TOPICS IN REPRESENTATION THEORY

ASSIGNMENT DUE ON 08/11/2006

(1) Are quotient modules or submodules of projective modules necessarily projective? Give proofs or explicit counterexamples.

(2) Let R be a left Noetherian and left Artinian ring. Prove that every projective R-module is a direct sum of the principal indecomposable R-modules.

(3) Consider a sequence $\lambda_1 < \lambda_2 < \cdots < \lambda_l$ of positive integers, and let $m_1, \ldots, m_l$ be any positive integers. Consider the $k[t]$-module

$$M_\lambda = (k[t]/t^{\lambda_1})^{m_1} \oplus \cdots (k[t]/t^{\lambda_l})^{m_l}.$$

Let $R = \mathrm{End}_{k[t]} M_\lambda$.

(a) What is the radical of R?
(b) Describe all the irreducible R-modules up to isomorphism.
(c) *Compute the Cartan matrix of R.

(4) Let k be an algebraically closed field and A be a finite dimensional k-algebra. Show that if A is semisimple then it is a symmetric algebra. What happens if k is not algebraically closed?