TOPICS IN REPRESENTATION THEORY

ASSIGNMENT DUE ON 01/09/2006

(1) Find a function from $s : \mathbf{Z}/5\mathbf{Z} \to \mathbf{Z}/25\mathbf{Z}$ such that $s(x_1x_2) = s(x_1)s(x_2)$ for all $x_1, x_2 \in \mathbf{Z}/5\mathbf{Z}$. Show that there can exist at most one such function. Note that this function is not a ring homomorphism.

(2) Find an element $q(t) \in \mathbf{R}[t]$ such that $(x^2+1)|(t - q(t))$ and $(x^2+1)^3|q(t)^2+1$.

(3) Compute the generalised Jordan canonical form for the matrix:
$$\begin{pmatrix} 0 & 1 & 1 & 0 \\ -7 & 0 & 0 & -11 \\ 5 & 0 & 0 & 8 \\ 0 & 1 & 1 & 0 \end{pmatrix}.$$
You may use a computer programme for this problem, in which case please give details.

(4) Express $\mathrm{diag}(3,3,4)$ as a polynomial in
$$\begin{pmatrix} 3 & 0 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix}.$$

(5) Compute the cardinality of the group
$$\mathrm{Aut}_{\mathbf{F}_q[t]}(\mathbf{F}_q[t]/(t^2) \oplus \mathbf{F}_q),$$
where $\mathbf{F}_q$ denotes the finite field with q elements, for some prime power q.

(6) For two Young diagrams λ and μ let S denote the set of all injective $k[t]$-module homomorphisms of k^λ into k^μ. The group $G_\mu(k)$ acts on this set by
$$g \cdot i = g \circ i, \text{ for } g \in G_\mu(k) \text{ and } i \in S.$$
Compute the number of orbits when $\lambda = (1)$ and $\mu = (1,2)$.