COMPLEX ANALYSIS

HOMEWORK 10

(1) In *information theory*,[1] a *discrete channel* is a system whereby a word in an alphabet $\{S_1, \ldots, S_n\}$ is transmitted from one point to another. Assume that the transmission of S_i takes t_i seconds. For a non-negative integer t, let $N(t)$ denote the total number of possible words that can be transmitted in t seconds. Show that

$$N(t) = N(t - t_1) + N(t - t_2) + \cdots + N(t - t_n).$$

Use the above recurrence relation to compute the (rational) generating function $f(z) = \sum_{n=0}^{\infty} N(t) z^t$. Let X_0 be the largest real solution to the equation:

$$X^{-t_1} + X^{-t_2} + \cdots + X^{-t_n} = 1.$$

The *capacity* of the channel is defined as:

$$C = \lim_{T \to \infty} \frac{\log N(T)}{T}.$$

Show that $C = \log X_0$.

(2) Explicitly compute a_n in terms of n when

$$\sum_{n=0}^{\infty} a_n z^n = \frac{1}{1 - z^2} \frac{1}{1 - z^3}.$$

(3) Explicitly compute $p(n, 3)$ as a function of n (for example, $p(n, 2) = n/2 + 1$ if $n \equiv 0 \mod 2$, and $(n + 1)/2$ if $n \equiv 1 \mod 2$).

(4) India has coins in denominations 1, 2, 5, and 10. Estimate (asymptotically) the number of distinct ways of paying a sum of n rupees, entirely in these coins, for n very large.

Date: Due on 24th October 2018.

[1]Claude Shannon, A Mathematical theory of Information, *The Bell System Technical Journal*, vol. 27, 1948.