde, Phys. Rev. Lett. **102**, 210402 (2009).

[8] J. L. Pienaar, T. C. Ralph, and C. R. Myers, Phys. Rev. Lett. **110**, 060501 (2013).

[9] D. Deutsch, Physical Review D **44**, 3197 (1991).

[10] S. Aaronson, Proc. Royal Soc. A **461**, 3473 (2005).

[11] J. J. Wallman and S. D. Bartlett, Foundations of Physics **42**, 656 (2012), arXiv:1005.2438.

[12] W. K. Wootters and W. H. Zurek, Nature **299**, 802803 (1982).

[13] D. Dieks, Physics Letters A **92**, 271272 (1982).

[14] T. A. Brun, M. M. Wilde, and A. Winter (2013), arXiv:1306.1795.

[15] E. Prugovecki, Int. J. Theor. Phys. **16**, 321 (1977).

[16] P. Busch, Int. J. Theor. Phys. **30**, 1217 (1991).

[17] J. M. Renes, R. Blume-Kohout, A. J. Scott, and C. M. Caves, Journal of Mathematical Physics **45**, 21712180 (2004).