

## TOPICS IN REPRESENTATION THEORY

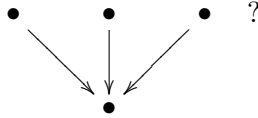
ASSIGNMENT DUE ON 15/09/2006

- (1) Let  $R$  be any ring, and  $M$  be any  $R$ -module. Show that if  $\text{End}_R M$  is a local ring, then  $M$  is indecomposable.
- (2) Let  $Q$  be the linear quiver with  $n$ -vertices:

$$1 \xrightarrow{\alpha_1} 2 \xrightarrow{\alpha_2} \dots \xrightarrow{\alpha_{n-1}} n .$$

Show that if  $(\pi, V)$  is an indecomposable representation of  $Q$  such that  $\pi(\alpha_j)$  is not surjective, then  $V_i = 0$  for all  $i \leq j$ .

- (3) Let  $Q$  be as in the previous exercise, and let  $(\pi, V) = [j, i]$  denote the indecomposable representation of  $Q$  where  $V_h \cong k$  for all  $j \leq h \leq i$ , and  $\pi(\alpha_h) = \text{id}$  for  $j \leq h < i$ . Compute  $\text{Hom}_Q([j, i], [j', i'])$ .
- (4) What are the irreducible representations of the quiver  $Q$  of the previous problem?
- (5) What is the representation type of the quiver



- (6) What is the representation type of the quiver



- (7) (Emmy Noether) A *left ideal* in a ring  $R$  is, by definition, a submodule of the the left  $R$ -module  $R$ . Show that a decomposition of a unital ring  $R$  as a direct sum

$$R = M_1 \oplus \dots \oplus M_n$$

of left ideals is equivalent to a decomposition of the unit 1 as a sum of rothogonal idempotents, i.e.,

$$1 = e_1 + \dots + e_n,$$

with

$$e_i^2 = e_i, \quad e_i e_j = 0 \text{ for } i \neq j, \text{ and } M_i = R e_i,$$

for all  $i$  and  $j$ .