$11\frac{1}{2}$

Numerical determination of character table:

Enough to decompose $\mathbb{C}[G]$ into irreducible reps. of $G\times G$ (the projection operators onto these subspaces will determine characters).

We will give an algorithm to find ~~an~~ a non-zero proper invariant subspace of a rep. that is not irreducible.

Principle: Suppose (ρ, V) is a rep. of G, $T \in \mathrm{End}_G V$. Then any eigenspace for T is an invariant subspace of V.

Pf: easy: $T\rho(g)v = \rho(g)Tv = \rho(g)\lambda v = \lambda\rho(g)v$.

By problem (2) (due 13/1/11), one of the $\dim V^2$ matrices E_{ij} will have

$$T = \sum_{g\in G} \rho(g)^{-1} E_{ij}\, \rho(g)$$

non-scalar if V is not simple.

An eigenspace of this matrix will give an invar. subspace of V.