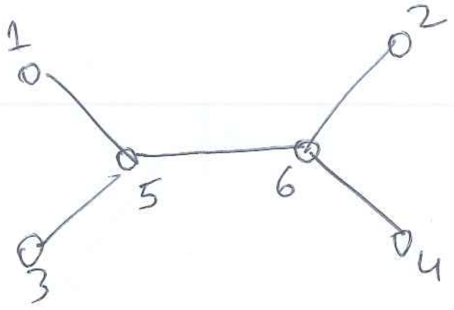
$\Gamma =$ 

vertices $\{1, \dots, n\}$

$$G = \mathrm{Aut}(\Gamma) = \{ w \in S_n \mid i - j \iff w(i) - w(j) \}$$

$$G_5 = \{ w \in G \mid w(5) = 5 \}$$ has 4 elements

$$\cong \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$$

$w1 = a = (1\,3)$

$w2 = b = (2\,4)$

$$G_5 = \{ a, b, \underset{\substack{\shortparallel \\ ba}}{ab}, id \}$$

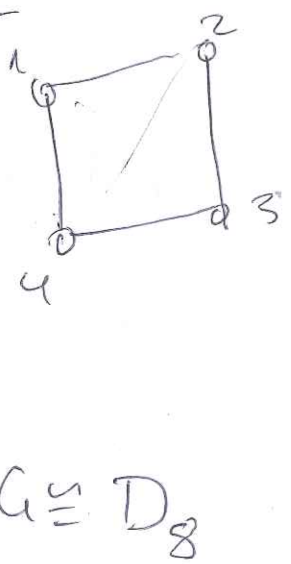
Suppose α interchanges 5 & 6.

Then $\{ w \in G \mid w(5) = 6,\ w(6) = 5 \}$

$= G\alpha =: \{ w\alpha \mid w \in G \}$

$\alpha = (1\,2)(3\,4)(5\,6)$.

So $|G| = 8$?? $\mathbb{Z}/2 \times \mathbb{Z}/2 \times \mathbb{Z}/2$

<u>Qn</u>: Is $G \cong D_8$ → 

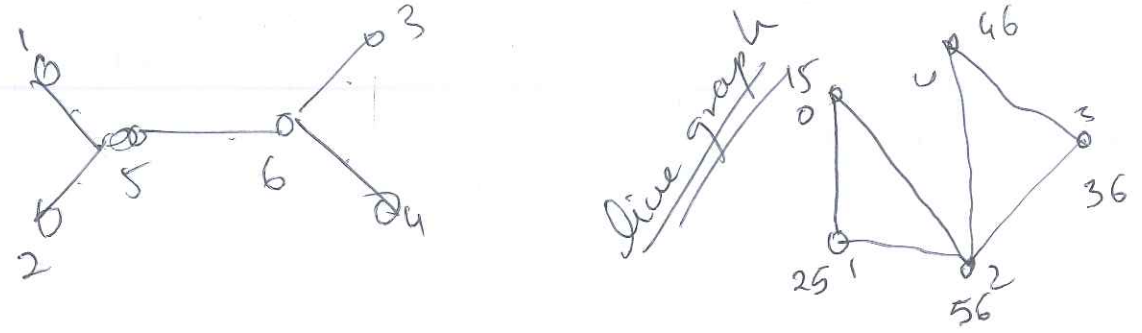
$\mathbb{Z}/8$

$\mathbb{Z}/4 \times \mathbb{Z}/2$

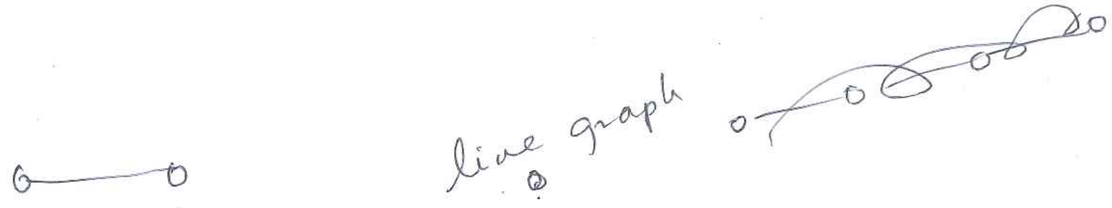
<u>HW</u>: Show (preferably cleverly) that $G \cong D_8$

LINE GRAPHS

(1)



(2)



Line graph of Γ — vertices are edges of Γ

— two edges are joined if they share a vertex.

$\mathrm{Aut}(\Gamma)$ need not be iso. to $\mathrm{Aut}(\mathrm{LineGraph}(\Gamma))$

But we do have a group homom.

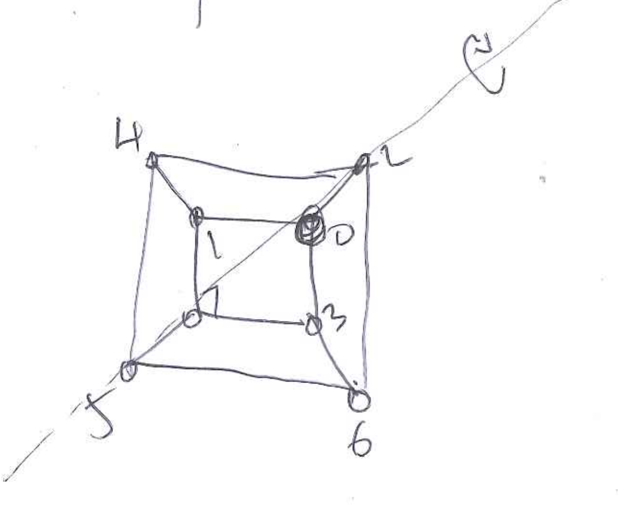
$\varphi : \mathrm{Aut}(\Gamma) \to \mathrm{Aut}(\mathrm{LineGraph}(\Gamma))$

<u>HW:</u> Does φ have to be surjective.

$|\mathrm{Aut}(\mathrm{CubeGraph})| = 48$

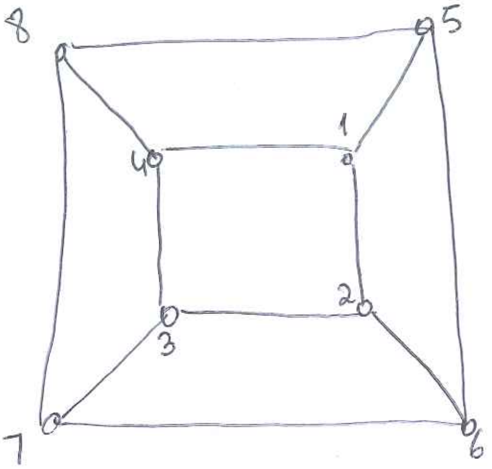
$|\mathrm{Aut}(\mathrm{Cube})| = 24 \cong S_4$.

$G(\mathrm{Cube})$



$\tilde{G}$ ||

Question: Is Aut(CubeGraph) $\cong \boxed{S_4 \times \mathbb{Z}/2\mathbb{Z}}$?



$(g, c)(g', c')$

(gg', cc')

$(g, 0)\,(1, c)$

(g, c)

$\alpha = (24)(68)$

$\beta = \cancel{(14)(23)(56)(78)}$
$(12)(34)(56)(78)$

G_C = cube gp.

$$\tilde{G} = \{ g\alpha \mid g \in G_c \} \sqcup \{ g \mid g \in G_c \}$$

$g = (1234)(5678)$

$(13)(24) = (1234)^2$

$\alpha g \neq g\alpha$

$ZG := \{ z \in G \mid zg = gz \ \forall\, g \in G \}$

centre of G

$$\cancel{ZG}\ Z(S_4 \times \mathbb{Z}/2\mathbb{Z}) = \underbrace{Z(S_4)}_{= id} \times \underbrace{Z(\mathbb{Z}/2\mathbb{Z})}_{= \mathbb{Z}/2\mathbb{Z}} = \mathbb{Z}/2\mathbb{Z}$$

$Z\tilde{G} = ?$