TOPICS IN REPRESENTATION THEORY

ASSIGNMENT DUE ON 11/08/2006

(1) Let R be a unital ring. Show that $\mathrm{End}_R R = R^{\mathrm{opp}}$.
(2) Let A and B be square matrices of the same size over a commutative ring. Show that if $AB = I$ then $BA = I$.
(3) Give an example where the result of the previous exercise fails to hold over a non-commutative ring.
(4) For which abelian groups A is the $R = \mathrm{End}(A)$ isomorphic to its opposite ring?
(5) Let G be a finite group. Let k be any field. Denote by $k[G]$ the set of all functions
$$f : G \to k.$$
Show that $k[G]$ is a ring when addition is defined pointwise and multiplication is defined as
$$fg(x) = \sum_{x_1 x_2 = x} f(x_1) g(x_2).$$
For any homomorphism $\pi : G \to GL(V)$ (invertibale linear transformations from a finite dimensional k-vector space V to V), show that V becomes a $k[G]$-module by the action:
$$\pi(f)v = \sum_{g \in G} f(g)\pi(g)v.$$