COMPLEX ANALYSIS

HOMEWORK 7

(1) Let $SL_2(\mathbf{C})$ denote the group of 2×2 complex matrices with determinant one. Given $A = \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in SL_2(\mathbf{C})$, and $z \in \hat{\mathbf{C}}$, let $A \cdot z$ denote the element $\frac{az+b}{cz+d} \in \hat{\mathbf{C}}$ for $z \neq \infty$, and $A \cdot z = a/c$ for $z = \infty$. Asssume $A \neq I$. Show that:
   (a) $A : \hat{\mathbf{C}} \to \hat{\mathbf{C}}$ has two fixed points if and only if the matrix A is diagonalizable.
   (b) Assume that A is diagonalizable. Show that, for every $z \in \hat{\mathbf{C}}$, the limits $\lim_{n\to\infty} A^n \cdot z$ and $\lim_{n\to-\infty} A^n \cdot z$ exist (and are the two fixed points of A) if ond only if A has an eigenvalue λ such that $|\lambda| \neq 1$. [Hint: one possible approach is to reduce to the case where A is actually a diagonal matrix, in which case its fixed points are 0 and ∞.]

Date: Due on 25th September 2018.