Detecting mixedness of quantum states using the uncertainty principle

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Abstract. We show that the uncertainty relation as expressed in the Robertson-Schrodinger generalized form can be used to detect the mixedness of three-level quantum systems in terms of measureable expectation values of suitably chosen observables when prior knowledge about the basis of the given state is known. In particular, we demonstrate the existence of observables for which the generalized uncertainty relation is satisfied as an equality for pure states and a strict inequality for mixed states corresponding to single as well as bipartite sytems of qutrits. Examples of such observables are found for which the magnitude of uncertainty is proportional to the linear entropy of the system, thereby providing a method for measuring mixedness.

Keywords: Uncertainty relation, mixedness/purity, Qutrit, Testable measure.

Motivation—In recent years certain important applications of uncertainty relations have been discovered in the realm of quantum information processing. The security of quantum key distribution protocols is based fundamentally on quantum uncertainty [5] and the amount of key extractable per state can be linked to the lower limit of entropic uncertainty [3]. The fine-grained uncertainty relation can be used to determine the nonlocality of the underlying physical system [4, 6]. The uncertainty principle has been used for discrimination between separable and entangled quantum states [7] and the Robertson-Schrodinger generalized uncertainty relation [1, 2] (GUR) has also been applied in this context [8].

In the present work our motivation is to investigate the role of GUR in the context of another important property, *viz.* the purity of quantum systems.

At the practical level the ubiquitous interaction with the environment inevitably affects the purity of a quantum system. A relevant issue for an experimenter is to ascertain whether a prepared pure state has remained isolated from environmental interaction in order to use it effectively as a resource for quantum information processing.

The purity of a given state is also related to the entanglement of a larger multipartite system of which it may be a part [9]. The mixedness of states can be characterized by the property of linear entropy, which is a non linear functional of the quantum state. The linear entropy can be extracted from the given state by tomography which usually is expensive in terms of resources and measurements involved. Bypassing a classical evaluation process, estimation of purity of a system using quantum networks has been suggested [10]. In this work we connect the Robertson-Schrodinger GUR to the property of mixedness of quantum states of discrete variables.

We show that the uncertainty relation as expressed in the Robertson-Schrodinger generalized form can be used to detect the mixedness of two and three-level quantum systems in terms of measureable expectation values of suitably chosen observables when prior knowledge about the basis of the given state is known. In particular, we demonstrate the existence of observables for which the generalized uncertainty relation is satisfied as an equality for pure states and a strict inequality for mixed states corresponding to single as well as bipartite sytems of qubits and qutrits. Examples of such observables are found for which the magnitude of uncertainty is proportional to the linear entropy of the system, thereby providing a method for measuring mixedness. For single qubit system spin measurements along two different directions yield necessary and sufficient condition for detection of purity. For two qubits we take pure states polarized along a specific known direction(say, z-axis) and one parameter mixed states comprising of that kind of pure states. For single qutrit systems a somewhat more elaborate procedure is required as may be expected from the richer structure of their state space. We find four settings for A and B so that for mixed states all these settings yield GUR as an inequality and pure states come out to be an equality. In this case we take single qutrit states upto 3 parameter family as classified in [11]. For bipartite case we consider a one-parameter class of two-qutrit mixed states including Werner and isotropic class of states. We find observables spanned by two selected Gellmann matrices yielding GUR as equality for pure states and inequality for mixed states.

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