HOMEWORK IX

ANALYSIS I

(1) Show that if f is a periodic function on $\mathbf{R}$ with period 2π and Riemann integrable on $[-\pi, \pi]$ and g is defined by

$$g(x) = c + f(x+s)$$

where c is complex and s is real, then $c_n(g) = c_n(f)e^{nis}$ for all $n \neq 0$ and $c_0(g) = c_0(f) + c$ (here $c_n(f)$ and $c_n(g)$ denote the nth Fourier coefficient of f and g respectively).

(2) Calculate the Fourier series of the periodic functions whose values on $[-\pi, \pi]$ are given by
   (a) $f(x) = -1$ for $-\pi \leq x \leq 0$ and $f(x) = 1$ for $0 < x < \pi$.
   (b) $g(x) = x + \pi$ for $-\pi \leq x \leq 0$ and $g(x) = x - \pi$ for $0 < x < \pi$.
   (c) $h(x) = (1 - re^{ix})^{-1}$, where $0 < r < 1$.

(3) Define

$$f(x) = \left(\int_0^x e^{-t^2} dt \right)^2, \quad g(x) = \int_0^1 \frac{e^{-x^2(t^2+1)}}{t^2+1} dt.$$

   (a) Show that $g'(x) + f'(x) = 0$ for all x and deduce that $g(x) + f(x) = \pi/4$.
   (b) Use (a) to prove that

$$\int_0^\infty e^{-t^2} dt = \frac{1}{2}\sqrt{\pi}.$$

   [Use the definition $\int_0^\infty = \lim_{x\to\infty} \int_0^x$.]

Date: due on 7 November 2005.