FINAL EXAM

FUNCTIONAL ANALYSIS

- (1) Show that two finite dimensional locally convex topological real vector spaces of the same dimension are homeomorphic.
- (2) Suppose that X and Y are normed linear spaces such that X is infinite dimensional and $Y \neq \{0\}$. Show that there exists a discontinuous linear operator $X \rightarrow Y$.
- (3) For each $f \in \mathcal{S}(\mathbb{R}^n)$, show that the map $x \mapsto T_x f$, where $T_x f(y) = f(y x)$ is a continuous function $\mathbb{R}^n \to \mathcal{S}(\mathbb{R}^n)$.
- (4) If $T \in S(\mathbf{R}^n)'$ is homogeneous of degree m, show that \hat{T} is homogeneous of degree -n m.
- (5) Show that the unit sphere in $L^{1}[0, 1]$ has no extreme points.

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