

## HOMEWORK XI

### ANALYSIS I

- (1) Prove the two variable Weierstrass approximation theorem: Every continuous function  $f : [a, b] \times [c, d] \rightarrow \mathbf{R}$  can be uniformly approximated by polynomials in two variables.
- (2) Let  $X$  and  $Y$  be two compact Hausdorff spaces. Suppose that  $f : X \times Y \rightarrow \mathbf{R}$  is a continuous function. Show that  $f$  can be uniformly approximated by functions of the form  $\sum_i f_i g_i$  where  $f_i : X \rightarrow \mathbf{R}$  and  $g_i : Y \rightarrow \mathbf{R}$  are continuous functions.
- (3) Let  $f$  be a continuous real-valued function on  $[0, 1]$ . The *moments* of  $f$  are the numbers  $\int_0^1 x^n f(x) dx$ , for  $n = 0, 1, 2, \dots$ . Prove that two continuous functions defined on  $[0, 1]$  are identical if they have the same sequence of moments.