Recall that U_λ was defined to be the unique simple which occurs in both $K[X_\lambda]$ and $K[X_{\lambda'}] \otimes \epsilon$. We wished to show that $U_\lambda \cong V_\lambda$.

Let $\tilde{\lambda}$ be the partition for which $U_\lambda \cong V_{\tilde{\lambda}}$. Since U_λ occurs in $K[X_\lambda]$, we know that $\tilde{\lambda} \leq \lambda$. By the same reasoning, $U_\lambda \cong V_{\tilde{\lambda}}$ does occur in $K[X_{\tilde{\lambda}}]$. By definition it occurs in $K[X_{\lambda'}]$. Therefore,

$$\dim \operatorname{Hom}_{S_n}(K[X_{\tilde{\lambda}}], K[X_{\lambda'}] \otimes \epsilon) > 0$$

which implies that $N_{\tilde{\lambda}\lambda'} > 0$. By a lemma proved in class, this is equivalent to $\lambda \leq \tilde{\lambda}$. Thus $\tilde{\lambda} = \lambda$.