

COMBINATORICS IN REPRESENTATION THEORY

ASSIGNMENT DUE ON 8TH MARCH 2011

- (1) Show that, if

$$a_0 + a_1x + \cdots + a_{n-1}x^{n-1} + x^n = (x - \lambda_1)(x - \lambda_2) \cdots (x - \lambda_n).$$

then every symmetric polynomial in $\lambda_1, \dots, \lambda_n$ can be expressed as a polynomial in a_0, a_1, \dots, a_{n-1} .

- (2) Compute the Schur polynomials of homogeneous degree three.
(3) Verify that the coefficient of the Schur polynomial s_ν in the expansion of power sum symmetric polynomial p_μ in terms of Schur polynomials is the value of the character of V_ν at the conjugacy class whose cycle decomposition is a partition of type μ .
(4) Let D_n denote the determinant of the character table of $\mathbf{Z}/n\mathbf{Z}$. Determine the primes p which divide D_n .