## HOMEWORK I

## ANALYSIS I

(1) If x = a/b is a solution to

$$p_0 x^n + p_1 x^{n-1} + \dots + p_{n-1} x + p_n = 0$$

then  $a|p_n$  and  $b|p_0$ .

(2) Show that if  $p_0 = p_n = 1$  and neither of

$$1 + p_1 + p_2 + \cdots$$
 and  $1 - p_1 + p_2 - \cdots$ 

is zero then the equation in the previous problem can have no rational root.

(3) Find the rational roots (if any) of

 $x^4 - 4x^3 - 8x^2 + 13x + 10 = 0.$ 

- (4) If a and b are rational then  $\sqrt{a} + \sqrt{b}$  can not be rational unless  $\sqrt{a}$  and  $\sqrt{b}$  are rational.
- (5) Show that  $\sqrt{2} + \sqrt{3}$  is algebraic.
- (6) A line AB is divided at C in such a way that  $AB \cdot AC = BC^2$ . Show that the ratio AC/AB is irrational.
- (7) The number  $10^{-1!} + 10^{-2!} + 10^{-3!} + \cdots$  is transcendental.
- (8) Given x > 0 show that there is an irrational number between 0 and x.
- (9) Given x > 0 show that there is a transcendental number between 0 and x.
- (10) Let S be a set of real numbers that is bounded above and let a be its least upper bound. Show that for any positive number  $\epsilon$  there is at least one x in S such that  $a \epsilon < x \leq a$ .

Date: due on 17 August 2004.