HOMEWORK XIV

FUNCTIONAL ANALYSIS

(1) If X is compact and $K \in C(X \times X)$, let $K_x(y) = K(x, y)$. Show that the map $X \to C(X)$ given by $x \mapsto K_x$ is continuous (here $C(X)$ is topologised by the supremum norm).

(2) Let X be as above, with a probability measure μ. Show that for any $f \in L^2(X, \mu)$, $T_K f \in C(X)$. Hint: use the previous problem along with the fact that the inclusion $C(X) \in L^2(X)$ is continuous and that $T_K f(x) = (f, \bar{K}_x)$.

(3) Let G be a compact group, $\pi : G \to GL(V)$ be a representation of G on a finite dimensional real (resp. complex) vector space V. Let $(\cdot, \cdot)$ be a Euclidean (resp. Hermitian) inner product on V. Show that

$$(x, y)_G = \int_G (\pi(g)x, \pi(g)y) d\mu(x),$$

where μ is the invariant probability measure on G defines a Euclidean (resp. Hermitian) inner product on V (the main thing to check is positive-definiteness).

Date: Solutions need not be submitted.