$$G_1 \cap G_2 < G$$

C : conjugacy class of G

Then $|C \cap G_1| = |C \cap G_2|$

$$k_1(i,j) = \begin{cases} 1 & \text{if } i=j \\ 0 & \text{o/w} \end{cases}$$

$$k_2(i,j) = \begin{cases} 1 & \text{if } i\neq j \\ 0 & \text{o/w}. \end{cases}$$

$$k_1 * k_1 = k_1$$

$$k_1 * k_2 = k_2 = k_2 * k_1$$

$$k_2 * k_2 = (n-1)k_1 + (n-2)k_2$$

$$\varepsilon_1 = \varepsilon = \alpha_1 k_1 + \alpha_2 k_2$$

$$\varepsilon_2 = k_1 - \varepsilon.$$

$$(\alpha_1 k_1 + \alpha_2 k_2)^2 = \alpha_1 k_1 + \alpha_2 k_2$$

$$\alpha_1^2 k_1 + 2\alpha_1\alpha_2 k_2 + \alpha_2^2\left[(n-1)k_1 + (n-2)k_2\right]$$

$$\underset{\alpha_1}{\underset{\|}{\left[\alpha_1^2 + (n-1)\alpha_2^2\right]}} k_1 + \left[\underbrace{2\alpha_1\alpha_2 + \alpha_2^2(n-2)}_{\alpha_2}\right] k_2$$

$$2\alpha_1 + \alpha_2(n-2) = 1$$

$$\alpha_1 = \frac{1 - \alpha_2(n-2)}{2}$$

$$\alpha_1 = \alpha_2 = \frac{1}{n} \qquad n\alpha_2 = 1$$

$$n\cancel{(n-1)}\,\alpha_2^{2} = \alpha_2\, 1$$

$$e_1 = \frac{1}{n}k_1 + \frac{1}{n}k_2$$

$$e_2 = \frac{n-1}{n}k_1 - \frac{1}{n}k_2 .$$

k_1 does nothing to a fn.

$$k_2 f(i) = \sum_{j \neq i} f(j)$$

$$\left.\begin{aligned} e_1 f(i) &= \frac{1}{n}\Big[f(i) + \sum_{j\neq i} f(j)\Big] \\ &= \frac{1}{n}\sum_j f(j) \end{aligned}\right\} \text{const fns. as image.}$$

$$\left.\begin{aligned} e_2 f(i) &= \frac{1}{n}\Big[(n-1)f(i) - \sum_{j\neq i} f(j)\Big] \\ &= \cancel{\tfrac{1}{n}\big[}\, n f(i) - \frac{\sum_j f(j)}{n} \\ &\Rightarrow f \mapsto f - av(f) \end{aligned}\right\} \text{sum zero fns. as image.}$$

$K[\binom{n}{2}]$

$\cancel{k_2(S}$ $k_i(S,T) = \begin{cases} 1 & \text{if } |S\cap T| = i \\ 0 & \text{o/w}. \end{cases}$

$k_2 = id.$

$k_1,$

$k_0.$

$k_i * k_j(S,T) = \#\left\{U \,\middle|\, \begin{array}{l} |S\cap U| = i \\ |T\cap U| = j \end{array}\right\}.$

$|S\cap T| = k$

<u>Combinatorial problem:</u>

Given i, j, k, take $S, T \ni |S\cap T| = k$.

Find no. of $U \ni |S\cap U| = i$, $|T\cap U| = j$.

$$\begin{tikzcd} & S \arrow[dl, dash, "k"'] \arrow[dr, dash, "i"] & \\ T \arrow[rr, dash, "j"'] & & U \end{tikzcd}$$

$$\begin{tikzcd} S \arrow[dr, dash] & & T \arrow[dl, dash] \\ & S\cap T & \end{tikzcd} \qquad \sum_{\ell=0}^{k} \cancel{\binom{S}{i-\ell}} \binom{\overset{|S|-k}{|S|}}{i-\ell}\binom{|T|}{j-\ell}\binom{k}{\ell}$$

$\ell = |S\cap T\cap U|$

$$\sum_{l=0}^{k} \binom{k}{l} \binom{s-k}{i-l} \binom{t-k}{j-l} \binom{n-(s+t-k)}{u-(j+i-l)}$$

$\uparrow$ $S \cap T \cap U$ $\quad$ $\uparrow$ $U \cap S - U \cap T$ $\quad$ $U \cap T - U \cap S$ $\quad$ $U - U \cap (S \cup T)$

$u - (j+i-l)$

$s+t-k$

In our example, $s = t = 2 = u$

Now ~~do th~~ compute the e_1, e_2, e_3 for $\mathrm{End}_{S_n} K[\binom{n}{2}]$.

$n = 3, \quad s, t, u = 2$

$k = 1, \quad i, q = 2, \quad j = 1$

$S = \{1, 2\}, \quad T = \{2, 3\}$

$\# \left\{ U \;\middle|\; \begin{matrix} |S \cap U| = 2 \\ |T \cap U| = 1 \end{matrix} \right\} \qquad U = S = \{1, 2\}$

$$\sum_{l=0}^{1} \binom{1}{l} \binom{2-1}{2-l} \binom{2-1}{1-l} \binom{\cancel{3-3}\ 0}{\cancel{2-(3-l)}\ l-1}$$