COMPLEX ANALYSIS

HOMEWORK 11

(1) Compute $\zeta(2)$, $\zeta(4)$ and $\zeta(6)$.
(2) Use the functional equation for the Riemann zeta function to show that
$$\zeta(-n) = (-1)^n \frac{B_{n+1}}{n+1}, \text{ for } n = 1, 2, 3, \ldots$$
(3) Let G be a finite group acting on a set X. Extend this action of G to the set X^n by $g \cdot (x_1, \ldots, x_n) = (g \cdot x_1, \ldots, g \cdot x_n)$ (this is called the *diagonal* action). Let a_n denote the number of orbits for the action of g on X^n. In particular, note that $a_0 = 0$. Show that $\sum_{n=0}^{\infty} a_n z^n$ represents a rational function. Express its radius of convergence in terms of the action of G on X. Determine the exponential growth rate of a_n.
(4) Take the special case of the previous exercise, where G is any finite group, and X is also G, the action being by conjugation. Determine the expnential growth rate of a_n in this case.
(5) Suppose $y(z) = \sum_{n=0}^{\infty} a_n z^n$ satisfies the functional equation:
   (a) $y = z(1 + y + y^2)$,
   (b) $y = ze^y$.

   In each case determine the exponential growth rate of a_n.

Date: Due on 5th November 2018.