Evolution of quasi-Bell states in a strongly coupled qubit-oscillator system and a study of complexity

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Abstract. We study evolution of bipartite entangled quasi-Bell states in a qubit-oscillator system in the strong coupling regime and also in the presence of a static bias. Using the adiabatic approximation the reduced density matrices for the qubit and the oscillator degrees of freedom are obtained in closed forms involving Jacobi theta functions. The entropy of the system quantifying the evolution of entanglement is computed. Using the Husimi Q distribution, the Wehrl entropy of the system is also studied.

Keywords: Adiabatic approximation, Quasi-Bell states, Entropy, Husimi Q-function, Complexity

1 Introduction

The coupled qubit-oscillator system is well-described by the Jaynes-Cummings model [1] in the context of the rotating wave approximation that holds for a small detuning and a tiny qubit-oscillator coupling compared to the qubit and oscillator frequency. Recently, however, a variety of experimental situations pertaining to stronger coupling domain, where the rotating wave approximation is no longer valid, have been investigated [2].

On the other hand, for the coupled qubit-oscillator system the nonclassical quasi-Bell states are of much interest. When the amplitude of the coherent states are large enough they are often called Schrödinger cat states as they introduce entanglement between a microscopic and a classical object. Moreover, it has been observed [3] that in the large coupling regime a state of the generic quasi-Bell type becomes the approximate ground state of the combined system. In the case of cavity electrodynamics, the static bias of a superconducting qubit, may be easily varied, say, by operating a magnetic flux on a Josephson junction [4]. In this work the evolution of quasi-Bell states in the strong coupling regime and also in the presence of a static bias is studied via the adiabatic approximation [5, 3].

2 The reduced density matrices

We study a coupled boson-qubit system with the Hamiltonian that reads in natural units ($\hbar = 1$) as follows:

$$H = -\frac{\Delta}{2}\sigma_x - \frac{\epsilon}{2}\sigma_z + \omega a^\dagger a + \lambda\sigma_z\,(a^\dagger + a), \tag{1}$$

where the harmonic oscillator has a frequency ω, the qubit is characterized by an energy splitting Δ and an external static bias ϵ, and the qubit-oscillator coupling strength is denoted by λ. In the regime of large detuning and strong coupling, the adiabatic approximation that relies on the separation of the time scales characterized by the high oscillator frequency and the (renormalized) low qubit frequency could be used. The fast-moving oscillator then adiabatically adjusts to the slow changes of the state of the qubit.

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The quasi-Bell initial state of the coupled system is chosen as

$$|\psi(0)\rangle^{(\pm)} = \frac{1}{\sqrt{2}}\left(|1,\alpha\rangle \pm |-1,-\alpha\rangle\right), \tag{2}$$

where $|\alpha\rangle$ is the coherent state. Corresponding to this state the evolution of the reduced density matrices of both the qubit and oscillator may be readily constructed. The explicit structure of the qubit reduced density matrix assumes the form

$$\rho_{\mathcal{Q}}^{(\pm)}(t) \equiv \mathrm{Tr}_{\mathcal{O}}\,\rho^{(\pm)}(t) = \begin{pmatrix} \frac{1}{2} \mp \zeta & \pm\xi^{(\pm)} \\ \pm(\xi^{(\pm)})^* & \frac{1}{2} \pm \zeta \end{pmatrix}, \tag{3}$$

where the components read

$$\begin{aligned}\zeta &= \tilde{\epsilon}\,\exp\left(-\varrho_{\widehat{\alpha}}^2\right)\sum_{n=0}^{\infty}(-1)^n\,\frac{\varrho_{\widehat{\alpha}}^{2n}}{n!}\,\delta_n\,\frac{\sin^2\chi_n t}{\chi_n^2}, &(4)\\ \xi^{(\pm)} &= \frac{1}{2}\,\exp(-\varrho_{\widehat{\alpha}}^2)\sum_{n,m=0}^{\infty}\frac{(\widehat{\alpha})^n\,(-\widehat{\alpha}^*)^m}{\sqrt{n!\,m!}}\,\mathfrak{C}_n^{(\pm)}\,\mathfrak{C}_m^{(\mp)}\,\times \\ &\quad\times\,\exp\big(-i(n-m)\omega t\big)\,\langle m_-|n_+\rangle, &(5)\end{aligned}$$

and the parameters are given by

$$\mathfrak{C}_n^{(\pm)} = \mathfrak{A}_n^{(\mp)}\exp(i\chi_n t) + \mathfrak{B}_n^{(\pm)}\exp(-i\chi_n t),$$

$$\mathfrak{A}_n^{(\pm)} = \frac{\chi_n + \tilde{\epsilon} \pm (-1)^n\delta_n}{2\chi_n}, \quad \mathfrak{B}_n^{(\pm)} = \frac{\chi_n - \tilde{\epsilon} \pm (-1)^n\delta_n}{2\chi_n},$$

$\delta_n = -\frac{\widetilde{\Delta}}{2}L_n(x)$, $\widetilde{\Delta} = \Delta\exp\left(-\frac{x}{2}\right)$, $x = (\frac{2\lambda}{\omega})^2$, $\tilde{\epsilon} = \frac{\epsilon}{2}$, $\chi_n = \sqrt{\delta_n^2 + \tilde{\epsilon}^2}$, $\widehat{\alpha} = \alpha + \lambda/\omega$, and $\varrho_{\widehat{\alpha}} = |\widehat{\alpha}|$.

Similarly the oscillator reduced density matrix can be constructed as

$$\begin{aligned}\rho_{\mathcal{O}}^{(\pm)}(t) &= \frac{1}{2}\exp(-\varrho_{\widehat{\alpha}}^2)\sum_{n,m=0}^{\infty}\frac{(\widehat{\alpha})^n\,(\widehat{\alpha}^*)^m}{\sqrt{n!m!}}\Big(\mathfrak{C}_n^{(\pm)}\mathfrak{C}_m^{(\pm)^*}|n_+\rangle\langle m_+| \\ &\quad+(-1)^{n+m}\mathfrak{C}_n^{(\mp)^*}\mathfrak{C}_m^{(\mp)}|n_-\rangle\langle m_-|\Big)\exp(i(m-n)\omega t).\end{aligned} \tag{6}$$

It can be observed that the reduced density matrix of the qubit with its two dimensional Hilbert space and that of

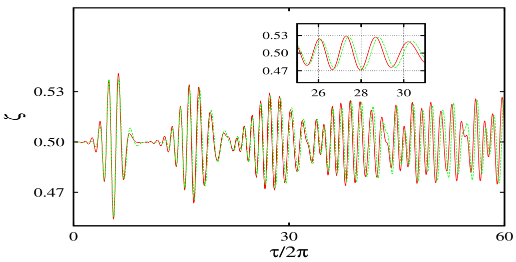

Figure 1: The series(red) and theta function(green dashed) evolution of the diagonal element of ρ_Q for the values of $\lambda = 0.15\omega, \Delta = 0.15\omega, \epsilon = 0.01\omega$ and $\alpha = 1.5$.

the oscillator endowed with the infinite dimensional Fock space satisfy the equality

$$\mathrm{Tr}\rho_{\mathcal{Q}}^{(\pm)}(t)^2 = \mathrm{Tr}\rho_{\mathcal{O}}^{(\pm)}(t)^2. \tag{7}$$

In an attempt to analytically evaluate the qubit density matrix in closed form we approximate the Poisson distribution with the corresponding Gaussian limit. The reduced density matrix for the qubit is expanded till the fourth power of coupling constant and is written in closed forms comprising of linear combinations of Jacobi theta functions. The analytical results based on the theta function evaluations are found to be in good agreement with their series counterparts for values $\alpha \approx 1-2$ and $\lambda \lesssim 0.2\omega$ (Fig.[1]). The entropy of the system quantifying the entanglement can be computed via the qubit reduced density matrix and also can be written in terms of theta functions.

3 The Q-function of the oscillator density matrix

The reduced density matrix of the oscillator is employed for obtaining the Husimi Q distribution:

$$\begin{aligned} Q^{(\pm)}(\beta) = \frac{1}{2\pi}\exp(-\varrho_{\widehat{\alpha}}^2)\sum_{n,m=0}^{\infty}\Big(&\exp(-\varrho_{\widehat{\beta}}^2)\frac{(\widehat{\alpha}\widehat{\beta}^*)^n(\widehat{\alpha}^*\widehat{\beta})^m}{n!m!} \times \\ &\times\ \mathfrak{C}_n^{(\pm)}\mathfrak{C}_m^{(\pm)*} + \exp(-\varrho_{\check{\beta}}^2)\frac{(\widehat{\alpha}\check{\beta}^*)^n\,(\widehat{\alpha}^*\check{\beta})^m}{n!m!} \times \\ &\times\ (-1)^{n+m}\mathfrak{C}_n^{(\mp)*}\mathfrak{C}_m^{(\mp)}\Big)\ \exp(-i(n-m)\omega t), \end{aligned} \tag{8}$$

where we abbreviate: $\widehat{\beta} = \beta + \lambda/\omega, \varrho_{\widehat{\beta}} = |\widehat{\beta}|$ and $\check{\beta} = \beta - \lambda/\omega, \varrho_{\check{\beta}} = |\check{\beta}|$. The Q-function at various time is given in Fig.[2]. It is, in turn, utilized for obtaining the expectation values of antinormally ordered operators in closed-form involving Jacobi theta functions. Long time behavior of the Heisenberg uncertainty for the state is found.

Our evaluation of the Husimi Q function allows us to study the complexity of the strongly coupled system. Complexity is a measure of delocalization of the Husimi distribution in phase space and it is computed, for instance by the inverse of second moment of the Husimi distribution [6]. The kinship in time dependence of the complexity, the Heisenberg uncertainty product and the

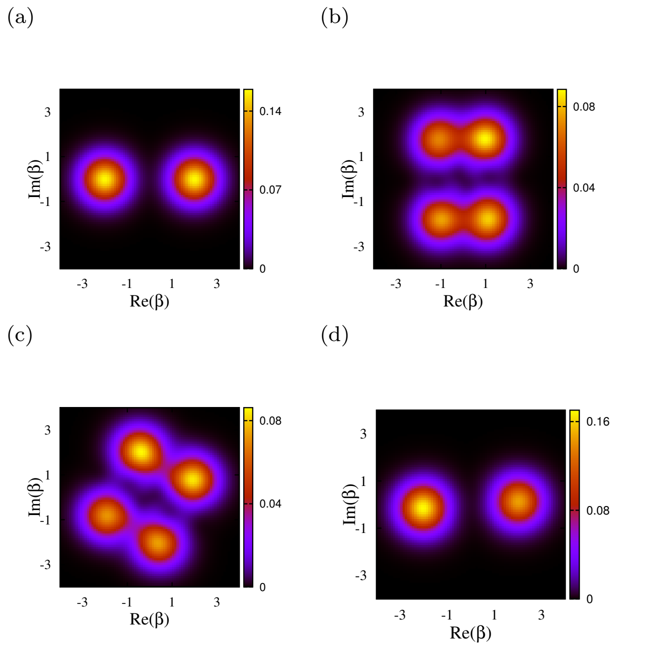

Figure 2: The $Q^{(+)}(\beta)$ function for $\lambda = 0.08\omega, \Delta = 0.15\omega, \epsilon = 0.01\omega$ and $\alpha = 2$ at various time (Natural unit) t (a)0 (b)300 (c)500 (d)900.

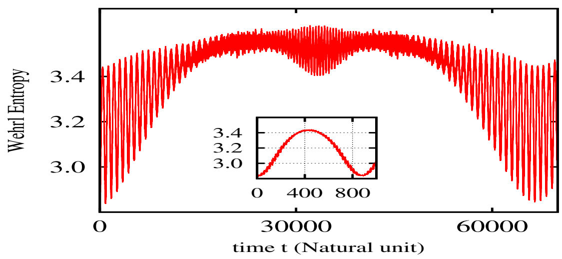

Figure 3: The evolution of the Wehrl entropy for the values of $\lambda = 0.08\omega, \Delta = 0.15\omega, \epsilon = 0.01\omega$ and $\alpha = 2$.

Wehrl entropy [7] is also verified. More delocalized phase space distributions corresponds to increase in the Wehrl entropy (Fig.[3]).

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