# ALMOST UNRAMIFIED AUTOMORPHIC REPRESENTATIONS FOR SPLIT GROUPS OVER $\mathbf{F}_q(t)$

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ABSTRACT. Let $G$ be a split reductive group over a finite field $\mathbf{F}_q$, $\mathbf{F}$ the field $\mathbf{F}_q(t)$ of rational functions in $t$ with coefficients in $\mathbf{F}_q$ and $\mathbf{A}$ the adèles of $\mathbf{F}$. We describe the irreducible automorphic representations of $G(\mathbf{A})$ which have non-zero vectors invariant under Iwahori subgroups at two places and maximal compact subgroups at all other places in terms of the irreducible square-integrable representations of an Iwahori-Hecke algebra associated to $G$ and the Satake isomorphism.

**Introduction.** Let $\mathbf{F}_q$ denote the finite field with $q$ elements. Let $G$ be a split reductive group over $\mathbf{F}_q$. Fix a Borel subgroup $B$ of $G$ defined over $\mathbf{F}_q$ and a maximal $\mathbf{F}_q$-split torus $T$ contained in $B$. Let $N$ be the unipotent radical of $B$. Consider the global field $\mathbf{F} = \mathbf{F}_q(t)$ with ring of adèles $\mathbf{A}$. For each valuation $v$ of $\mathbf{F}$ let $F_v$ denote the corresponding completion of $\mathbf{F}$. Let $O_v$ denote the ring of integers in $F_v$. Denote by 0 and $\infty$ the valuations uniformized by $t$ and $t^{-1}$ respectively. Set

$$K_v = \begin{cases} \text{pre-image of } B(\mathbf{F}_q) \text{ under } G(O_v) \to G(\mathbf{F}_q) & \text{if } v = \infty \text{ or } 0, \\ G(O_v) \text{ otherwise.} \end{cases}$$

Let $\mathbf{K} = \prod_v K_v$. Let

$$M = L^2(G(\mathbf{F})\backslash G(\mathbf{A})/\mathbf{K}).$$

Denote by $H_v$ the convolution algebra of compactly supported complex-valued measures on $G(F_v)$ that are left and right invariant under $K_v$. For $v = \infty$ or 0, $H_v$ is an *Iwahori-Hecke algebra*. For all the other $v$'s, $H_v$ is a *spherical Hecke algebra*. $H_v$ has a right action on $M$ given by

$$m \cdot \omega_v(x) = \int_{G(F_v)} m(xg^{-1}) d\omega_v(g) \text{ for all } m \in M,\ \omega_v \in H_v. \tag{1}$$

The convolution algebra of compactly supported measures on $G(\mathbf{A})$ which are left and right invariant under $\mathbf{K}$ is a *restricted tensor product* of the algebras $H_v$ (see e.g., [4, Example 2]). Denote this algebra by $\otimes_v H_v$. For each irreducible representation $(\pi, V)$ of $\otimes_v H_v$ occurring discretely in $M$ there exist irreducible representations $(\pi_v, V_v)$ of $H_v$ and vectors $\mathbf{v}_v \in V_v$ for each $v$ such that $(\pi, V)$ is a restricted tensor product of the $(\pi_v, V_v)$'s with respect to $\{\mathbf{v}_v\}$ in the sense of [4]. The isomorphism classes of the factors $(\pi_v, V_v)$ are completely determined by the isomorphism class of $V$. The representation $\pi_v$ is called the *local constituent* of $\pi$ at $v$.

The main theorem (Theorem 10) of this article describes the local constituents of the irreducible representations of $\otimes_v H_v$ that occur in the discrete part of $M$.

*Date*: 28 October, 2002.

2000 *Mathematics Subject Classification.* 11F70, 22E55.

For each $(\pi, V)$ occurring discretely in $M$ the representations $(\pi_v, V_v)$ are explicitly described in terms of the irreducible square-integrable representations of an Iwahori-Hecke algebra for $v = \infty, 0$ in [12]. In this article the local constituents of $(\pi, V)$ for the remaining $v$ are obtained by showing first that a function in $M$ is determined by its constant term along $N$, and then showing that the constant terms lie in a representation of $\otimes_v H_v$ where the eigencharacter for the center of $H_\infty$ of an eigenvector for the action of the center of $\otimes_v H_v$ determines the eigencharacter for the center of $H_v$ for any other $v$.

The automorphic representations described here lie in the *residual discrete spectrum* coming from the residues of Eisenstein series associated to unramified automorphic characters of $T$ (this can be deduced from Lemmas 2 and 3). For classical groups over number fields, such representations have been studied extensively by Mœglin and Waldspurger in [10] and by Mœglin in [8] and [9]. A more detailed overview of these articles may be found in the introduction of [12]. The techniques used in [12] and in this paper use only general results about the structure of split reductive groups and therefore also provide, for the first time, an opportunity to understand the residual discrete spectrum for all the exceptional groups.

**The constant term along $N$.** Given a function $m \in M$, its *constant term along* $N$ is the function

$$m_N(x) = \int_{N(\mathbf{F})\backslash N(\mathbf{A})} m(nx)dn \text{ for each } x \in G(\mathbf{A}),$$

where $dn$ is the Haar measure on the compact group $N(\mathbf{F})\backslash N(\mathbf{A})$ normalized to give $N(\mathbf{F})\backslash N(\mathbf{A})$ total measure one. Clearly $m_N$ is a complex-valued function on $T(\mathbf{F})N(\mathbf{A})\backslash G(\mathbf{A})/\mathbf{K}$ and the map $m \mapsto m_N$ is a homomorphism of $\otimes_v H_v$-modules.

The following lemma shows that every irreducible module of $\otimes_v H_v$ occurring discretely in $M$ also occurs as a submodule of the $\otimes_v H_v$-module $\tilde{M}$ consisting of all complex valued functions on $T(\mathbf{F})N(\mathbf{A})\backslash G(\mathbf{A})/\mathbf{K}$:

**Lemma 2.** *Let $m$ be a complex-valued function on $G(\mathbf{F})\backslash G(\mathbf{A})/\mathbf{K}$. If $m_N \equiv 0$ then $m \equiv 0$.*

*Proof.* Let $W = N_{G(\mathbf{F}_q)}T/T(\mathbf{F}_q)$. Fix a section (which need not be a homomorphism of groups) $\phi : W \to N_{G(\mathbf{F}_q)}T$. It induces a bijection

$$W \to B(\mathbf{F}_q)\backslash G(\mathbf{F}_q)/B(\mathbf{F}_q).$$

Composing $\phi$ with the natural inclusion $G(\mathbf{F}_q) \to G(O_v)$ gives a function $\phi_v : W \to G(O_v)$. Let $X_*(T)$ denote the lattice $\mathrm{Hom}(\mathbf{G}_m, T)$ of rational cocharacters of $T$. For $\mu \in X_*(T)$ and $a$ a unit in a ring $A$, let $a^\mu$ denote the element $\mu(a) \in T(A)$. The Weyl group $W$ has a natural action on $X_*(T)$. The semidirect product $\tilde{W} = X_*(T) \rtimes W$ is known as the *extended affine Weyl group* of $G$. Recall the following theorem [11, Theorem 3.1.1] (see also [13])

**Theorem** (Birkhoff Decomposition)**.** *The map $\tilde{\phi}_\infty : \tilde{W} \to G(\mathbf{A})$ defined by*

$$\mu \rtimes w \mapsto (t^{-1})^\mu \phi_\infty(w) \in G(F_\infty) \subset G(\mathbf{A})$$

*induces a bijection of sets*

$$\tilde{W} \tilde{\to} G(\mathbf{F})\backslash G(\mathbf{A})/\mathbf{K}.$$

By abuse of notation we will write $w$ instead of $\tilde{\phi}_\infty(w)$ for each $w \in \tilde{W}$. Let $t_w = 1_{G(F)\phi_\infty(w)\mathbf{K}}$. These functions form a basis for the space of compactly supported functions in $M$. For $w, w' \in \tilde{W}$, we have

$$\begin{aligned}(t_{w'})_N(w) &= \int_{N(\mathbf{F})\backslash N(\mathbf{A})} t_{w'}(nw)dn \\ &= \text{measure } (N(\mathbf{F})\backslash(N(\mathbf{A}) \cap G(\mathbf{F})w'\mathbf{K}w^{-1}))\end{aligned}$$

The group $\tilde{W}$ is endowed with a partial ordering "$\leq$" known as the *Bruhat ordering*. Note that $N(\mathbf{A}) \subset G(\mathbf{F})\mathbf{K}$. Moreover, formula for convolving a basis vector $t_w$ with a generator of the algebra $H_\infty$ in [12, §3.c] implies that

$$G(\mathbf{F})\mathbf{K}w \subset \sqcup_{w' \leq w} G(\mathbf{F})w'\mathbf{K}.$$

Therefore, $N(\mathbf{A}) \cap G(\mathbf{F})w'\mathbf{K}w^{-1}$ is non-empty only when $w' \leq w$. Moreover, for each $w \in \tilde{W}$, $w^{-1}N(\mathbf{A})w \cap \mathbf{K}$ has positive measure in $w^{-1}N(\mathbf{A})w$ so that measure $(N(\mathbf{F})\backslash(N(\mathbf{A}) \cap G(\mathbf{F})w\mathbf{K}w^{-1})) > 0$. It follows from the above remarks that if $m$ is a compactly supported function in $M$ such that $m_N \cong 0$ then $m \cong 0$. Lemma 2 then follows from the fact that the compactly supported functions in $M$ are dense in $M$. □

The following lemma is the key to understanding the relationship between the local constituents at different valuations of the irreducible representations of $\otimes_v H_v$ occurring in $\tilde{M}$:

**Lemma 3.** *For every $m \in \tilde{M}$, $t_0 \in \prod_v T(O_v)$ and $g \in G(\mathbf{A})$,*

$$m(t_0 g) = m(g).$$

*Proof.* Let $g \in G(\mathbf{A})$. By the Iwasawa decomposition $g$ can be written as $tnk_0$, where $t \in T(\mathbf{A})$, $n \in N(\mathbf{A})$ and $k_0 \in \prod_v G(O_v)$. By the Bruhat decomposition, $k_0$ can be written as $k_0 = n_0\phi_\infty(w_\infty)\phi_0(w_0)k$ for some $n_0 \in \prod_v N(O_v)$, $w_\infty, w_0 \in W$ and $k \in \mathbf{K}$. Hence every double coset in $T(\mathbf{F})N(\mathbf{A})\backslash G(\mathbf{A})/\mathbf{K}$ has a representative of the form $t\phi_\infty(w_\infty)\phi_0(w_0)$. Therefore it suffices to prove Lemma 3 for $g$ of the form $t\phi_\infty(w_\infty)\phi_0(w_0)$.

For $t_0 \in \prod_v T(O_v)$ and $m \in \tilde{M}$,

$$m(t_0 t\phi_\infty(w_\infty)\phi_0(w_0)) = m(t\phi_\infty(w_\infty)\phi_0(w_0))$$

since $\phi_\infty(w_\infty)\phi_0(w_0)$ normalizes $\prod_v T(O_v)$, which in turn is contained in $\mathbf{K}$. □

**Hecke algebras.** Recall some basic properties of the Hecke algebras $H_v$. Let $Z_v$ denote the center of $H_v$ for each $v$ and $R_v$ denote the algebra $\mathbf{C}[T(F_v)/T(O_v)]$. Let $M_v$ denote the space of complex-valued functions on $T(O_v)N(F_v)\backslash G(F_v)/K_v$ for each $v$. $M_v$ is a right $H_v$-module, where the action of $\omega_v \in H_v$ on $m \in M_v$ is given by (1). $M_v$ is also a left $R_v$-module under the action

$$r_v \cdot m_v(x) = \int_{T(F_v)} m_v(t^{-1}x)\delta_B^{\frac{1}{2}}(t)r_v(t)dt \text{ for } r_v \in R_v \text{ and } m_v \in M_v. \tag{4}$$

**Proposition 5.** *For each $v$ there exists an isomorphism*

$$B_v : R_v^W \to Z_v$$

*such that $m_v \cdot B_v(r_v) = r_v \cdot m_v$ for each $m_v \in M_v$ and $r_v \in R_v^W$.*

*Proof.* Suppose that $v$ is not $\infty$ or 0. Let $dg$ denote the Haar measure on $G(F_v)$ which gives $K_v$ volume one. Then for any $m_v \in M_v$, $\omega_v = h(g)dg \in H_v$ and $x \in T(F_v)$,

$$\begin{array}{rcll} m_v \cdot \omega_v(x) & = & \int_{G(F_v)} m_v(xg^{-1})h(g)dg & \\ & = & \int_{B(F_v)} \int_{K_v} m_v(xb^{-1}k^{-1})h(kb)d_r b dk & [1, (5)] \\ & = & \int_{B(F_v)} m_v(xb^{-1})h(b)d_r b & \\ & = & \int_{T(F_v)} \int_{N(F_v)} m_v(xt^{-1}n^{-1})h(nt)dndt & [1, (2)] \\ & = & \int_{T(F_v)} m_v(t^{-1}x) \int_{N(F_v)} h(nt)dndt & \\ & = & \int_{T(F_v)} m_v(t^{-1}x)\delta_B^{\frac{1}{2}}(t) \left(\delta_B^{-\frac{1}{2}}(t) \int_{N(F_v)} h(nt)dn\right) dt & \end{array}$$

Here $d_r b$ is the right Haar measure on $B$ normalized so that $B(O_v)$ has total measure one, and $dk$ (resp. $dn$, $dt$) is a Haar measure on $K_v$ (resp. $N(F_v)$, $T(F_v)$) giving $K_v$ (resp. $N(O_v)$, $T(O_v)$) measure one. The map $H_v \to R_v$ defined by

$$\omega_v \mapsto \left( t \mapsto \delta_B^{-\frac{1}{2}}(t) \int_{N(F_v)} h(nt)dn \right)$$

is well-known to be an isomorphism onto $R_v^W$ (it is known as the *Satake isomorphism* [1, (19)]). Thus $R_v$ is commutative, i.e., $Z_v = R_v$. Moreover, by the Iwasawa decomposition, $m \in M_v$ is completely determined by its restriction to $T(F_v)$. Therefore, taking $B_v$ to be the inverse of the Satake isomorphism completes the proof of Proposition 5 for $v$ different from $\infty$ and 0.

If $v$ is 0 or $\infty$, then it may be shown that for a dominant $\mu \in X_*(T)$ (we say that $\mu \in X_*(T)$ is *dominant* if $\pi_\infty^\mu(N(F_\infty) \cap K_\infty)\pi_\infty^{-\mu} \subset N(F_\infty) \cap K_\infty$)

$$1_{\pi_v^\mu} \cdot 1_{T(O_v)N(F_v)K_v} = \delta_B^{\frac{1}{2}}(\pi_v^\mu) 1_{T(O_v)N(F_v)K_v} \cdot \Theta_\mu,$$

where $\Theta_\mu(g) = 1_{K_v \pi_v^\mu K_v}(g)dg$ and $dg$ is the Haar measure on $G(F_v)$ which gives $K_v$ total measure one. It follows that we may take $B_v$ to be the isomorphism of the center of $H_v$ onto $R_v^W$ attributed to Bernstein [7, Theorem 8.1]. $\square$

**Conditions on central characters.** We now show how Lemma 3 can be used to show that the eigencharacter for $Z_\infty$ of an eigenvector in $\tilde{M}$ determines its eigencharacters for $Z_v$ for all $v$. The map $X_*(T) \to T(F_v)/T(O_v)$ given by $\mu \mapsto \mu(\pi_v)$, where $\pi_v$ is a uniformizing element for $F_v$, is independent of the choice of $\pi_v$ and determines an isomorphism $f_v : R \to R_v$, where $R := \mathbf{C}[X_*(T)]$.

**Lemma 6.** *Suppose that $m \in \tilde{M}$ is an eigenvector for the action of $\otimes_v Z_v$ with eigencharacter $\psi_v : Z_v \to \mathbf{C}$ for each $v$. Then for any valuation $v$,*

$$\psi_v(B_v \circ f_v(r)) = \psi_\infty(B_\infty \circ f_\infty(r^{[\deg(v)]})) \text{ for each } r \in R^W$$

*where, for any integer $d$, $r^{[d]}$ denotes that element of $R = \mathbf{C}[X_*(T)]$ for which $r^{[d]}(\mu) = r(d\mu)$.*

*Proof.* For each $\mu \in X_*(T)$, the elements $\pi_\infty^{\deg(v)\mu}$ and $\pi_v^\mu$ are congruent in $T(\mathbf{A})$ modulo $T(\mathbf{F}) \prod_v (T(O_v))$. It follows from Lemma 3 that for any $m \in \tilde{M}$

$$m(\pi_v^{-\mu}x) = m(\pi_\infty^{-\deg(v)\mu}x).$$

Therefore, for any $r \in R^W$, we have

$$f_v(r) \cdot m = f_\infty(r^{[\deg(v)]}) \cdot m.$$

Thus

$$\begin{aligned}\psi_v(B_v \circ f_v(r))m &= m \cdot (B_v \circ f_v(r)) \\ &= f_v(r) \cdot m \\ &= f_\infty(r^{[\deg(v)]}) \cdot m \\ &= \psi_\infty(B_\infty \circ f_\infty(r^{[\deg(v)]})).\end{aligned}$$

□

Let $\hat{T}$ denote the group of characters $X_*(T) \to \mathbf{C}^\times$. Then $\hat{T}$ is a complex torus which may be viewed as a maximal torus in the *Langlands dual group* of $G$. It is canonically isomorphic to spec$R$. Therefore, $\hat{T}/W$ is canonically isomorphic to spec$(R^W)$. For $s \in \hat{T}/W$, let $\chi_v^s$ be the character of $Z_v$ defined by

$$\chi_v^s(B_v \circ f_v(r)) = s(r) \text{ for each } r \in R^W.$$

Lemma 6 implies

**Proposition 7.** *Suppose $m \in M$ is an eigenvector for the action of $Z_\infty$ with eigencharacter $\chi_\infty^s$. Then for each valuation $v$, it is an eigenvector for the action of $Z_v$ with eigencharacter $\chi_v^{s^{\deg(v)}}$.*

**Local constituents at $\infty$ and 0.** let $F$ denote the field $\mathbf{F}_q((\pi))$ of Laurent series in a variable $\pi$. Let $O$ denote the ring $\mathbf{F}_q[[\pi]]$ of integers in $F$. Let $I$ be the Iwahori subgroup of $G(F)$ which is the pre-image of $B(\mathbf{F}_q)$ under the map $G(O) \to G(\mathbf{F}_q)$ and let $H$ denote the corresponding Iwahori-Hecke algebra. The assignment $\pi \mapsto t$ (resp. $\pi \mapsto t^{-1}$) defines an isomorphism $F \to F_0$ (resp. $F \to F_\infty$) and hence an isomorphism $\theta_0 : H \to H_0$ (resp. $\theta_\infty : H \to H_\infty$). Let

$$N = L^2(\Delta G(F) \backslash (G(F)^2)/I^2).$$

Then $N$ is an $H \otimes H$ module whose discrete part is known by elementary and standard considerations (see e.g., [12, Proposition 6.1]):

$$N_{\text{discrete}} = \bigoplus_{(\pi,V)\in\hat{H}} (\pi \otimes \tilde{\pi}, V \otimes \tilde{V}).$$

Here $\hat{H}$ denotes the set of isomorphism-classes of irreducible square-integrable representations of $H$. Moreover, by [12, Theorem 1.2], there is an isometry of Hilbert spaces

$$I : N \to M$$

such that

$$I(n \cdot (h \otimes h')) = I(n) \cdot (\theta_\infty \circ \iota(h) \otimes \theta_0 \circ \iota \circ \overline{\kappa}(h')) \text{ for } h, h' \in H,\ n \in N.$$

where $\iota$ is the Iwahori-Matsumoto involution (introduced at the very end of [5]; or see [14, §5.3]) and $\overline{\kappa}$ is an involution (defined in [12, §5.a]), which when the derived group of $G$ is adjoint, takes a representation to its contragredient. It follows that as an $H_\infty \otimes H_0$-module

$$M_{\text{discrete}} = \bigoplus_{(\pi,V)\in\hat{H}} (\pi \circ \iota \circ \theta_\infty \otimes \tilde{\pi} \circ \overline{\kappa} \circ \iota \circ \theta_0, V \otimes \tilde{V}). \tag{8}$$

**The Iwahori-Matsumoto involution.** The following theorem (which is interesting in its own right) is the final ingredient in the proof of our main theorem (Theorem 10):

**Theorem 9.** *Let $(\pi, V) \in \hat{H}$. The central character of $(\pi, V)$ equals the central character of $(\pi \circ \iota, V)$.*

*Proof.* As in the paragraph preceding Proposition 7, the characters of the center of $H$ are parameterized by points in $\hat{T}/W$. Let $s \in \hat{T}/W$ be the central character of $H$ on $(\pi, V)$. Let $(r(\pi), U)$ denote the representation of $R$ that is obtained by taking the $T(O)$-invariants of the normalized Jacquet module (see [14, §2.3] for the correct normalization) of the irreducible representation of $G(\mathbf{F})$ whose $I$ invariants form the irreducible representation $(\pi, V)$ of $H$. By [3, Proposition 2.4] the canonical projection $V \to U$ is an isomorphism. Moreover, it follows easily from the definition of $B_v$, [3, Proposition 2.5] and [2, Lemma 1.5.1] that the restrictions to $R^W$ of the weights of $r(\pi)$ is also $s$. By [14, Proposition 13], weights of $r(\pi)$ and $r(\pi \circ \iota)$ on $U$ differ by conjugation by the longest element in $W$. Therefore, their restrictions to $R^W$ are the same. This means that the central character of $(\pi, V)$ equals that of $(\pi \circ \iota, V)$. ☐

It is erroneously stated in [12] that the Iwahori-Matsumoto involution takes $s$ to $s^{-1}$. In view of Theorem 9, $s^{-1}$ should be replaced by $s$ in the statements of Theorems 1.3 and 1.5 in [12].

**The discrete spectrum.** For $(\pi, V) \in \hat{H}$, let $s(\pi) \in \hat{T}/W$ correspond to the central character of $\pi$ on $V$. Let $\chi_v^s$ be as in Proposition 7.

**Theorem 10** (Main Theorem)**.** *As an $\otimes_v H_v$-module,*

$$M_{\text{discrete}} = \bigoplus_{(\pi,V)\in\hat{H}} (\pi \circ \iota \circ \theta_\infty \otimes \tilde{\pi} \circ \overline{\kappa} \circ \iota \circ \theta_0 \otimes (\otimes_{v\neq 0,\infty} \chi_v^{(s(\pi)^{\deg(v)})}), V \otimes \tilde{V}). \tag{11}$$

*Proof.* Theorem 10 is a direct consequence of Proposition 7, (8) and Theorem 9. ☐

**Acknowledgments.** We thank Robert Kottwitz for some extremely useful discussions related to this article. We thank the *Centre de Recherches Mathématiques* in Montréal and the *Max-Planck-Institut für Mathematik* in Bonn for their support during the preparation of this article. We are grateful to *The University of Chicago* for its hospitality during the final stages of the preparation of this article.

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