Improvement of quasi-optimum quantum receiver for M-ary PSK coherent-state signals

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Abstract. When the average number of photons of the signals is large, quasi-optimum quantum receivers proposed in Refs.[1, 2, 3] approach optimum quantum receivers in performance. However when signals are very weak, they perform worse than homodyne or heterodyne receivers. We describe improvements in quasi-optimum quantum receivers for M-ary phase shift keying coherent-state signals and show that the improved receivers always outperform heterodyne receivers.

Keywords: quantum communications, quasi-optimum quantum receiver, PSK, coherent state signals

1 Introduction

One important topic of study in quantum information theory is the development of optimum or quasi-optimum quantum receivers. In pioneering this area in 1973, Kennedy proposed a quasi-optimum quantum receiver (so-called Kennedy receiver) for binary coherent-state signals[1]. Later, the Kennedy receiver was adapted to detect QPSK coherent-state signals[2] and subsequently to M-ary coherent-state signals[3]. Although quasi-optimum quantum receivers, described in Refs.[2, 3], approach the performances of optimum quantum receivers when the average number of photons in the signals is large, they perform worse than heterodyne receivers when signals are very weak.

To combat this problem, Tamori showed that the Kennedy receiver performs better if the amplitude of the local oscillator light is regulated; such receivers always outperform homodyne receivers[4]. Takeoka et al. rediscovered Tamori's result and showed that the receiver is further improved by squeezing the coherent-state signal[5].

In this paper, we improve on the quasi-optimum quantum receivers for M-ary PSK coherent-state signals by applying the method in Refs.[4, 5] and show that the improved receivers always outperform heterodyne receivers[1].

2 Quasi-optimum quantum receiver

In the following, we consider Fig.1 of Ref.[2]. In explanation, we express the detection process of the local oscillator using a shift operator $\hat{D}(\cdot)$ on the Hilbert space which corresponds to the whole signal duration. Note that a certain conversion is necessary for an exact expression. Let $|\alpha_m\rangle$ $(m = 0, 1, \cdots, M-1)$ be the input signal state to the receiver. Here, $\alpha_m = \alpha \mathrm{e}^{2\mathbf{i}m\pi/M}$ is the complex amplitude of the coherent state and we assume that α is a positive real number. The shift operator $\hat{D}(-\alpha)$, by which the signal $|\alpha_0\rangle$ is transformed to the vacuum state, is first applied to the input state; the shifted signal is then input to the detector. If a photon is detected, the result is fed back to the phase shifter and the phase of the local oscillator is shifted. If no photon is detected, we set the signal to i when feedback is applied i times. The feedback is applied at most $M-1$ times for M-ary signals.

3 Improvement of quantum receiver

3.1 Improvement by regulating amplitude and phase of the local oscillator

Following Refs.[4, 5], we optimize the amplitude and phase of the local oscillator of the quantum receiver for M-ary signals. We replace the k-th shift operator by $\hat{D}(-\gamma_k \mathrm{e}^{\mathbf{i}\theta_k \pi})$, where $k = 0, 1, \cdots$ and γ_k (and θ_k) is (are) optimized to minimize the average error probability. Then the average number of photons for signal m after applying the k-th shift operator is

$$Ns_{k,m}^{MPSK} = \left| \alpha \mathrm{e}^{2\mathbf{i}m\pi/M} - \gamma_k \mathrm{e}^{\mathbf{i}\theta_k \pi} \right|^2, \tag{1}$$

3.2 Improvement by increasing feedback

To consider further improvements in the quantum receiver for M-ary signals, we increase the number of times of feedback is applied to more than $M-1$ times assuming this to be $N-1$ times with $N > M$. If no photon is detected, we set the signal to $k(\mathrm{mod}\ M)$ when feedback has been applied $k(< N-1)$ times. Next, we consider the error probability. We assume that the duration of the signal is T and the time the k-th shift operator is applied is $t_{k-1}(t_0 = 0)$.

We derive the probability that no photon has been detected after applying the k-th shift operator when the input signal is $|\alpha_m\rangle$. This is the joint probability for when no photon is detected in $[0, t_1), (t_1, t_2), \cdots, (t_{k-1}, t_k]$ and when a photon is detected at $t_1, t_2, \cdots, t_{k-1}$:

$$\begin{aligned} P_{ND}(k,m) \quad = \quad & \frac{1}{T^{k-1}} \mathrm{e}^{-Ns_{k,m}^{MPSK}} \prod_{s=1}^{k-1} Ns_{s,m}^{MPSK} \\ & \int_{t_{s-1}}^{T} \mathrm{e}^{-(Ns_{s,m}^{MPSK} - Ns_{s+1,m}^{MPSK}) t_s/T} dt_s. \end{aligned} \tag{2}$$

We decide that input signal is $N(\mathrm{mod}\ M)$ when a photon is detected after the time t_{N-1}.

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[1]We partly reported partial results at a domestic conference and a journal[6, 7].

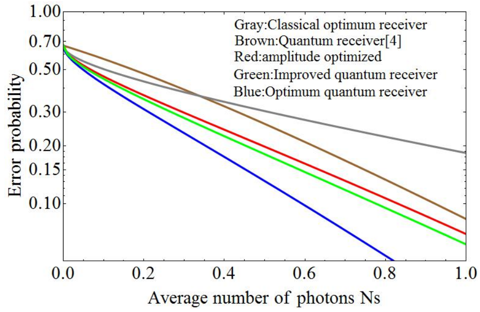

Figure 1: Average error probabilities of existing receivers and improved receivers for 3-ary PSK coherent-state signals.

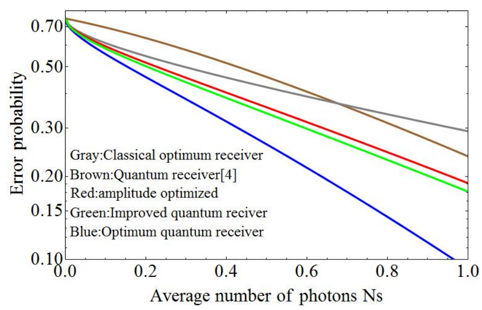

Figure 2: Average error probabilities of existing receivers and improved receivers for 4-ary PSK coherent-state signals.

4 Performance

4.1 3PSK

Figure 1 shows the average error probabilities of the classical optimum receiver, the optimum quantum receiver, the quasi-optimum receiver[3], one of the improved quantum receivers in which the amplitude of the shift operator is optimized, and an improved receivers in which the shift operator is optimized with increased feedback for 3-ary PSK coherent-state signals. Here, the number of feedback applications is increased from two to three. We see that the improved quasi-optimum quantum receiver always outperforms the classical optimum receiver. The quasi-optimum quantum receiver is further improved by optimizing the phase of the shift operator and by increasing the number of feedback applications. From Fig.1, feedback is more effective than phase optimization for 3-ary PSK signals.

4.2 4PSK

Figure 2 shows a comparison of the improved receivers with existing receivers. As for the 3-PSK signals, the improved quasi-optimum quantum receivers always outperform the classical optimum receiver. For 4-PSK signals, we increased the number of feedback applications from three to four. For the 4-PSK signals, if the increase in feedback applications is only one, the improvement in error performance is very small. However, the phase optimization is effective. Indeed, the effect of phase optimization for the 4-PSK signals is greater than that for the 3-PSK signals (Fig.3).

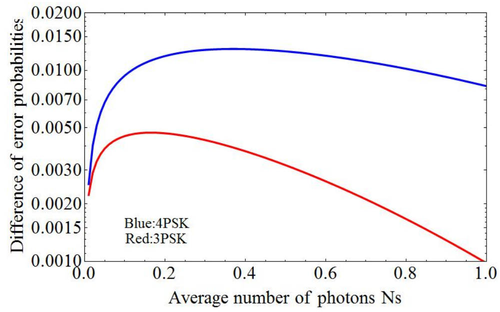

Figure 3: Difference of average error probabilities of the improved quasi-optimum quantum receivers with and without phase optimization.

5 Conclusion

We have shown that error performances of the quasi-optimum quantum receivers were improved by optimizing a shift operator and by increasing feedback applications. As a result, the improved receivers always outperform the classical optimum receiver. It is expected that phase optimization of the shift operator is more effective when the number of signals increases. On the other hand, if the number of signals increases, further increase in feedback is desired.

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